Comparative Analysis of Some 2D Transformation Models for 3rd Order Planimetric Mapping

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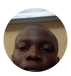
1 AUTHOR:



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COMPARATIVE ANALYSIS OF SOME 2-D TRANSFORMATION MODELS FOR THIRD ORDER ACCURACY PLANIMETRIC MAPPING.

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ABSTRACT:

Considering the ease and speed of planimetric mapping from aerial photographs, transformation of resulting image-space vector of co-ordinates into real-time object space co-ordinates becomes an essential engineering problem for photogrammetric evaluation. A careful evaluation of three (3) different 2-Dimention co-ordinate transformation models is herein presented. The similarity model showed least suitability while the polynomial model proved maximally effective for third-order accuracy planimetric mapping.

Key Words: Third-Order Accuracy, Planimetric Mapping, Transformation Models.

1.0 INTRODUCTION

Since ground methods of planimetric mapping are cumbersome and time consuming, aerial or terrestrial photographs provide a quick alternative to the problem whenever its usage seems cost effective. Once such photographs have been acquired, the photogrammetrist is left with the task of relating the image space to the corresponding object space. This sets the stage for an engineering problem i.e. relating the resulting 2-D image-space to the actual 3-D object space (Olaleye, 2013). Several methods exist for solving this problem but in this paper we shall consider using 2D transformation models.

This process otherwise referred to as image Registration is the process of finding the geometrical transformation that aligns the images in such a way that the points in the two images corresponding to the same physical region of the scene being imaged (Wang and Zhang, 2009).

Image registration is one of the basic image processing operations in remote sensing. Registering two different images acquired during different times or by different sensors; can be used in various applications like change detection, image fusion (A.S.Kumar, 2003).

A transformation model attempts to mathematically relate two surfaces together by describing the relationship between them using empirically defined parameters. This therefore requires that control points be selected all-over the image whose co-ordinate values in both systems would be known. Once the control points have been chosen, the choice of model to be used becomes another decision that the photogrammetrist is required to make.

The effect of transformation on a group of points defining a 2D Polygon or 3D Object varies from simple changes of location and orientation (without) any change in shape or size to uniform scale change (No change in shape) and finally to changes in shape and size of different degrees of non-linearity (Mikhail, 1976). Three transformation models have been selected for comparative analysis in this paper.

2.0 MATHEMATICAL MODELS:

The mathematical illustrations given in the following sections are as presented by Deakin, R. E (2004) with slight modifications by the author.

2.1 HELMERT 2D (SIMILARITY) TRANSFORMATION:

2.1.1 Similarity Transformations Involving Only Rotations

$x = u \cos\theta + v \sin\theta$ Equ. 1(a)

$y = -u \sin\theta + v \cos\theta$ Equ. 1(b)

In Matrix notations, $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$ Equ. 2

Note that rotation matrices are orthogonal.

2.1.2 Transformations involving uniform scaling and Rotation

$x = u(s \cos\theta) + v(s \sin\theta)$ Equ. 3(a)

$y = -u(s \sin\theta) + v\,(s\cos\theta)$ Equ. 3(b)

Equations 3(a) and (b) can be expressed in matrix notation as shown below:

$\begin{bmatrix} x \\ y \end{bmatrix} = s \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$ Equ. 4(a)

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$ Equ. 4(b)

Where $s = \sqrt{a^2 + b^2}$

$\theta = \tan^{-1}(b/a)$.

2.1.3 Transformation involving Scaling, Rotation and Translation (FULL SIMILARITY TRANSFORMATION)

$x = u(s \cos\theta) + v(s \sin\theta) + tx$ Equ. 5(a)

$$y = -u(s \sin \theta) + v\ (s \cos\theta) + ty. \qquad \text{Equ. 5(b)}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = s \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} Tx \\ Ty \end{bmatrix} \qquad \text{Equ. 6(a)}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = sR \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} Tx \\ Ty \end{bmatrix} \qquad \text{Equ. 6(b)}$$

It worthy of note to mention that "translation" has no effect on area and shape.

2.2 AFFINE 6-PARAMETER TRANSFORMATION

This is a six (6) parameter transformation with the following unknown parameters: a, b, c, d, e and f.

It modifies the orthogonal type by using different scale factors in the x and y directions. It correct for shrinkage by means of scale factor then applies the translation to the shift of the origin and also performs rotation through angle θ (plus a small angular correction for non-orthogonality to orient the axes in the u,v photo system). Unlike orthogonal projection, affine projection allows oblique projection to an image plane (C. Stamatopoulos and C. S. Fraser, 2011).

The affine transformations and their associated special cases are sometimes represented by means of *homogeneous coordinates*. While the use of homogeneous coordinates does not produce any extra power or generality for rigid transformations, it does simplify notation, especially when rigid transformations must be combined with rational/projective transformations.

For series captured by high altitude imaging satellites with narrow angular field of view (FOV) of a relatively flat terrain, the mathematical relationship between the coordinates of conjugate points in the reference and input images can best be described by an Affine Transformation (Rami Al-Ruzouq, 2004).

$$x = au + bv + e \qquad \text{Equ. 7(a)}$$

$$y = cu + dv + f. \qquad \text{Equ. 7(b)}$$

These can be represented in Matrix Notation as:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix} \qquad \text{Equ. 8}$$

Where:

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ are scalar Quantities, $\begin{bmatrix} e \\ f \end{bmatrix}$ are Translation Parameters.

$a = Horizontal\ Scaling\ (x)$

$b = Vertical\ Scaling\ (y)$

$c = Rotation\ Angle$

$d = Skew\ Angle$

According to Deakin, R. E (2004), the Equ. 8 could be simplified as shown below:

$$\begin{bmatrix} x \\ y \end{bmatrix} = s \begin{bmatrix} Su\ cos\theta & SvCos\ \theta Sin\ \alpha + SvSin\theta Cos\ \alpha \\ -Susin\theta & -SvSin\ \theta Sin\ \alpha + SvCos\theta Cos\ \alpha \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix} \qquad \text{Equ. 9}$$

Where:

$Su = \sqrt{a^2 + b^2}$

$Sv = \sqrt{b^2 + d^2}$

Where Rotation $\theta = \tan - \frac{c}{a}$

Skew $\alpha = \tan(\theta + \alpha) = \frac{d}{b}$

2.3 2ND ORDER POLYNOMIAL TRANSFORMATION:

$$x = P(u, v) = \sum_{m=0}^{p} \sum_{n=0}^{q} Cnm\ U^m V^n$$

$$y = P(u, v) = \sum_{m=0}^{p} \sum_{n=0}^{q} dnm\ U^m V^n$$

$$x = C_0 + C_1U + C_2V + C_3UV + C_4U^2 + C_5V^2 + C_6U^2V + C_7UV^2 + C_8U^3 + C_9V^3 + C_{10}U^3V \ldots$$

$$y = d_0 + d_1U + d_2V + d_3UV + d_4U^2 + d_5V^2 + d_6U^2V + d_7UV^2 + d_8U^3 + d_9V^3 + d_{10}U^3V \ldots$$

While the first parameters in both cases take care of the translations in the x and y axes respectively, the other parameters model the several other variables required for accurate transformation which include the scale, rotation and skew in all three axes. The more the number of parameters modelled the better the resolution obtained.

However, after a certain polynomial order is reached, the model no longer effectively transforms the image to object space but rather begins to deform the result obtained (This is not investigated in this research).

3.0 SOLVING FOR TRANSFORMATION PARAMETERS:

Each transformation requires a minimum number of control points (2 points for Similarity, 3 for affine, 6 for second order polynomial and 9 for third order polynomials) for estimating its required parameters. If more points are selected, the residuals and the derived Root Mean Square error (RMSE) or Sigma may be used to obtain the best estimates.

Several methods abound for estimating the transformation parameters. George B et al used Single Layer Neural Networks (SL-NN) to compute transformational parameters for an affine transform. Besides, the Singular Value Decomposition and Interval Arithmetic method are also alternative methods used to obtain approximate values of the parameters.

However, traditionally, the task of determination of transformation parameters is solved via least squares. The resulting observation equations are arranged into a design matrix and evaluated as follows:

$$V = AX + L^b \qquad \text{Equ. 3.18}$$

$$X = (A^T PA)^{-1} A^T L^b \qquad \text{Equ. 3.19}$$

Where:

X = Parameters to be estimated

V = Vector of Residuals

A = Design Matrix

L = Vector of observations

4.0 DATA USED:

The Image used for this experiment is an aerial photograph of part of Minna, Niger-State. The Photograph was captured on 20th April 2013, during the recently conducted surveying camping exercise of the Surveying and Geo-Informatics Students, Federal University of Technology Minna as part of their exposure to the practicality of Digital Photogrammetry.

Figure. 1(a): Aerial Photo of part of Minna, Niger State.

Two sets of control points were used within the same image frame. The first being a multi-shaped figure and the second a simple four sided figure. The essence for selecting two (2) different figures is to enhance examination of the strengths, weakness and reliability of each of the examined models on different types of shapes.

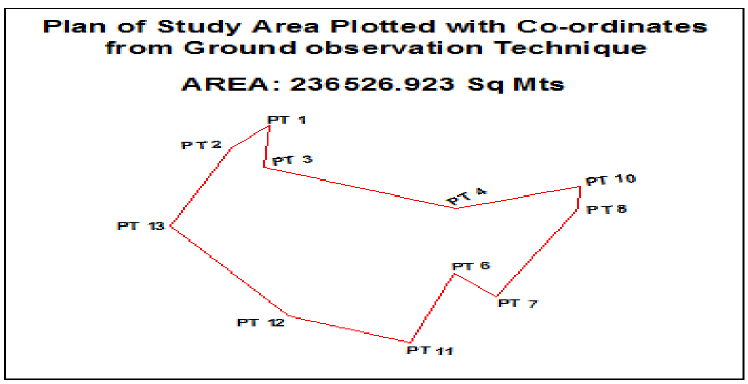

Figure 1(b): Study Area 1

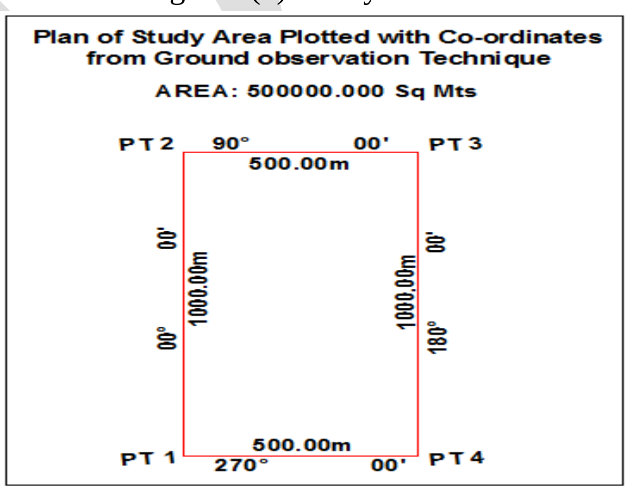

Figure 1(c): Study Area 2

5.0 METHODOLOGY:

All three transformation models earlier discussed were used in this research to determine the most suitable model for third-order accuracy for 2-D planimetric mapping. The obtained image was then read into MATLAB environment (Trial Version) where digital co-ordinate values were assigned for all control points.

The design matrix was formulated alongside the all other relevant vectors and then MATLAB codes written to solve for the transformation parameters and subsequently generate co-ordinate values for subsequent points within the image space.

The list of Ground controls and their corresponding Digital Photo controls are as shown below:

Table 1: Showing list of control points and their corresponding digital photo co-ordinates.

Ground Co-ordinates		**Digital Photo Co-ordinates**	
226230.128	1059899	231	161
226163.936	1059828	207	172
226219.49	1059766	232	177
226562.352	1059635	388	179
226559.683	1059554	402	189
226559.787	1059433	428	210
226634.523	1059360	487	221
226713.313	1059506	478	192
226779.621	1059636	468	173
226780.687	1059706	455	166
226480.357	1059212	455	276
226263.845	1059297	287	269
226056.051	1059581	153	213

Source (Author)

6.0 DISCUSSION OF RESULTS:

6.1 RESULTS FROM FIGURE 1(b) (STUDY AREA 1)

Table 2: Derived Transformation Parameters

DERIVED TRANSFORMATION PARAMETERS (FIRST FIGURE)							
SIMILARITY TRANSFORM		AFFINE TRANSFORM		POLYNOMIAL TRANSFORM		POLYNOMIAL TRANSFORM	
PARAMETER	*VALUE*	*PARAMETER*	*VALUE*	*PARAMETER*	*VALUE*	*PARAMETER*	*VALUE*
Scale	1.2287	Scale_X	2.58065	C1	3.8892	D1	-1.5413
Rotation	0.532	Scale_Y	1.80575	C2	-4.1364	D2	-19.4947
Tx	225906.9576	Skew	-0.29325	C3	-0.0119	D3	0.0026
Ty	1059493.3733	Rotation	1.19731	C4	0.0008	D4	0.0006
		Tx	226123.4514	C5	0.0154	D5	0.0315
		Ty	1060753.1526	Tx (C0)	226008.1315	Ty (D0)	1062445.5368

Source (Authors' Research)

6.1.1 Transformation Parameters:

The table above shows the obtained transformation parameters for each of the three models based on figure 1(b).

6.1.2 Test of Model Results (Re-Computation of Control Points)

The parameters were then used to re-compute the boundary points from their digital photo co-ordinates and the results obtained is as summarised in the table below:

Table 3: Obtained residuals for each control point after re co-ordinating them using each of the Models

ANALYSIS OF RESIDUALS OF COMPUTATION (FIRST FIGURE)									
Ground Co-ordinates		Digital Photo Co-ordinates		Similarity Residual		Affine Residual		Polynomial Residual	
226230.128	1059899	231	161	-21.790	379.337	-57.630	191.446	-8.155	1.970
226163.936	1059828	207	172	-69.428	281.870	-58.820	-2.501	-2.740	3.647
226219.49	1059766	232	177	-43.462	230.326	-42.526	55.291	5.921	1.921
226562.352	1059635	388	179	132.978	194.030	4.880	685.572	3.299	-5.789
226559.683	1059554	402	189	109.251	111.405	-7.327	668.040	0.025	-9.753
226559.787	1059433	428	210	68.733	-15.460	-20.740	663.366	5.648	5.441
226634.523	1059360	487	221	74.141	-64.133	-40.040	872.295	-6.619	8.202
226713.313	1059506	478	192	180.540	107.818	5.864	990.284	-10.019	-0.320
226779.621	1059636	468	173	269.282	251.350	58.489	1080.955	2.149	-4.113
226780.687	1059706	455	166	288.476	320.324	72.363	1090.713	5.895	5.340
226480.357	1059212	455	276	-80.432	-290.439	-37.814	539.002	6.360	-6.426
226263.845	1059297	287	269	-114.698	-302.358	55.480	-193.372	-6.938	5.811
226056.051	1059581	153	213	-145.698	-42.732	7.640	-535.459	5.172	-5.929

Source (Authors' Research)

The charts shown below further describe the discrepancy between the actual ground co-ordinate and the computation-obtained co-ordinates from each of the models:

SIMILARITY TRANSFORM:

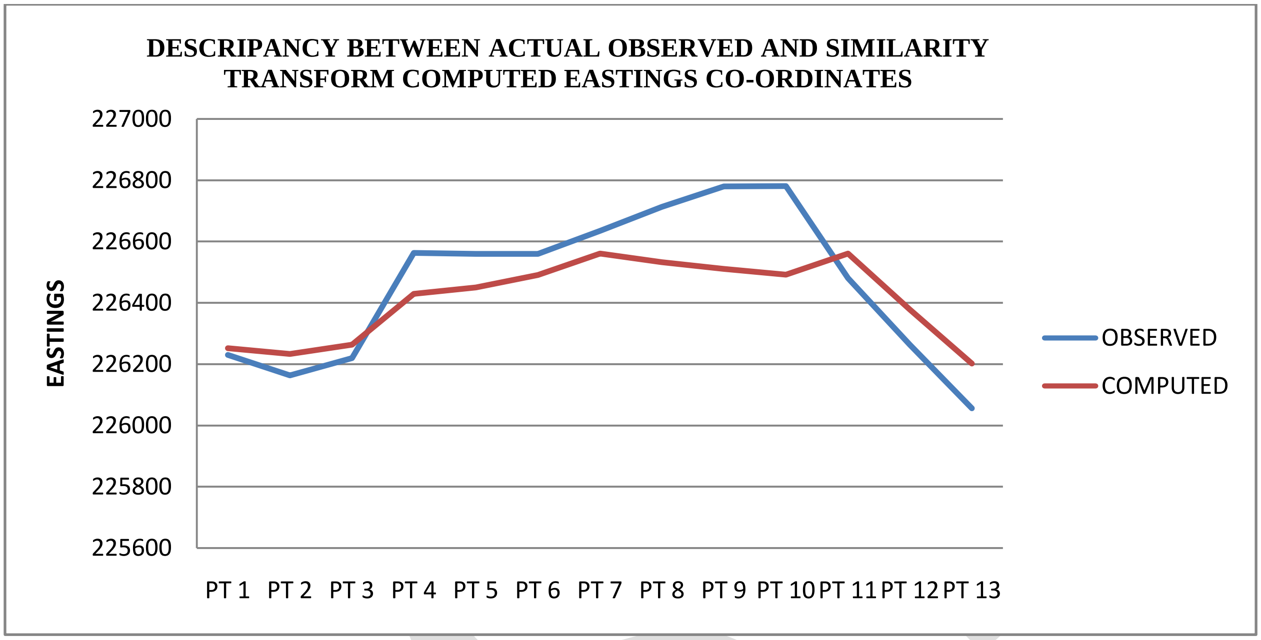

Figure 3(a): Similarity Model – Deviation Along The Eastings Axes.

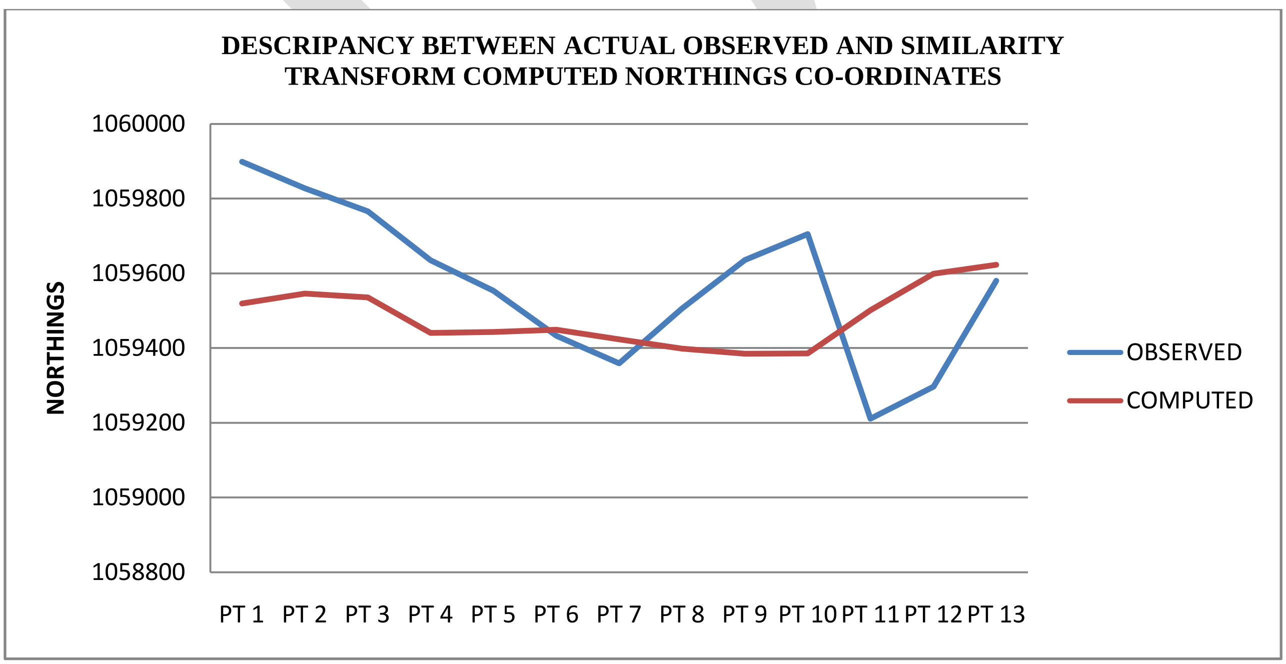

Figure 3(b): Similarity Model – Deviation Along The Northing Axes.

AFFINE TRANSFORM:

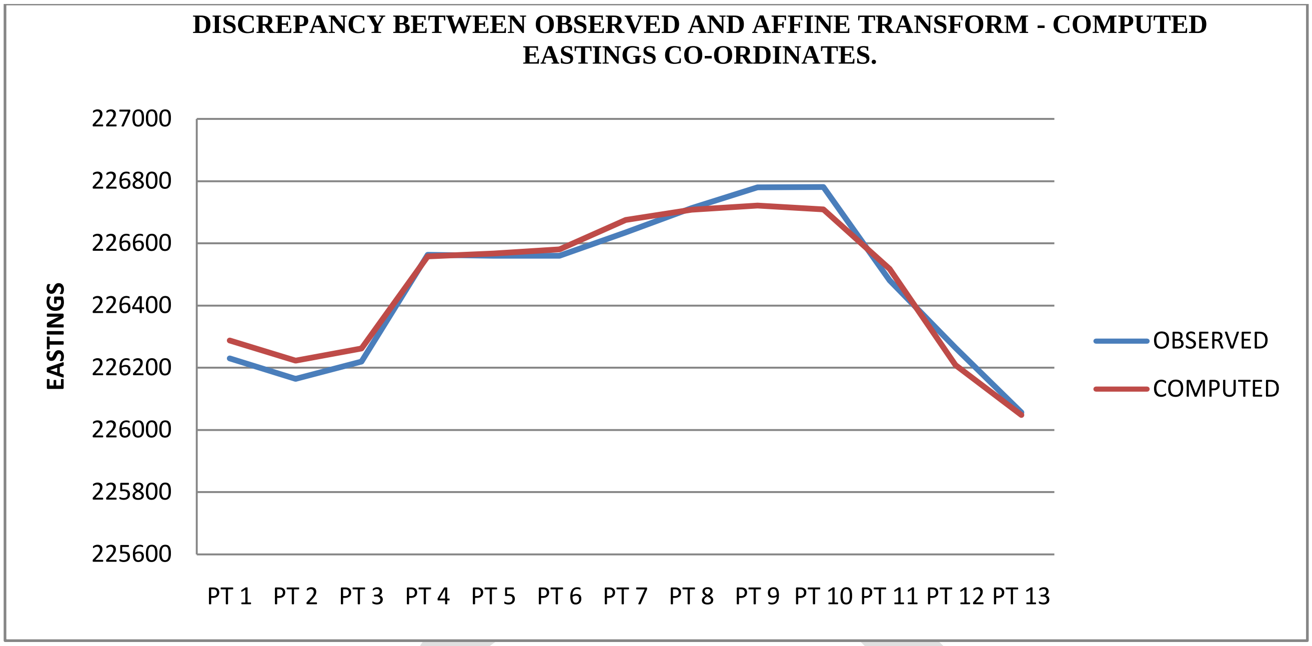

Figure 4(A): Affine Model – Deviation Along The Eastings Axes.

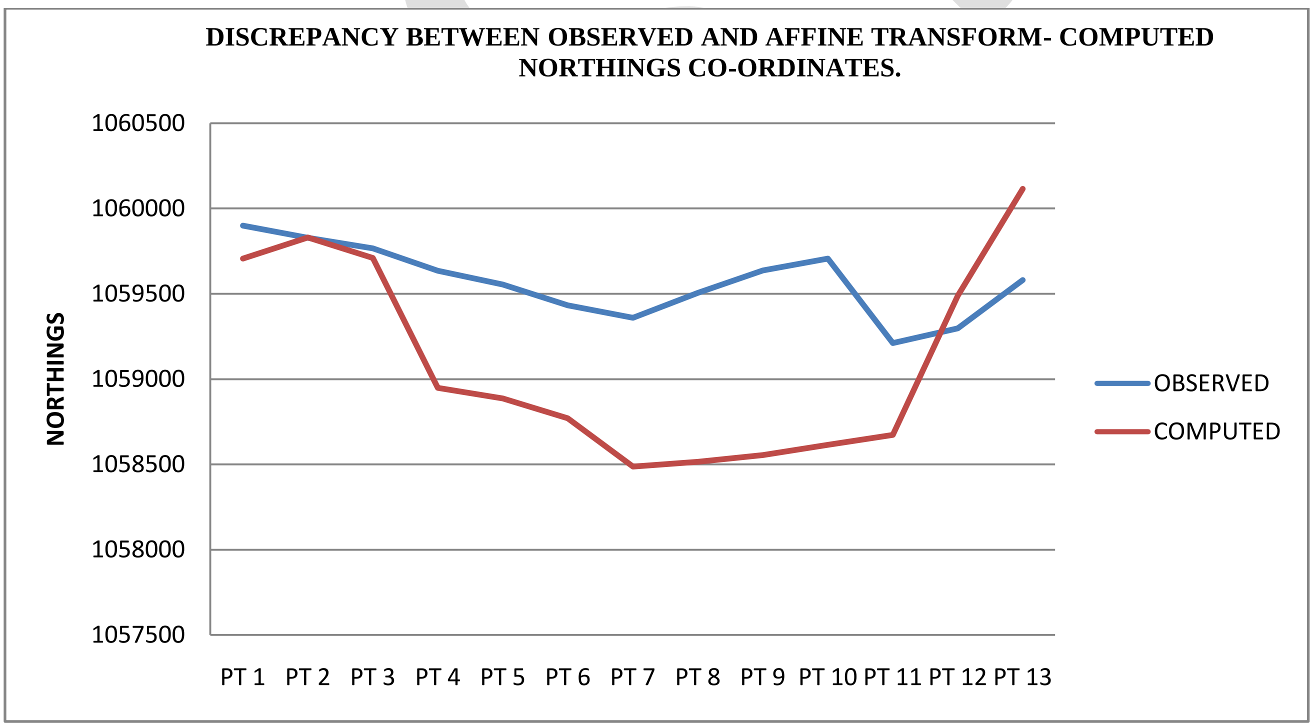

Figure 4(b): Affine Model – Deviation Along The Northings Axes.

POLYNOMIAL TRANSFORM (2ND ORDER):

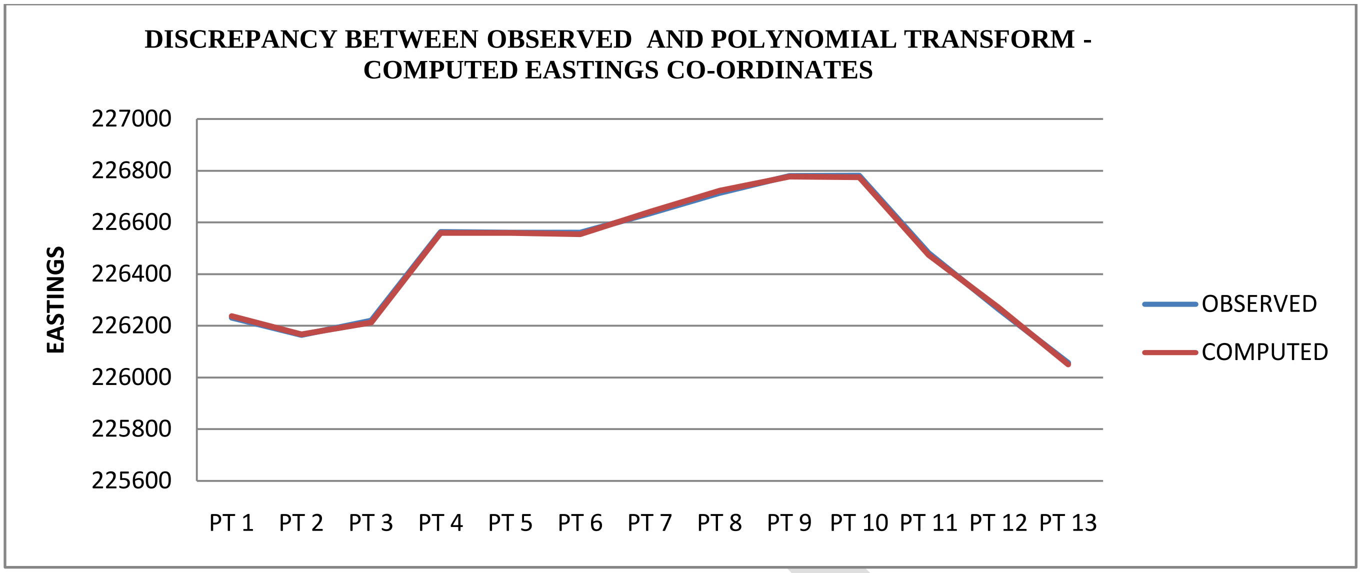

Figure 5(a): Polynomial Model – Deviation Along The Eastings Axes.

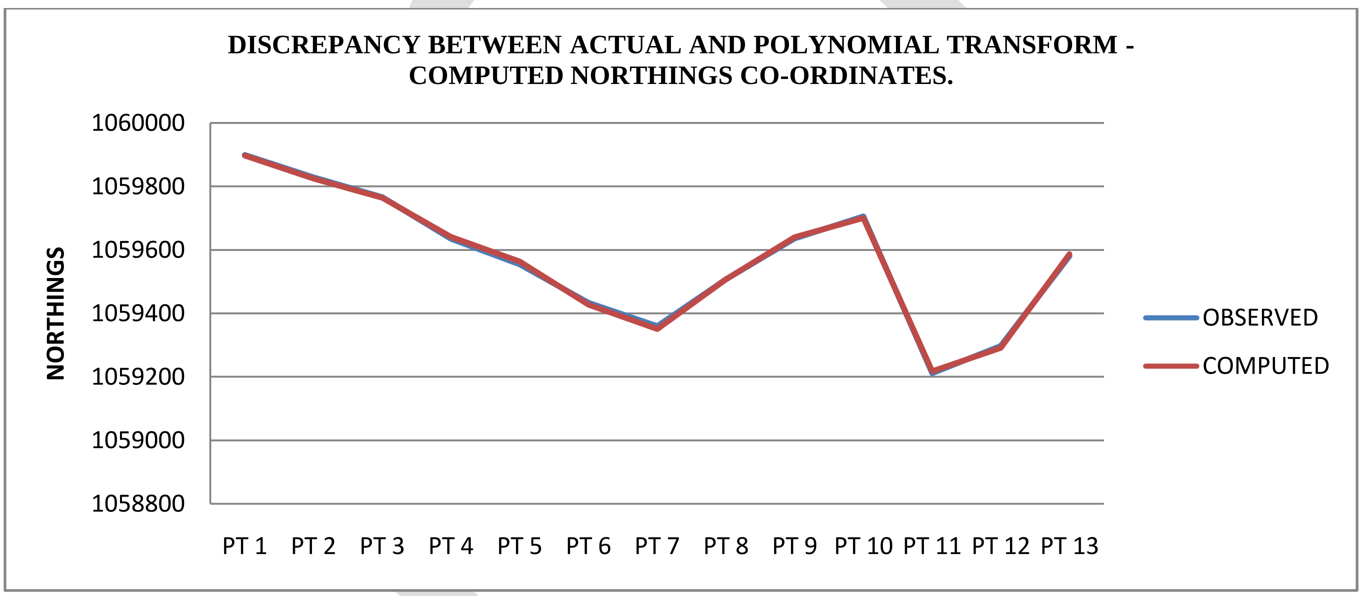

Figure 5(b): Polynomial Model – Deviation Along The Northings Axes.

6.1.3 2-D Comparison of Obtained Results:

The figures below show a plot of the results obtained by plotting the model derived co-ordinates:

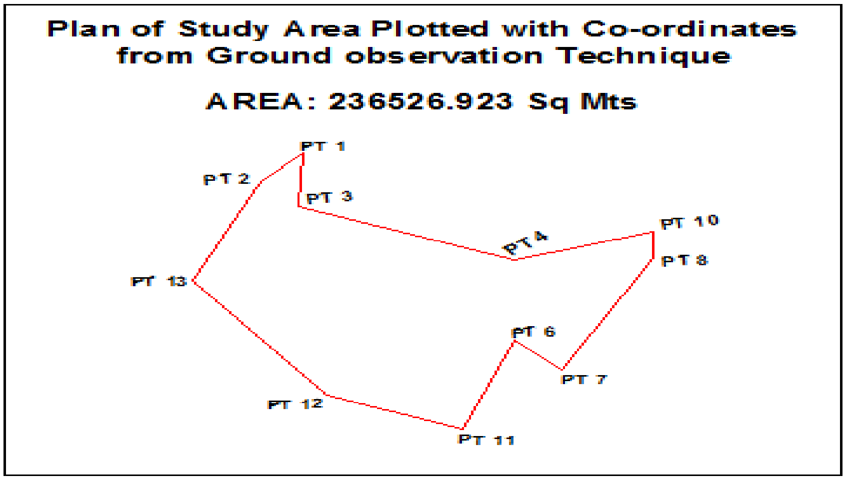

Figure 6(a): Plot of Actual Co-ordinates of Control Points.

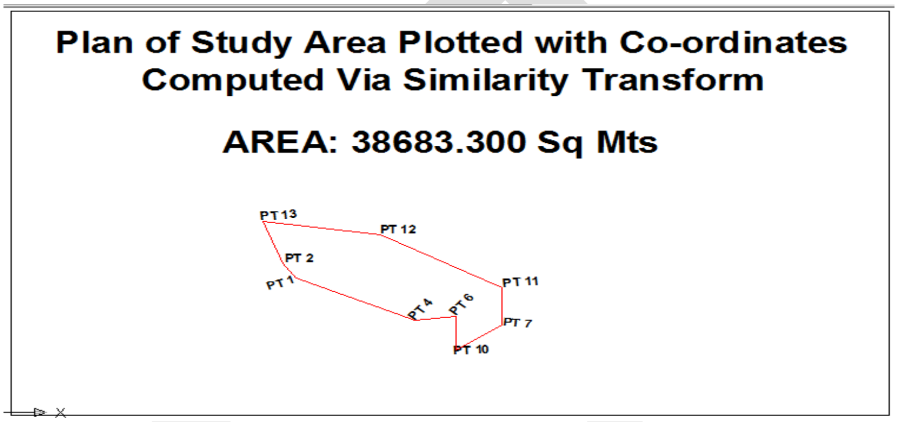

Figure 6(b): Plot of Similarity-Model Computed Co-ordinates of Control Points.

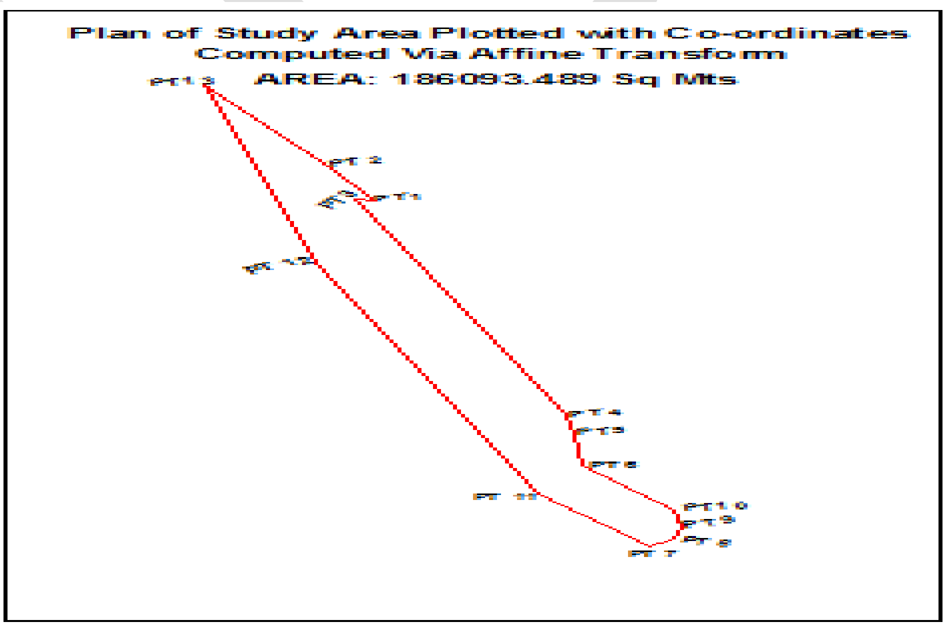

Figure 6(c): Plot of Affine-Model Computed Co-ordinates of Control Points.

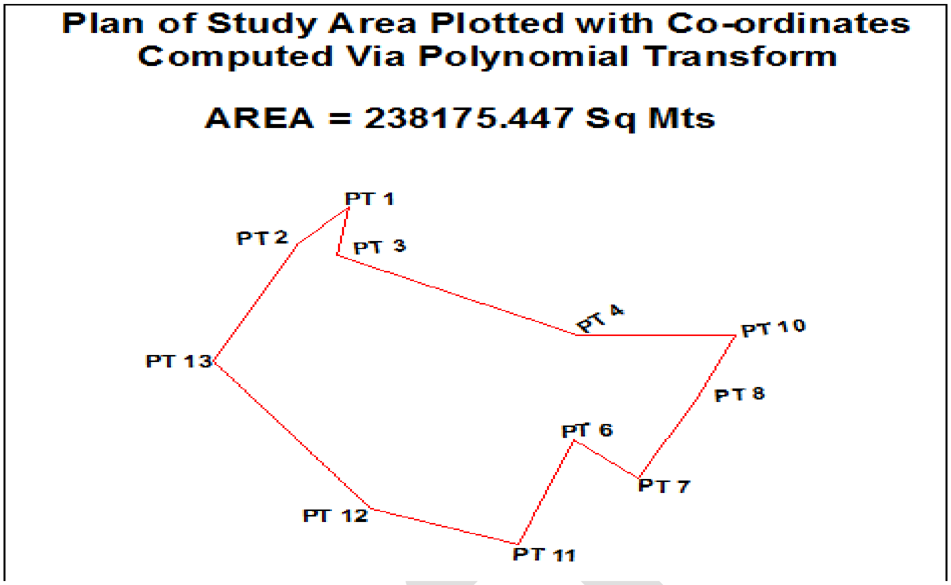

Figure 6(d): Plot of Polynomial-Model Computed Co-ordinates of Control Points.

6.2 RESULTS FROM FIGURE 1(c) (STUDY AREA 2)

6.2.1 Transformation Parameters:

The derived transformation parameters for the second figure are as shown below:

Table 4: Derived transformation parameters

DERIVED TRANSFORMATION PARAMETERS (SECOND FIGURE)							
HELMERT TRANSFORM		AFFINE TRANSFORM		2D POLYNOMIAL TRANSFORM		2D POLYNOMIAL TRANSFORM	
PARA.	***VALUE***	***PARA.***	***VALUE***	***PARA.***	***VALUE***	***PARA.***	***VALUE***
Scale	10.6633	Scale_X	6.7111	C1	6.486486486	D1	-0.000000000466
Rotation	-1.2457	Scale_Y	28.5716	C2	-0.321750322	D2	28.571
Tx	231808.9533	Skew	-0.6040	C3	0.002574003	D3	0.000000000004
Ty	1058520.7169	Rotation	0	Tx (C0)	232177.3662	Ty (D0)	1056960.207
		Tx	232157.666				
		Ty	1056960.207				

Source (Authors' Research)

6.2.2 Test of Model Results (Re-Computation of Control Points)

The parameters were then used to re-compute the boundary points from their digital photo co-ordinates and the results obtained is as summarised in the table below:

Table 5: Obtained residuals for each control point (Second Figure) after re co-ordinating them using each of the Models

ANALYSIS OF RESIDUALS OF COMPUTATION (SECOND FIGURE)									
Ground Co-ordinates		Digital Photo Co-ordinates		Similarity Residuals		Affine Residual		Polynomial Residual	
226230.128	1059899.166	50	70	151.238	-310.611	1.667	0.000	0.000	0.000
226163.936	1059828.309	51	105	143.158	316.109	-1.689	0.000	0.000	0.000
226219.49	1059766.473	125	105	-145.906	310.648	1.689	0.000	0.000	0.000
226562.352	1059635.038	125	70	-148.489	-316.146	-1.667	0.000	0.000	0.000

Source (Authors' Research)

Find below further charts illustrating the deviations between the actual object space values of the control points and their corresponding model-Computed co-ordinates.

SIMILARITY TRANSFORM:

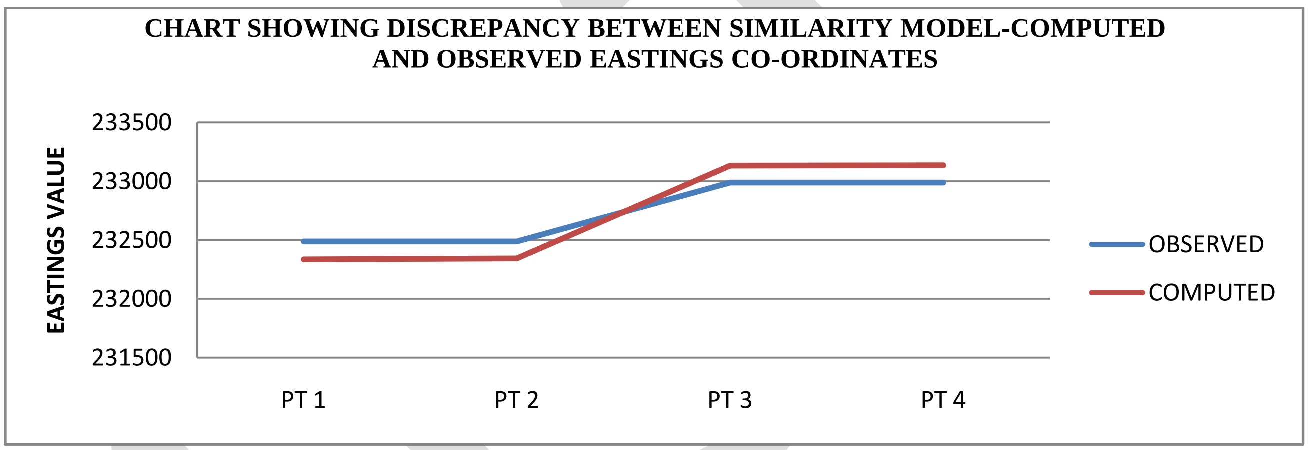

Figure 7(a): Similarity Model – Deviation Along The Eastings Axes.

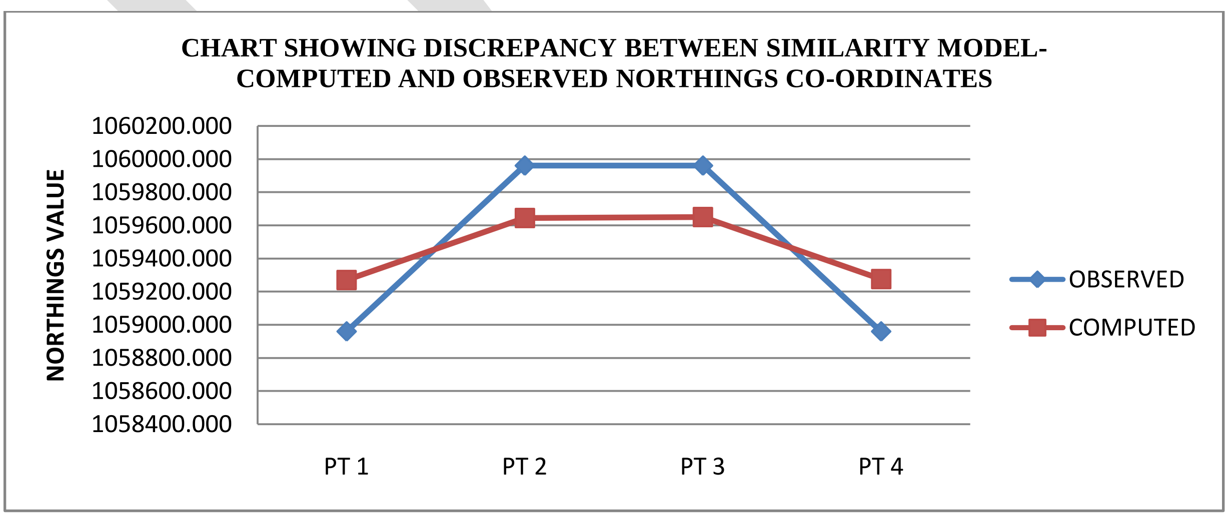

Figure 7(b): Similarity Model – Deviation Along The Northings Axes.

AFFINE TRANSFORM:

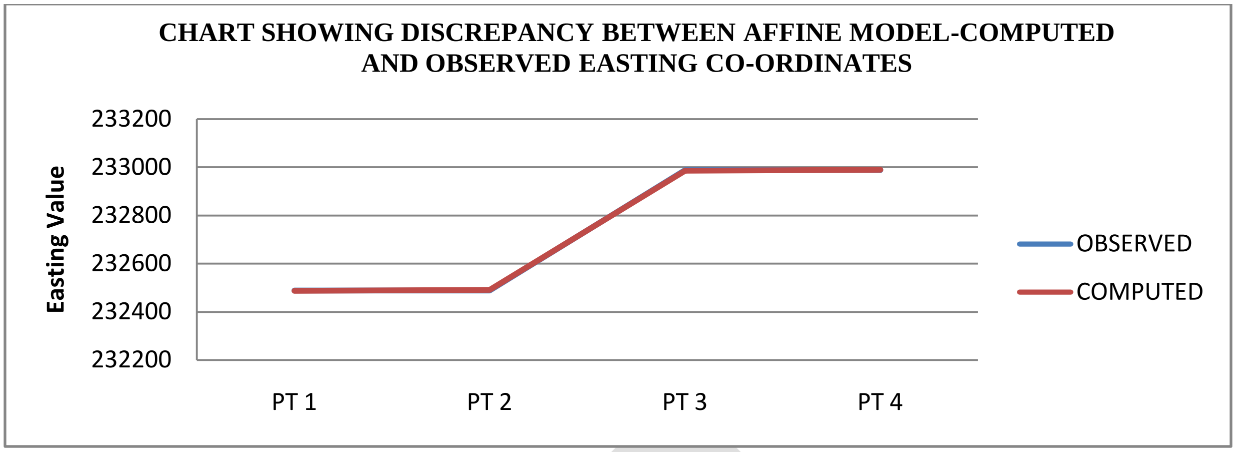

Figure 8(a): Affine Model – Deviation Along The Eastings Axes.

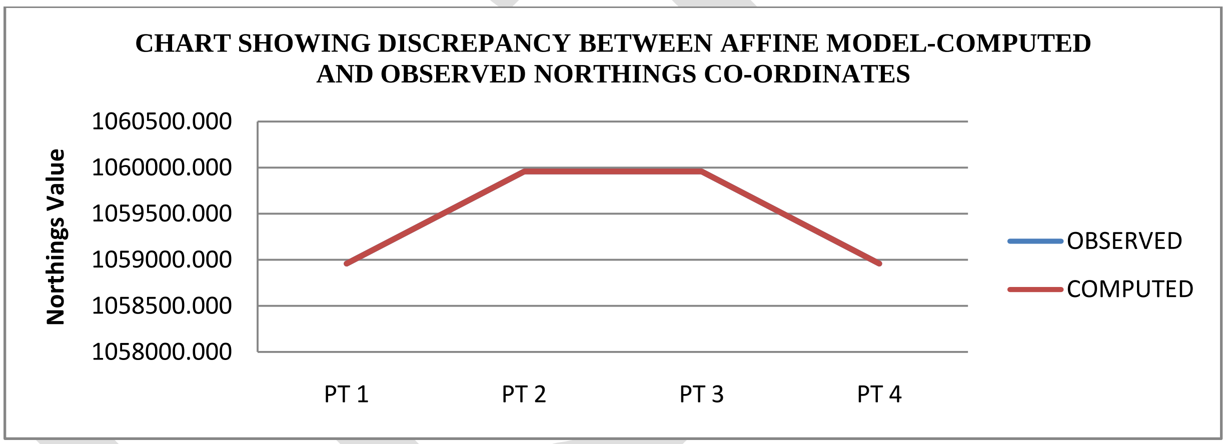

Figure 8(b): Affine Model – Deviation Along The Northings Axes.

POLYNOMIAL TRANSFORM (1ST-ORDER):

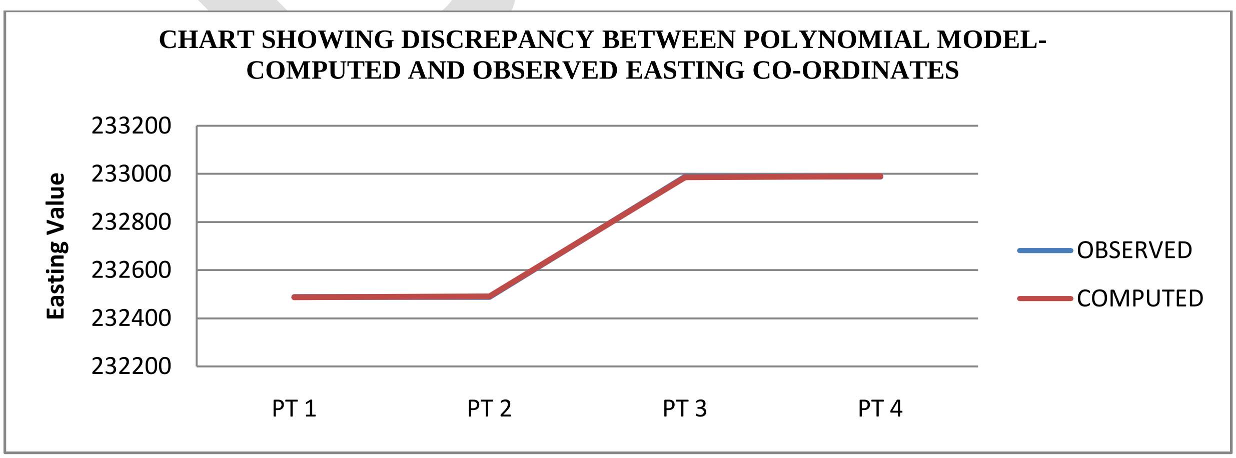

Figure 9(a): Polynomial Model – Deviation Along The Eastings Axes.

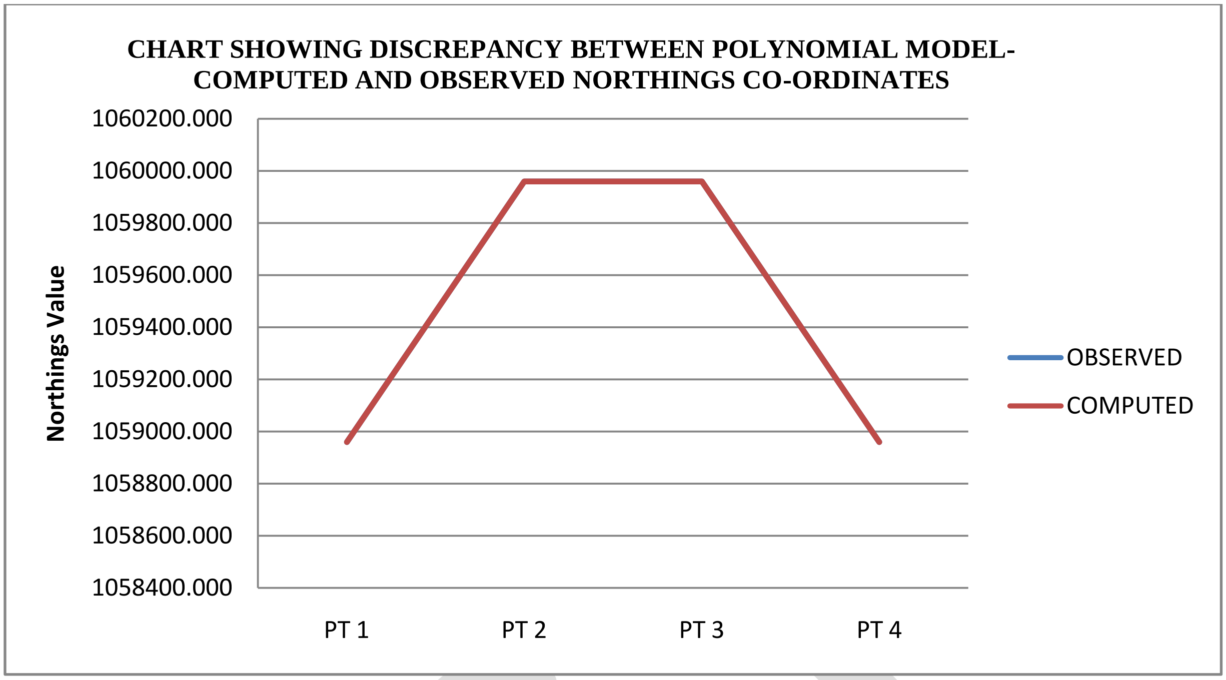

Figure 9(b): Polynomial Model – Deviation Along The Northings Axes

6.2.3 2-D Comparison of Obtained Results:

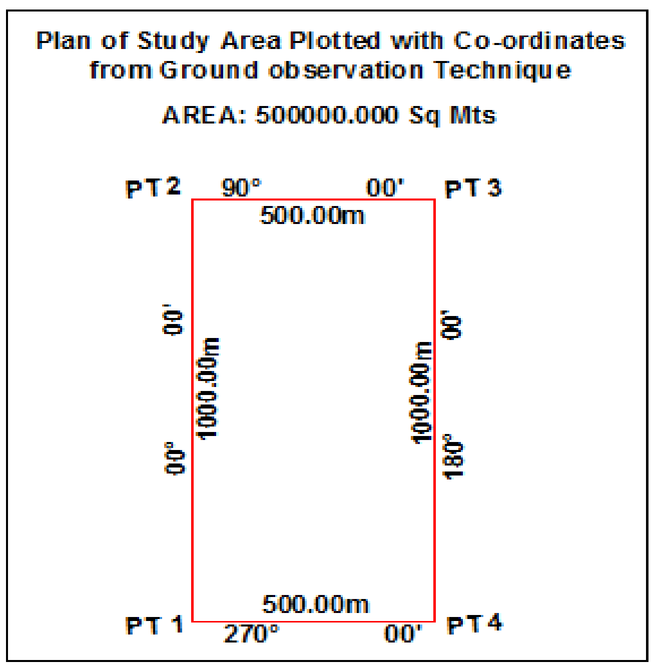

Figure 10(a): Plot of Actual Observed Co-ordinates of Control Points.

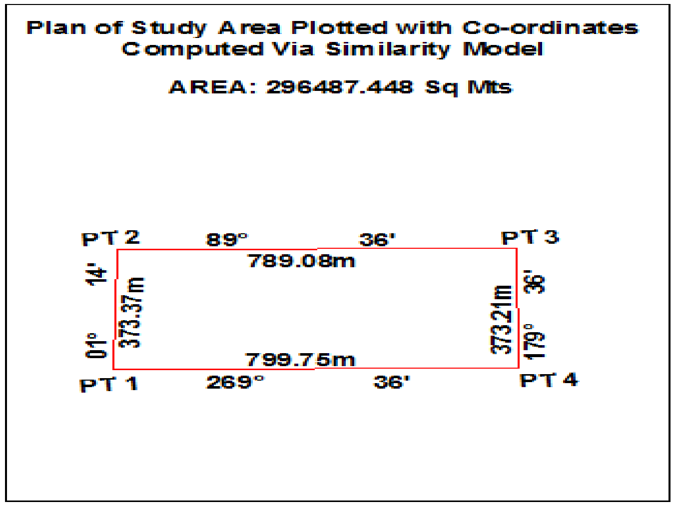

Figure 10(b): Plot of Similarity-Model Computed Co-ordinates of Control Points.

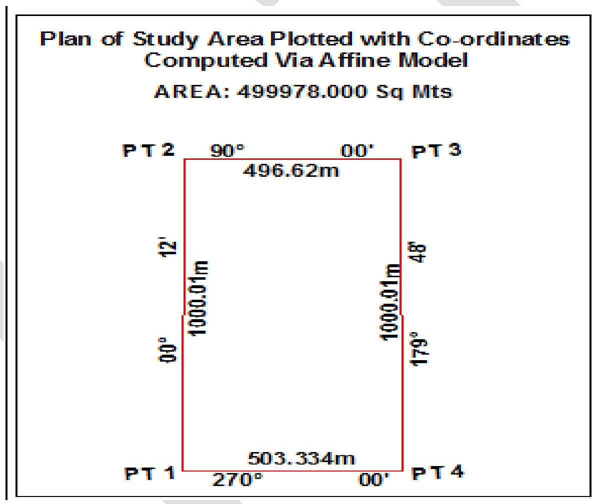

Figure 10(c): Plot of Affine-Model Computed Co-ordinates of Control Points.

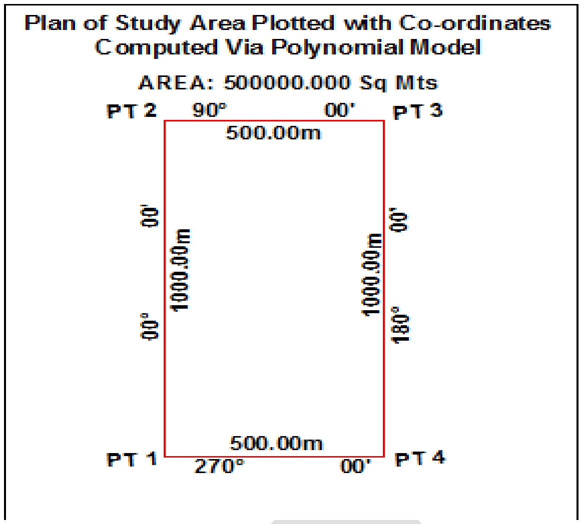

Figure 10(d): Plot of Polynomial-Model Computed Co-ordinates of Control Points.

The table below gives a clear summary of the findings of this research

Table 6: Analysis of Results

ANALYSIS OF RESULTS								
	COMPLEX MULTI-SHAPED FIGURES				**SIMPLE SHAPED FIGURES**			
PROPERTY	***OBJECT***	**SIMILARITY**	**AFFINE**	**POLYNOMIAL**	***OBJECT***	**SIMILARITY**	**AFFINE**	**POLYNOMIAL**
LENGHTS	Varies	NP	NP	PP	500.00	789.08 (NP)	496.62 (NP)	500 (P)
BREATH	Varies	NP	NP	PP	1000.00	373.37 (NP)	1000.01 (P)	1000 (P)
ANGLES	Varies	NP	NP	NP	90 Degrees	Varies (NP)	90 Degrees (P)	P
SCALE	1.00	NP	NP	SP	1.00	NP	P	P
SHAPE	Irregular	NP	NP	P	Rectangle	NP	P	P
AREA	23.65 Hect	3.87 Hect (NP)	18.61 Hect (NP)	23.82 Hect (P)	50 Hect	29.6 Hects (NP)	49.99 Hect (P)	50 Hects (P)
OVERALL COMMENT		*NOT SUITABLE*	*NOT SUITABLE*	***SUITABLE***		*NOT SUITABLE*	***SUITABLE***	***SUITABLE***

Source (Author's Research)

Where: NP = Not Preserved

PP = Partly Preserved

P = Preserved

7.0 CONCLUSION:

It has been discovered in the course of this research that the Polynomial model is most suitable for 2-D transformation of photographic images into their real world co-ordinates where third – order accuracy is required. This is because it most suitably preserves angular relationships and does not show trace of deformations of polygons of whichever shape.

It should be noted however that the research has not considered the maximum order of polynomial suitability for this purpose, however, it is expected that after certain order, the polynomial model will begin to deform the size and shape of the resulting image.

The Affine transforms ranks next to the polynomial transform in terms of overall suitability, though it is not a suitable transformation model for multi-shaped polygons as it only preserves parallel lines.

The similarity model however has proven completely un-suitable for 3rd-order planimetric mapping from photogrammetric images, though Deakin, R. E (2004) suggested that an equal area transform be imposed on it to obtain refined results.

REFERENCES

[1] A. S. Kumar; Merging Multispectral and PAN images by A-Trous wavelets. International Journal of Remote Sensing, 2003.

[2] C. Stamatopoulos and C. S. Fraser; An orthogonal projection model for photogrammetric orientation of long focal length imagery, 2011.

[3] Deakin, R. E (2004). Co-ordinate Transformations in Surveying and Mapping. Lecture Notes on Geospatial Science, RMIT.

[4] George Bebis, Michael Georgiopoulos, Niels da Vitoria Lobo and Mubarak Shah. "Learning Affine Transformations". Available on-line @ http://www.crcv.ucf.edu/papers/learn_affine_paper.pdf

[5] Mikhail, E, M, 1976. Observations and Least Squares. IEP – A Dun-Donnelley, New York.

[6] Olaleye, J. B (2013); Unpublished Lecture notes on Digital Photogrammetry, University of Lagos, Nigeria.

[7] Rami Al-Ruzouq., "Semi Automatic Registration of Multi-Source Satellite Imagery with Varying Geometric Resolutions" Ph.D Thesis, Department of Geomatics Engineering, University of Calgary, Calgary, Alberta, 2004.

[8] www.wikipedia.com

[9] Wang Anna, Wang Tingjun, Zhang Jinjin, Xue Silin (2009). "A Novel Method of Medical Image Registration Based on DTCWT and NPSO," icnc, vol. 5, pp.23-27, 2009 Fifth International Conference on Natural Computation, 2009