Covenant Journal of Physical & Life Sciences (CJPL) Vol. 7 No. 1, June 2019
ISSN: ***p. 2354 – 3574 e. 2354 – 3485*** **DOI: 10.20370/v09m-vp86**

An Open Access Journal Available Online

Magnetohydrodynamics (MHD) Flow of a Third Grade Fluid through a Cylindrical Pipe in an Inclined Plane with Radiation

Bolarin G., Yusuf A., Emmanuel O. & Aiyesimi Y. M.

Department of Mathematics, Federal University of Technology, Niger State, Nigeria
*g.bolarin@futminna.edu.ng

Received: 12.09.2018 Accepted: 14.05.2019 Date of Publication: June, 2019

Abstract: This work considered the natural convection of a one-dimensional heat generation and viscous dissipation model of a Magnetohydrodynamics (MHD) third grade fluid in an inclined cylindrical pipe with radiation. The governing dimensional equations of the momentum and energy were first non-dimensionalized and then solved analytically using the Homotopy Analysis Method (HAM). The results obtained are displayed on graphs. From the graphs, we observe that increase in the grashof number Gr leads to increase in both the velocity and the temperatue of the fluid. Also, the temperature of the fluid increases with increase in the radiative heat flux and drops gradually as the radiative heat flux decreases. The effect of various parameters on the velocity and temperature profiles are reported graphically for both cases of the extended models to elucidate special features of the solutions.
Keywords: Natural Convection, MHD, Homotopy Analysis Method, Third Grade Fluid, Radiation.

Introduction

MHD flow of non–Newtonian fluid through pipes plays significant role in different areas of science and technology such as petroleum industry, biomechanics, and heat exchangers and so on. Due to the nonlinearity of the governing equations, exact solutions are difficult to be obtained and are few in number under certain conditions. As a consequence of different physical structures of non–Newtonian fluids, there is not a single constitutive model which can predict all its salient features. Generally, there are three non–Newtonian fluid models, namely empirical models, differential models and integral models. But the most famous amongst them are the last two models.

In recent years the study of convection heat and mass transfer in non-Newtonian fluids has received much attention and this is because the traditional Newtonian fluids cannot precisely describe the characteristics of the real fluids. In addition, progress has been considerably made in the study of heat and mass transfer in magneto hydrodynamic flow of non-Newtonian fluids due to its application in many devices, like the MHD power generator, aerodynamics heating, and electrostatic precipitation and Hall accelerator. Yurusoy and Pakdermirli [1] analyzed the heat transfer flow of third-grade fluid in a pipe. Massoudi and Chiristie [2,3] studied the fully developed flow of an incompressible, thermodynamically compatible fluid of grade three in a pipe. Also, Aiyesimi *et al.* [4] made an investigation into unsteady MHD thin film flow of a third grade fluid down an inclined plane with no slip boundary condition. The regular perturbation method was used to reduce the governing nonlinear partial differential equations to linear partial differential equations which were solved using the method of separation of variables and eigen-functions expansion. Aiyesimi *et al.* [5] investigated the combined effects of magnetic field on the MHD flow of a third grade fluid through inclined channel in the presence of a uniform magnetic field with the consideration of heat transfer.

Convective heat transfer involves both heat diffusion known as conduction and by bulk heat transfer of fluid known as advection. Convection heat transfer may be natural, forced or mixed. Convection phenomena play a key role in practical life. The chemical transportation in packs bed reactors, drying of porous solid and solar power collectors to mention just a few are the practical applications of mixed convection phenomena. The concept of mixed convection had been extensively studied by many scientists. Heckel *et al*. [6] studied mixed convection flow along slender vertical cylinder by considering variable surface temperature. Also, the characteristics of heat transfer along mixed convection flow of non-Newtonian fluid past a vertical plate was considered by Wang [7]. Chen [8] examined the laminar mixed convection flow over a vertical continuously stretching sheet. Seddeek *et al*. [9] reported mixed convection flow past a continuously stretching vertical cylinder by considering variable viscosity and thermal diffusivity. The properties of mixed convection flow past a permeable vertical cylinder along surface heat flux were provided by Bachok and Ishak [10]. Hsiao [11] examined the magnetohydrodynamic mixed convection flow of viscoelastic fluid over a porous wedge. Mukhopadhyay and Ishak [12] explored the mixed convection flow past a stretching cylinder by means of thermal stratification. The mixed convection flow of viscoelastic fluid over a stretching cylinder was discussed by Hayat et al. [13].

The study of MHD flow with radiation is also significant in many cases. Some problems related to radiation effect are mentioned here. Radiation effect on free convection flow past a semi-infinite vertical plate was analyzed by Soundalgekar and Takhar [14]. Radiation effect on mixed convection along a vertical plate with uniform surface temperature was considered by Hossain and Takhar [15]. Takhar et al. [16] have analyzed radiation effect on MHD free convection flow of a radiating gas past a semi-infinite vertical plate. Raptis and Massals [17] have investigated magnetohydrodynamics flow past a plate in the presence of radiation.

To the best of our knowledge, the use of Homotopy Analysis method to obtain the solution of a MHD flow of a third grade fluid through an inclined cylindrical pipe with radiation has remained unexplored. The objective of the current work is to study the effect of various parameters that may occur on the velocity and temperature profile.

Materials and Methods

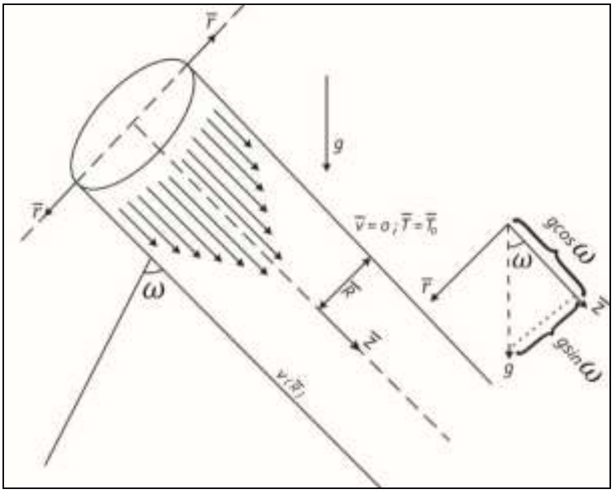

Figure 1: Physical Model and Coordinate System

Consider a long cylindrical pipe with the steady incompressible flow of a third grade fluid. The physical configuration is as shown in Figure 1. The equations for the velocity and the temperature, given by Massoudi and Christe [2,3] as well as Yurosoy and Pakdemirli [1], Jayeoba and Okoya [18], may be extended to incorporate a magnetic and radiative heat flux term given by

Mass balance

$$\frac{\partial \bar{v}_{\bar{z}}}{\partial \bar{z}} = 0 \qquad (1)$$

Momentum equation

$$\frac{1}{\bar{r}}\frac{d}{d\bar{r}}\left(\bar{r}\bar{\mu}\frac{d\bar{v}}{d\bar{r}}\right) + \frac{1}{\bar{r}}\frac{d}{d\bar{r}}\left(2\bar{r}\beta_3\left[\frac{d\bar{v}}{d\bar{r}}\right]^3\right) - \sigma{\beta_0}^2\bar{v} + g\beta_g(\bar{T} - \bar{T}_0)\cos\omega = 0 \qquad (2)$$

Energy equation

$$K\left(\frac{1}{\bar r}\frac{d}{d\bar r}\left(\bar r\frac{d\bar T}{d\bar r}\right)\right)+\bar\mu\left(\frac{d\bar v}{d\bar r}\right)^2+2\beta_3\left(\frac{d\bar v}{d\bar r}\right)^4+\overline{QC_0}(\bar T-\bar T_0)-(4\alpha(\bar T-\bar T_0))=0 \qquad (3)$$

The required boundary conditions to solve equations (2) and (3) are

$$\bar v(\bar R)=\bar T(\bar R)=0\ ,\ \frac{d\bar v}{d\bar r}(0)=\frac{d\bar T}{d\bar r}(0)=0 \qquad (4)$$

The angle of inclination ω throughout this work is kept constant at 60^0. Also, the last term in the energy equation represents the radiative heat flux as given by Cogley *et al.* [19]. Where all symbols used in this work are as defined in the table of nomenclature (Section5). The corresponding dimensionless equations for equations (2)-(3) lead to the following:

$$\frac{d\mu}{dr}\frac{dv}{dr}+\frac{\mu}{r}\left(\frac{dv}{dr}+r\frac{d^2v}{dr^2}\right)+\frac{\Lambda}{r}\left[\frac{dv}{dr}\right]^2\left(\frac{dv}{dr}+3r\frac{d^2v}{dr^2}\right)-Hv+Gr\theta\cos\omega=0 \qquad (5)$$

$$\frac{d^2\theta}{dr^2}+\frac{1}{r}\frac{d\theta}{dr}+\Gamma\left(\frac{dv}{dr}\right)^2\left(\mu+\Lambda\left(\frac{dv}{dr}\right)^2\right)+\delta\theta-Ra\theta \qquad (6)$$

$$\theta(1)=0v(1)=0,\quad \text{and}\quad \frac{dv(0)}{dr}=\frac{d\theta(0)}{dr}=0 \qquad (7)$$

The equations of the forms (5) and (6) depend on the viscosity model, and the viscosity μ is assumed to be a function of temperature. We now present the Reynold's model case as can be found in Massoudi and Christe [3], Nadeem and Ali [20], Jayeoba and Okoya [18]. Here,

$$\bar\mu(\bar T)=\bar\mu_0 exp(-\bar M(\bar T-\bar T_0)) \qquad (8)$$

It is well known that Reynolds viscosity decreases with increasing temperature for liquids whenever M is positive, and it increases with increasing temperature for gas whenever M is negative. When M is large, then the effect of variable viscosity can be neglected. The corresponding non-dimensional form of (8) is

$$\mu=exp(-\rho\theta) \qquad (9)$$

which after using the Maclaurin's series can be written as

$$\mu=exp\,(-\rho\theta)=1-\rho\theta+O(\rho^2) \qquad (10)$$

Results We now apply the HAM to establish analytical solutions to determine the velocity and temperature distribution using Reynolds models of viscosity.

For HAM solution, we select

$$v_0(r)=\frac{-Gr(1-r^2)}{4}\quad \text{And}\quad \theta_0(r)=\frac{\Gamma Gr^2(1-r^4)}{64} \qquad (11)$$

as the initial approximations of $v(r)$ and $\theta(r)$ respectively, which satisfy the linear operator and corresponding boundary conditions. We use the method of higher order differential mapping, to choose the auxiliary linear operator L we follow the approach of Van Gorder, and Vajravelu [21]

$$L=\frac{d}{dr}^2 \qquad (12)$$

which satisfies the following relation

$$L[A_0 r + A_1] = 0 \tag{13}$$

where A_0 and A_1 are constants.

Let $R = \frac{d}{dr}$ (14)

Next, we construct the homotopy

$$H_v[v^*(r,p); v_0(r); H(r); h; p] = (1-p)L[v^*(r,p) - v_0(r)] - phH(r)N_v[v^*(r,p); \theta^*(r,p)] \tag{15}$$

$$H_\theta[\theta^*(r,p); \theta_0(r); H(r); h; p] = (1-p)L[\theta^*(r,p) - \theta_0(r)] - phH(r)N_\theta[v^*(r,p); \theta^*(r,p)] \tag{16}$$

where $h \neq 0$ is an auxiliary parameter, $H(r) \neq 0$ is an auxiliary function, $p \in [0, 1]$ is an embedding parameter, $v_0(r)$ and $\theta_0(r)$ are the initial approximations to the solution that satisfies the given initial or boundary conditions, $v^*(r,p)$ and $\theta^*(r,p)$ satisfy the initial or boundary conditions, and L is some linear operator. The zero-order deformation equations are given by the following relations

$$(1-p)L[v^*(r,p) - v_0(r)] = phH(r)N_v[v^*(r,p); \theta^*(r,p)] \tag{17}$$

$$(1-p)L[\theta^*(r,p) - \theta_0(r)] = phH(r)N_\theta[v^*(r,p); \theta^*(r,p)] \tag{18}$$

The mth order deformation equation is

$$L[V_m(r) - \chi_m v_{m-1}(r)] = hH(r)\zeta_m(v_{m-1}) \tag{19}$$

$$L[\theta_m(r) - \chi_m \theta_{m-1}(r)] = hH(r)\zeta_m(\theta_{m-1}) \tag{20}$$

with boundary conditions

$$v_m(1) = \theta_m(r) = 0, \text{ and } \frac{dv_m}{dr}(0) = \frac{d\theta_m}{dr}(0) = 0 \tag{21}$$

Where

$$\zeta_m(v_{m-1}) = Lv_{m-1}(r) + \frac{1}{r}Rv_{m-1}(r) - \rho\sum_{k=0}^{m-1} Lv_{m-1-k}(r)\theta_k(r) - \frac{\rho}{r}\sum_{k=0}^{m-1} Rv_{m-1-k}(r)\theta_k(r) - \rho\sum_{k=0}^{m-1} Rv_{m-1-k}(r)R\theta_k(r) + \frac{\Lambda}{r}\sum_{k=0}^{m-1} Rv_{m-1-k}(r)\sum_{j=0}^{k} Rv_{k-j}(r)Rv_j(r) + 3\Lambda\sum_{k=0}^{m-1} Rv_{m-1-k}(r)\sum_{j=0}^{k} Rv_{k-j}(r)Lv_j(r) - Hv_{m-1}(r) + Gr\theta_{m-1}(r)\cos\omega \tag{22}$$

$$\zeta_m(\theta_{m-1}) = -L\theta_{m-1}(r) - \frac{1}{r}\theta_{m-1}(r) - \Gamma\sum_{k=0}^{m-1} Rv_{m-1-k}(r)Rv_k(r) + \rho\Gamma\sum_{k=0}^{m-1} Rv_{m-1-k}(r)\sum_{j=0}^{k} Rv_{k-j}(r)\theta_j(r) - \Lambda\Gamma\sum_{k=0}^{m-1} Rv_{m-1-k}(r)\sum_{j=0}^{k} Rv_{k-j}(r)\sum_{l=0}^{j} Rv_{j-l}(r)Rv_l(r) - \delta\theta_{m-1}(r) + Ra\theta_{m-1} \quad (23)$$

And the nonlinear operators are

$$N_v[v^*(r,p);\theta^*(r,p)] = \frac{d^2v^*}{dr^2} + \frac{1}{r}\frac{dv^*}{dr} - \rho\frac{d^2v^*}{dr^2}\theta^* - \frac{\rho}{r}\frac{dv^*}{dr}\theta^* - \rho\frac{dv^*}{dr}\frac{d\theta^*}{dr} + \frac{\Lambda}{r}\left(\frac{dv^*}{dr}\right)^3 + 3\Lambda\left(\frac{dv^*}{dr}\right)^2\left(\frac{d^2v^*}{dr^2}\right) - Hv + Gr\theta\cos\omega = 0 \quad (24)$$

$$N_\theta[v^*(r,p),\theta^*(r,p)] = \frac{d^2\theta^*}{dr^2} + \frac{1}{r}\frac{d\theta^*}{dr} + \Gamma\left(\frac{dv^*}{dr}\right)^2 - \rho\Gamma\left(\frac{dv^*}{dr}\right)^2\theta^* + \Lambda\Gamma\left(\frac{dv^*}{dr}\right)^4 + \delta\theta^* - Ra\theta \quad (25)$$

$$\chi_m = \begin{cases} 0, & m \le 1 \\ 1, & m > 0 \end{cases} \quad (26)$$

For conveniences, we used Maple-17 to compute the solutions and by Van Gorder and Vajravelu [21], $-0.9 \le h \le -0.1$. For simplicity, we choose $H = 1$. The analytical expressions for the velocity, temperature is for $m = 1$ is given as

$$v(r) = \frac{-Gr(1-r^2)}{4} + h\left(\frac{1}{2}Grr^2 - \frac{1}{64}\rho Gr^3\Gamma\left(-\frac{1}{30}r^6 + \frac{1}{2}r^2\right) + \frac{1}{960}\rho Gr^3r^6\Gamma + \frac{1}{24}\Lambda Gr^3r^4 + \frac{1}{4}HGr\left(-\frac{1}{12}r^4 + \frac{1}{2}r^2\right) + \frac{1}{64}Gr^3\Gamma\cos\omega\left(-\frac{1}{30}r^6 + \frac{1}{2}r^2\right) - h\left(\frac{1}{2}Gr - \frac{1}{160}\rho Gr^3\Gamma + \frac{1}{24}\Lambda Gr^3 + \frac{5}{48}HGr + \frac{7}{960}Gr^3\Gamma\cos\omega\right)\right) \quad (27)$$

$$\theta(r) = \frac{\Gamma Gr^2(1-r^4)}{64} + h\left(\frac{1}{256}\Gamma^2\rho Gr^4\left(-\frac{1}{56}r^8 + \frac{1}{12}r^4\right) - \frac{1}{480}\Lambda\Gamma Gr^4r^6 - \frac{1}{64}\delta\Gamma Gr^2\left(-\frac{1}{30}r^6 + \frac{1}{2}r^2\right) + \frac{1}{64}\delta\Gamma Gr^2\left(-\frac{1}{30}r^6 + \frac{1}{2}r^2\right)\right) - h\left(\frac{11}{43008}\Gamma^2\rho Gr^4 - \frac{1}{480}\Lambda\Gamma Gr^4 - \frac{7}{960}\delta\Gamma Gr^2 + \frac{7}{960}Ra\Gamma Gr^2\right) \quad (28)$$

Discussion

The effects of the physical parameters on the velocity and temperature distributions are shown in Figures 2-12.

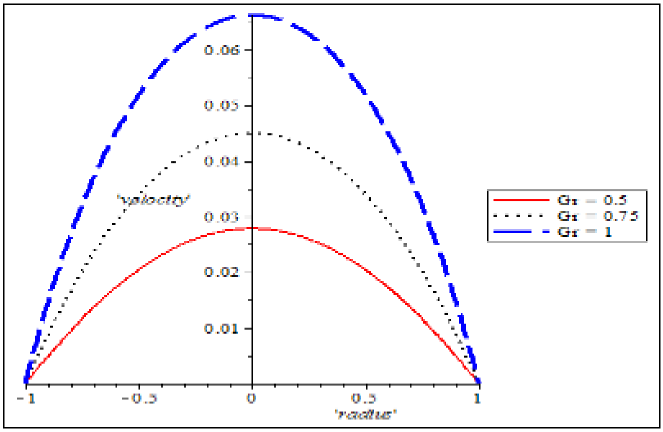

Figure 2: Graph of velocity v against the radius r for values of Grasof number Gr with $H = \Gamma = \Lambda = \rho = 1$

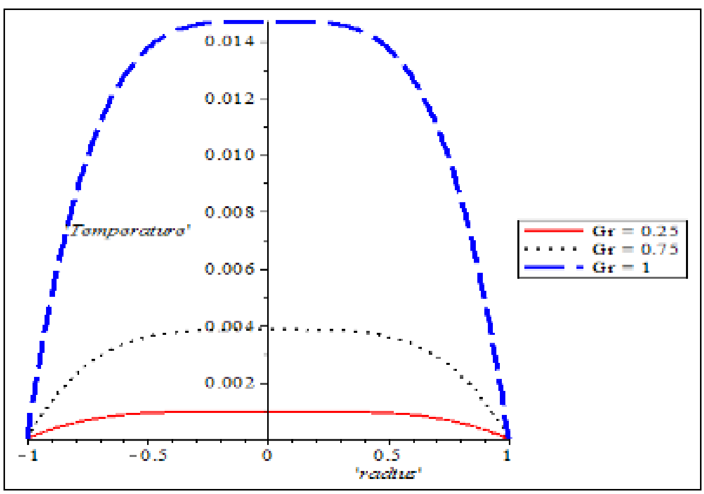

Figure 3: Graph of temperature θ against the radius r for values of Grasof number Gr with $Ra = \delta = \Gamma = \Lambda = \rho = 1$

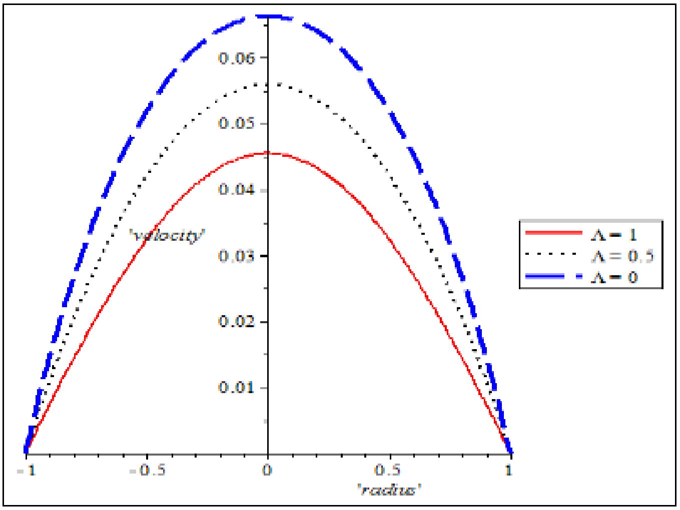

Figure 4: Graph of velocity v against the radius r for values of the non-Newtonian Material parameter of the Fluid Λ with $Gr = H = \Gamma = \rho = 1$

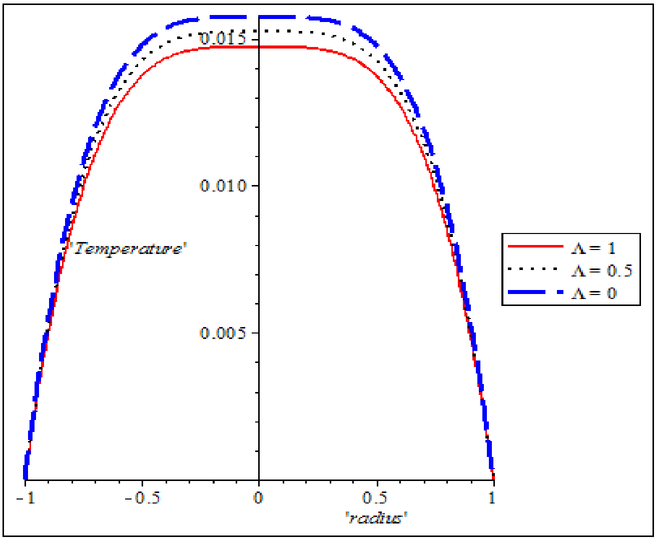

Figure 5: Graph of temperature θ against the radius r for values of non-Newtonian Material parameter of the Fluid Λ with $Gr = Ra = \delta = \Gamma = \rho = 1$

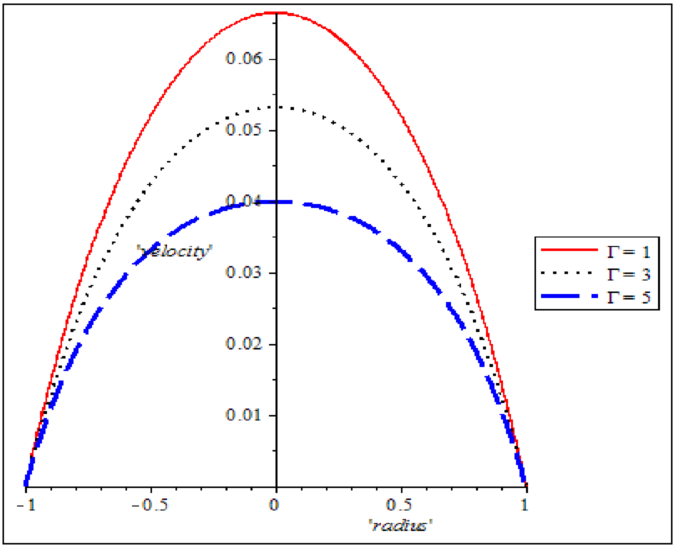

Figure 6: Graph of velocity $\boldsymbol{v}$ against the radius $\boldsymbol{r}$ for values of the Viscous dissipation parameter $\boldsymbol{\Gamma}$ of the Fluid with $\boldsymbol{Gr = H = \Lambda = \rho = 1}$

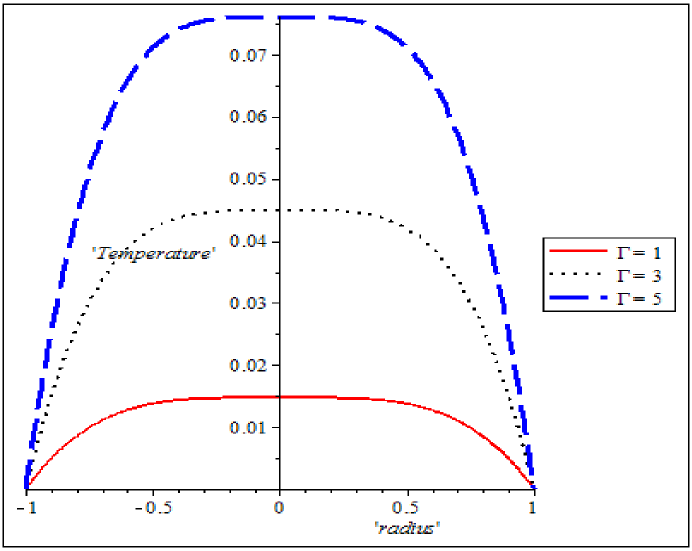

Figure 7: Graph of temperature $\boldsymbol{\theta}$ against the radius $\boldsymbol{r}$ for values of Viscous dissipation parameter $\boldsymbol{\Gamma}$ of the Fluid with $\boldsymbol{Gr = Ra = \delta = \Lambda = \rho = 1}$

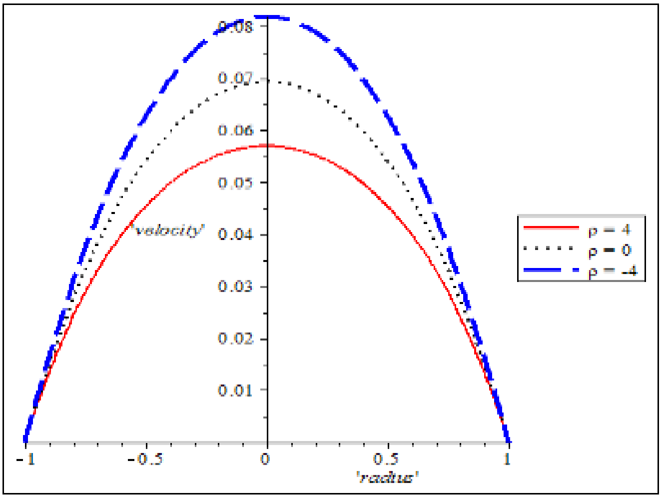

Figure 8: Graph of velocity $\boldsymbol{v}$ against the radius $\boldsymbol{r}$ for values of the Reynold's Viscosity Variational parameter $\boldsymbol{\rho}$ of the fluid with $\boldsymbol{Gr = H = \Lambda = \Gamma = 1}$

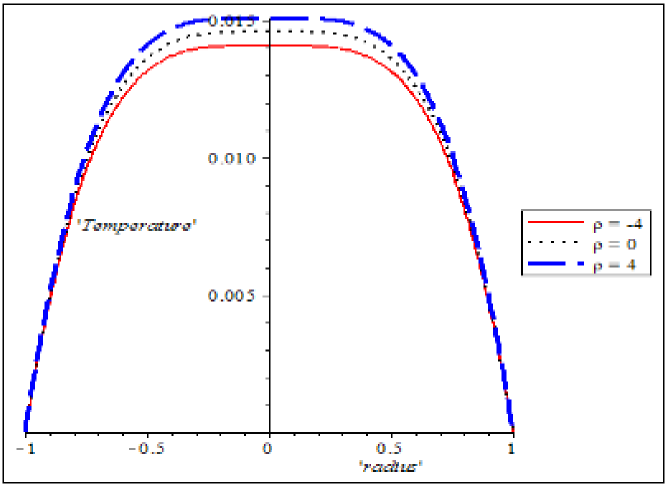

Figure 9: Graph of temperature $\boldsymbol{\theta}$ against the radius $\boldsymbol{r}$ for values of the Reynold's Viscosity Variational parameter $\boldsymbol{\rho}$ fluid with $\boldsymbol{Gr = Ra = \delta = \Lambda = \Gamma = 1}$

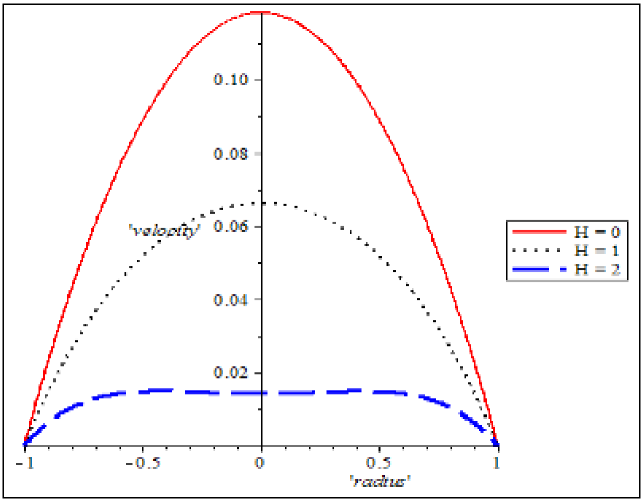

Figure 10: Graph of velocity $\boldsymbol{v}$ against the radius $\boldsymbol{r}$ for values of the Magnetic parameter $\boldsymbol{H}$ of the fluid with $\boldsymbol{Gr = \rho = \Lambda = \Gamma = 1}$

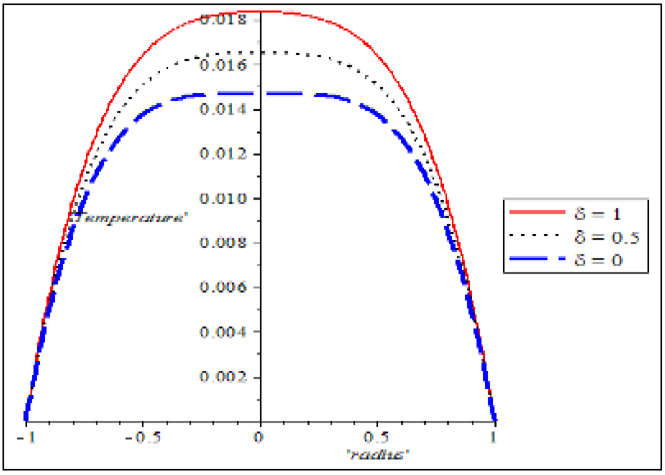

Figure 11: Graph of temperature $\boldsymbol{\theta}$ against the radius $\boldsymbol{r}$ for values of the heat generation parameter $\boldsymbol{\delta}$ of the fluid with $\boldsymbol{Gr = Ra = \rho = \Lambda = \Gamma = 1}$

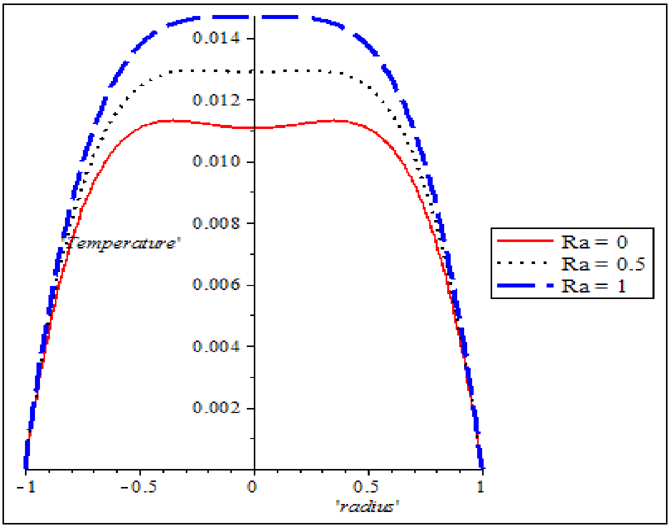

Figure 12: Graph of temperature θ against the radius r for values of the heat radiation parameter Ra of the fluid with $Gr = \delta = \rho = \Lambda = \Gamma = 1$

Figure 2 to 3, show respectively the effects of Grashof number Gr on the velocity and temperature of the fluid. We observe that both the velocity and temperature of the flow increase with increase in Gr and decreases vice versa.

Figure 4 to 5, show the effect of non-Newtonian material parameter on the velocity and temperature of the fluid. We observe that at the point where the non-Newtonian parameter, $\Lambda = 0$ maximum velocity and temperature profile was attained. This holds true for the present study as well as that of Jayeoba and Okoya [18]. With increase in the non-Newtonian parameter, both the velocity and temperature of the fluid decrease.

Figure 6 to 7 respectively show the effect of viscous dissipation parameter, Γ on the velocity and temperature profile. We observe from Figure 6 that increase in Γ, leads to decrease in the velocity profile while decrease in Γ, leads to increase in the velocity profile. In Figure 7 the temperature of the fluid increases with increase in Γ, this is due to the irreversible conversion of the mechanical energy to thermal energy.

Figure 8 to 9 show the effect of the viscous variation parameter ρ on the velocity and the temperature of the fluid. In Figure 8, we observed that the velocity drops for positive values of ρ and increases for negative values of, ρ. While in Figure 9, the temperature of the fluid increases with increase in the value of ρ and decreases as the value of ρ decreases.

Figure 10 shows the effect of the magnetic parameter H, H has a decelerating effect on the velocity profile. We observe that increase in the magnetic parameter contributes to slowing down the velocity of the fluid. This decrease in the velocity of the fluid is due to the resistive Lorentz's force which comes into play as a result of the interactions of the magnetic field with conducting fluid. Again in the absence of the magnetic parameter $H = 0$ we observed a monumental increase in the velocity especially at the center of the pipe.

Figure 11 shows the influence of heat generation parameter δ on the temperature of the fluid. The case where, $\delta = 0$, the least temperature distribution was observed. As the amount of heat generation parameter increases, the maximum temperature of the fluid increases gradually.

Figure 12 shows the effect of the radiation parameter Ra, on the temperature of the fluid. We observe that increase in Ra increases the temperature of the fluid and vice versa.

Conclusion

The problem of natural convective MHD flow of a third-grade fluid through an inclined cylindrical pipe with radiation was considered. The steady state one-dimensional heat transfer model was formulated and solved analytically for the Reynold's viscosity model using the Homotopy analysis method. The effects of various values of the thermo-physical parameters are presented in a graphical form. It is observed that increase in Gr and Ra leads to both increase in the velocity and temperature profile.

NOMENCLATURE

$\bar{r}$	Dimensional perpendicular distance from pipe axis
$r = \bar{r}/\bar{R}$	Dimensionless perpendicular distance from pipe axis
$\bar{R}$	Radius of the pipe
$\bar{v}(\bar{R})$	Dimensional velocity component in the $\bar{z}$ axis
$v = \bar{v}/\bar{v}_0$	Dimensionless velocity component in the $\bar{z}$ axis
$\bar{v}_0$	Dimensional reference velocity
$\bar{z}$	Axis of the cylinder
$\bar{T}$	Dimensional temperature
$\bar{T}_0$	Dimensional reference temperature
$\bar{T}_f$	Scaled temperature
g	Acceleration due to gravity
K	Constant thermal conductivity
$\bar{Q}$	Heat generation constant
$\overline{C_0}$	Initial concentration of reactant species
$\alpha_1, \alpha_2, \beta_3$	Material constant coefficients
$\bar{\mu}$	Dynamic shear viscosity
$\mu = \bar{\mu}/\bar{\mu}_0{}^e$	Dimensionless viscosity
ϕ	Rotational direction

$\rho = \bar{M}\bar{T}_f$	Reynolds Viscosity Variational Parameter
$\theta = \frac{(\bar{T}-\bar{T}_0)}{\bar{T}_f}$	Dimensionless temperature
ω	The angle which the cylinder makes with wall
$H = \frac{\sigma \bar{R}^2 \beta_0{}^2}{\bar{\mu}_0}$	Magnetic parameter
$\Lambda = \frac{2\beta_3 \bar{v}_0{}^2}{\bar{\mu}_0 \bar{R}^2}$	Non-Newtonian parameter
$\Gamma = \frac{\bar{\mu}_0 \bar{v}_0{}^2}{K\bar{T}_f}$	Viscous dissipation parameter
$\delta = \frac{Q\bar{C}_0}{K}$	Heat generation parameter
$Gr = \frac{g\beta \bar{R}^2 \bar{T}_f}{\bar{v}_0 \bar{\mu}_0}$	Grashof number
$Ra = \frac{4\alpha \bar{R}^2}{K}$	Radiation parameter

Acknowledgement
The Author acknowledged the reviewers for making this work better than the original form.

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