INVESTIGATION OF THE EFFECT OF SEA LEVEL VARIATION ON VERTICAL REFERENCE FRAMES BASED ON A DESIGNED EXPERIMENT

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The Mean Sea level as a result of its "near-coincidence" with the equipotential surface of the geoid is universally adopted as a reference surface for the physical realization of the vertical reference system. Unfortunately, the effect of climate change has continued to alter the value of the mean sea level across the globe by as much as 10mm per annum at certain locations. The Gauss-Markov functional model has been used in this study to determine the effect of Sea Level variation on sea-related physical heights along the ZTT-control series in Lagos state using the different International Association of Geodesy (IAG) standard geo-potential values as representative indicators of sea level rise. Results obtained show very minimal effect of MSL variation on the VRF with a standard deviation of ∓ 0.000000000015 m

Keywords: *Climate Change, functional Models, GNSS Levelling, Height Systems, Vertical Reference Frames (VRF).*

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INTRODUCTION

Most environmental hazards (such as flooding, erosion, air-pollution, earthquakes, e.t.c) are spatially related phenomena that have tendencies of spreading into neighboring environments depending on the spatial characteristics of its immediate surroundings; the most significant characteristic that determines the extent and nature of such spread being the terrain undulation. Terrain undulation refers to the height variation of an area of land, depicting the low and high lands in the area with reference to a reference measurement surface. Diverse reference surfaces for height measurement have been adopted over the years by several countries (Rizos, 2015b) some of which include the Mean High Water Neaps (MHWN), Mean High Water Level (MHWL), Mean High Water Springs (MHWS), Mean Sea Level (MSL), e.t.c (Figure 1).

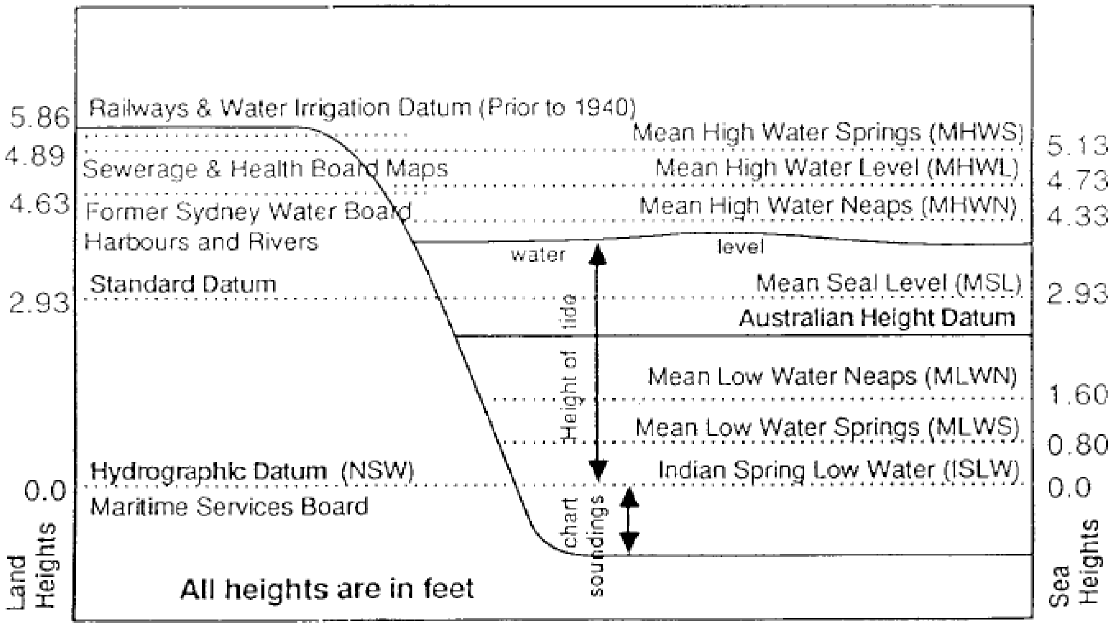

Figure 1: Some vertical datums used for height definition: An example of Sydney (Rizos, 2015b)

However, the "near-coincidence" of the MSL with the geoid has over the years popularized the adoption of sea-related heights (Tide Guage Datums) for the realization of several national vertical reference frames (Torge, 2001; Agren, 2015). This practice of relating heights to sea level are generally prominent because of it is practically easy to achieve and also due to the vast presence of the global oceans. Good as it may be, time dependence of the MSL (MSL Variation) due to climate change, non-parallelism of the equipotential surface and inconsistence in the exact definition of the zero reference level (Fotopulous, 2003; Rizos, 2015(a & b)) makes sea dependent heights a rather unstable choice for height definition. To overcome these problems vertical reference frames across the world should be tied to the geoid.

The geoid is best defined as the equi-potential surface of the earth's gravity field which nearly coincides with the surface of the MSL of the global oceans and is everywhere perpendicular to the plumbline (Listings,1873). The Mean Sea Level is defined as the average level of the global oceans taken over a period of 19.6 years (Ojinaka, 2007). Reference systems are introduced in order to model geodetic observations as a function of unknown parameters of interest. The coordinate systems are defined in terms of orientation, metrics, and curvature i.e they are three-dimensional in principle (Heitz 1988, Torge, 2001). A reference system defines the origin and the orientation of fundamental planes or axes of the system. It also includes the underlying fundamental mathematical and physical models (Seeber, 2003). Therefore, a Reference system could simply be described a set of parameters and idealized theoretical descriptions/model for an intended real world positioning system. On the other hand, a reference frame means the practical realization of a reference system through observations. It consists of a set of identifiable fiducial points on the sky (e.g. stars, quasars) or on Earth's surface (e.g. fundamental stations) (Torge, 2001; Seeber, 2003; Rizos, 2015a)

Since climate change has cause the position of the MSL to change over the years, the implication of maintaining a vertical reference Framework that is MSL dependent is that heights within the framework may lose positional integrity over time and might need to be re-observed. This research looks at the effects of MSL variation on Lagos vertical Reference Frame work from the geodetic perspective by taking advantage of the direct relationship between heights and earth surface potentials.

PHYSICAL HEIGHTS FROM A GEODETIC PERSPECTIVE

Height is the distance of a point above a specified surface of constant potential; the distance measured along the direction of gravity (Meyer et al, 2005, Odumosu et al, 2015). This can be mathematically expressed as equation 1:

$$H = {(W_0 - W_P)}/{g} \quad (1)$$

Where:

g = gravity Vector

W_0 = Potential at Reference Point

W_P = Potential at Observation Point

But g and W are inter-dependent as shown in equation 2

$g = \nabla W = grad\, W$ (2)

Now, we can show that potential depends only on positions (equations 3 – 5)

$W = V + \phi$ (3)

Where:

V = Gravitational Potential

ϕ = Centrifugal Potential

$\therefore W = G \iiint_v \frac{\rho}{l} dv + \frac{1}{2} w^2(x^2 + y^2)$ (4)

$\oint W = - \oint g.dn = 0$ (5)

To avoid path dependence, spirit leveled height differences are converted to potential differences as given by Argen, 2015 (equation 6 - 7) and shown in Figure 2.

$dW = W_B - W_A = -\int_A^B g.dn \approx \sum_A^B g.\delta n$ (6)

$\therefore H_A = {C_A}/{g_A}$ (7)

Where $C_A = W_0 - W_A$

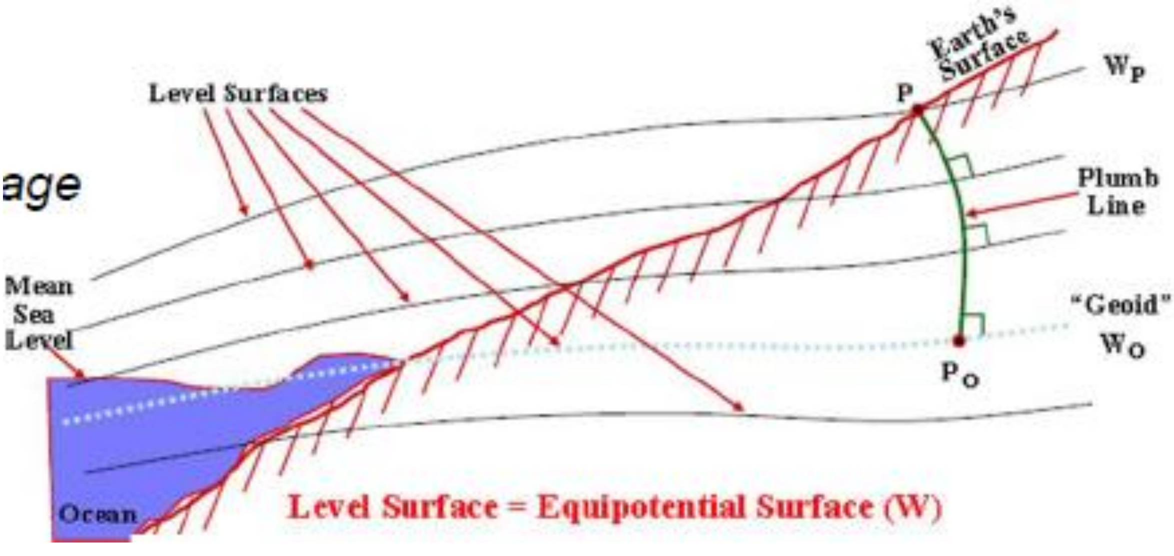

Figure 2: Definition of Height from gravimetric perspective (Argen, 2015)

EFFECT OF SEA LEVEL RISE ON VERTICAL REFERENCE FRAME

The changes in sea Level with time results in a shift in the origin of the Vertical Reference Frame (VRF) since the equipotential surface used as the zero height reference will change in position (Kotsakis, 2012). Although, this effect might be constant at the origin (δW_0), its effect on individual points within the frame work will vary due to the non-parallelism of the equipotential surfaces (equation 8):

$$H_t - H_i = \frac{\delta W_0}{g_t} - \frac{\delta g}{\delta H} \cdot \frac{\delta W_0{}^2}{2g_t{}^3} + \ldots\ldots \quad (8)$$

From Equations 7 and 8 therefore, a change in the MSL results in a change in the adopted equipotential surface (Sanchez, 2013). This by implication changes the value of our reference potential (W_0) as shown in Figure 3.

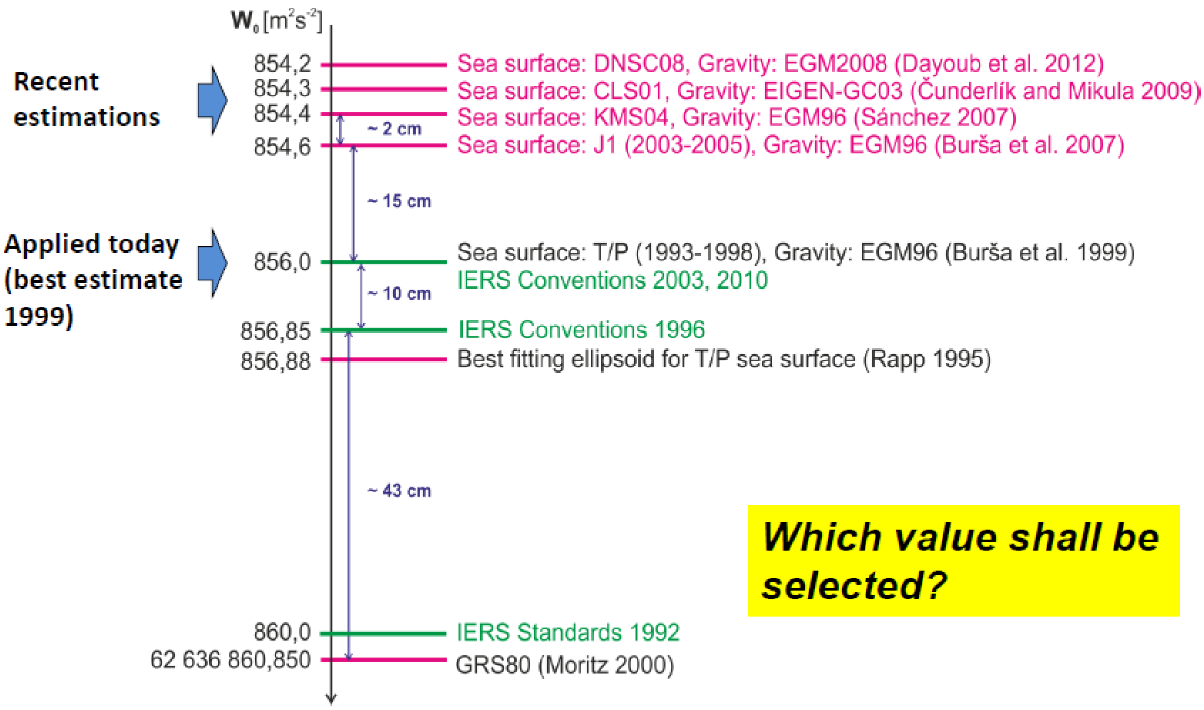

Figure 3: Some examples of W_0 values (Sanchez, 2013)

METHODOLOGY / MODEL FORMULATION

Due to lack of absolute gravity data for the study area, actual gravity is replaced in equations 7 and 8 with normal gravity since normal gravity potential accounts for approximately 99.9995% of the total potential (Jekeli, 2007). Therefore, equation 7 is re-written as equation 9

$$H_A = {(W_0 - W_A)}/{\gamma_A} \qquad (9)$$

$$\gamma = 978032.67714\left(\frac{1+0.00193185138639\, sin^2\varphi}{1-0.00669437999013\, sin^2\varphi}\right) \qquad (10)$$

Where: γ = Normal gravity

φ = Geodetic Latitude

Given the initial orthometric height of the 6 points used in this study, normal gravity values was computed at each point using equation (10) and an initial W_0 was selected. The geo-potential value for each of the 6 points (W_A) was thereafter computed using equation (9).

Since geo-potential and normal gravity values are position dependent and do not change with time, the newly computed geo-potential value for each point and their corresponding normal gravity value are then used to recomputed the orthometric height for each point using equation (9).

From Figure 3, we have adopted W_0 values that correspond to about 50cm rise in MSL in this study. The adopted values are as shown:

(Adopted) Initial W_0 Value: 62 636 860.0 m^2s^{-2}

(Adopted Final) W_0 Value: 62 636 854.0 m^2s^{-2}

GAUSS MARCOV MODEL

The classical Gauss Marcov functional model has been used in this study to determine the height implication of a 50cm sea Level variation on the VRF within the study area. Given the stochastic formulation for the Gauss Marcov in equation 11 (Helmert, 1924; Fotopulous, 2003)

$$C_v = \sigma^2{}_0 Q \qquad (11)$$

If the given set of orthometric heights of points are described as H_p and the re-computed set of orthometric height values (signifying the effect of sea level rise) described as H_n; then the resulting framework of condition equations for determining the variance-covariance of adjusted observations between both VRF's (previous heights and new heights) shall be as equation 12

$$[(H_2 - H_1) + (H_3 - H_2) + (H_1 - H_3)]_P = [(H_2 - H_1) + (H_3 - H_2) + (H_1 - H_3)]_n = 0 \quad (12)$$

Equation 12 shall be satisfied in loops through-out the entire leveling Network or Vertical Reference Frame. The Leveling Network is as shown in Figure 4

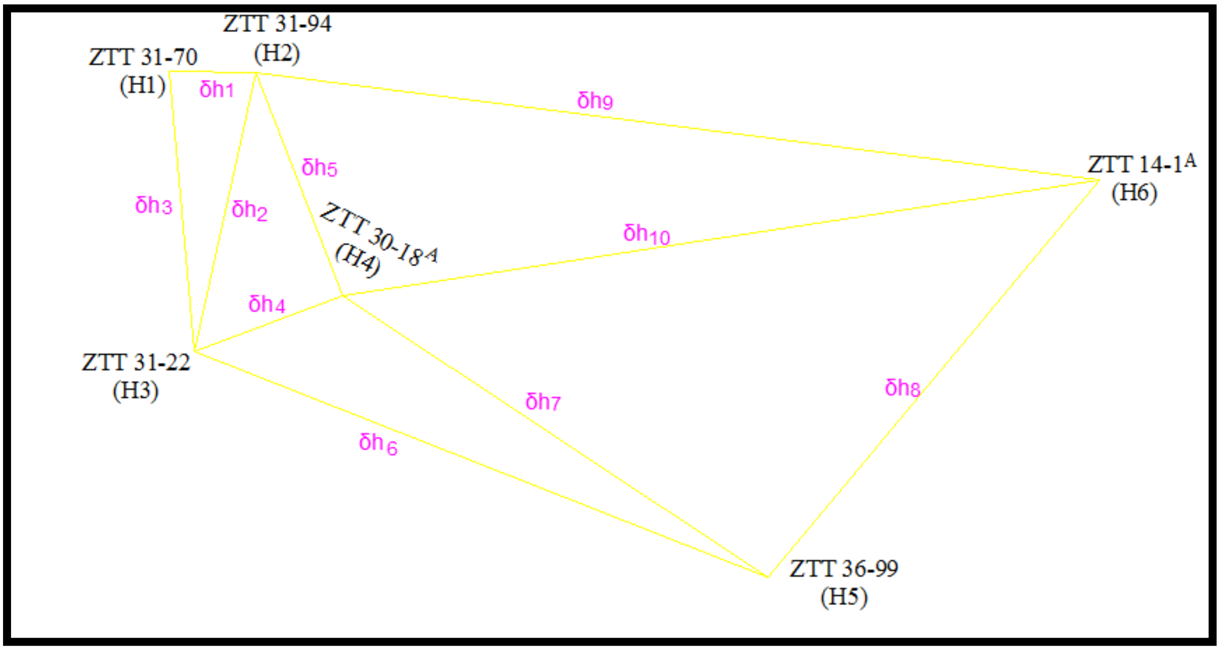

Figure 4: Leveling Network / VRF

Let

$$[H_2 - H_1]_p = \Delta h_{1_p} \quad (13a)$$

$$[H_2 - H_1]_n = \Delta h_{1_n} \quad (13b)$$

$$\therefore \Delta h_{1_p} - \Delta h_{1_n} = \delta h_1 \quad (14)$$

Equation (12) then becomes (15)

$$\delta h_1 + \delta h_2 + \delta h_3 = 0 \quad (15)$$

Eleven (11) condition equations were formed from the leveling Network following Equation (15) leading to the formulation of an 11 by 10 design matrix (B) and a uniform scaled weight matrix (P) was adopted.

Equation (11) then transforms into equations 16 - 18 (Ayeni, 1985)

$$C_v = \sigma^2{}_0 \, Q_{L^a} \tag{16}$$

$$\sigma^2{}_0 = {V^T PV}/{r} \tag{17}$$

Where:

V = Observational Residuals

r = Number of conditions

$$Q_{L^a} = P^{-1} - P^{-1} B^T M^{-1} B P^{-1} \tag{18}$$

STUDY AREA

The control points selected being part of the ZTT 1 – 39 Series covering the Central and Eastern parts of Lagos State. The information was collected from the office of the Surveyor General of Lagos State. Being a coastal state, it is mainly low lying and highly vulnerable to inundation due to MSL variations.

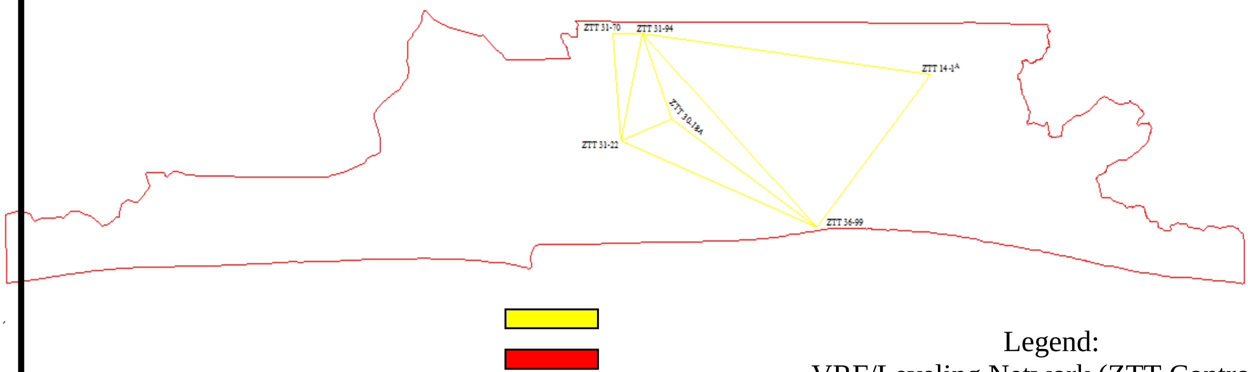

Figure 5: Study Area.

Legend:
VRF/Leveling Network (ZTT Control Series)

DATA USED

Presented in table 1 below is a summary of the data used

S/N	DATA	SOURCE	ACCURACY
1.	Initial Orthometric Heights (H_p)	Office of the Surveyor General of Lagos State – Interspatial Surveys. (6 points within the 2nd Order State-wide Controls Network)	2nd – Order Accuracy
2.	Normal Gravity	Computed using Equation (10)	N/A
3.	Station Geo-potential	Computed using Equation (9)	N/A
4.	Final Orthometric Heights (H_n)	Computed using Equation (9)	N/A

RESULTS

Presented in Table 2 are the initial and newly computed orthometric heights of the selected controls within the VRF.

Table 2: Heights of selected Control points within the VRF both before and after MSL rise.

STATn_ID	EASTINGS (m)	NORTHINGS (m)	Initial Wo (m2/s2)	Final Wo (m2/s2)	Station Potential	Initial Ht (m)	Final Ht (m)	Residual
ZTT31-70	556698.923	737215.7	62 636 860.0	62 636 854.0	17640164.58	46.002	46.001970	-0.000030
ZTT31-94	561055.077	737189.968	66 636 860.0	66 636 854.0	31558211.47	31.773	31.772977	-0.000023
ZTT31-22	557938.52	723141.589	63 636 860.0	63 636 854.0	58366291.08	4.366	4.365995	-0.000005
ZTT30-18A	565399.781	725965.523	64 636 860.0	64 636 854.0	45288512.01	17.736	17.735998	-0.000002
ZTT36-99	586740.558	711778.59	65 636 860.0	65 636 854.0	58010263.45	4.730	4.729993	-0.000007
ZTT14-1A	603337.562	731753.378	67 636 860.0	67 636 854.0	56825701.69	5.941	5.940993	-0.000007

A graphical plot of the residuals obtained is presented in Figure 6 – 7 to provide a brief impression of the spatial relationship between MSL variation and heights within the study area.

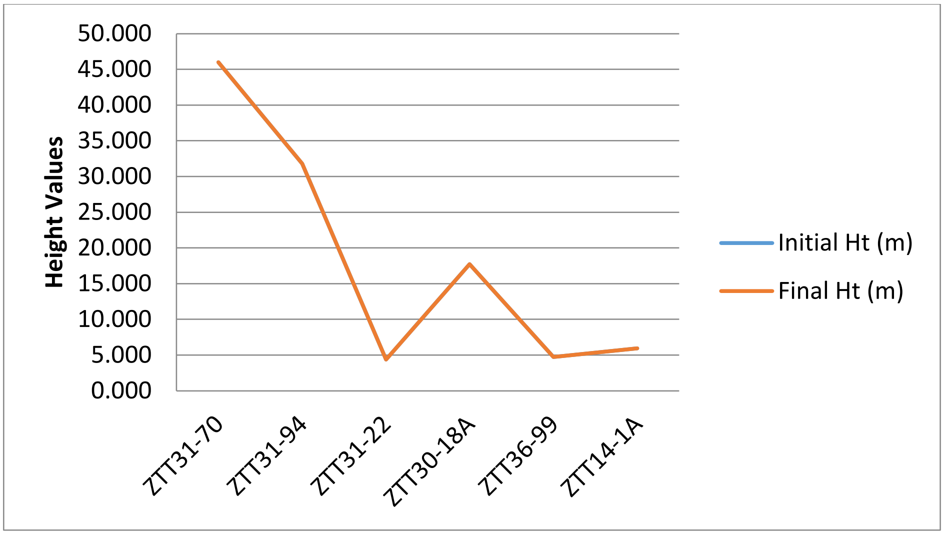

Figure 6: An Overlay Plot of heights of Selected Controls Points before and after MSL rise

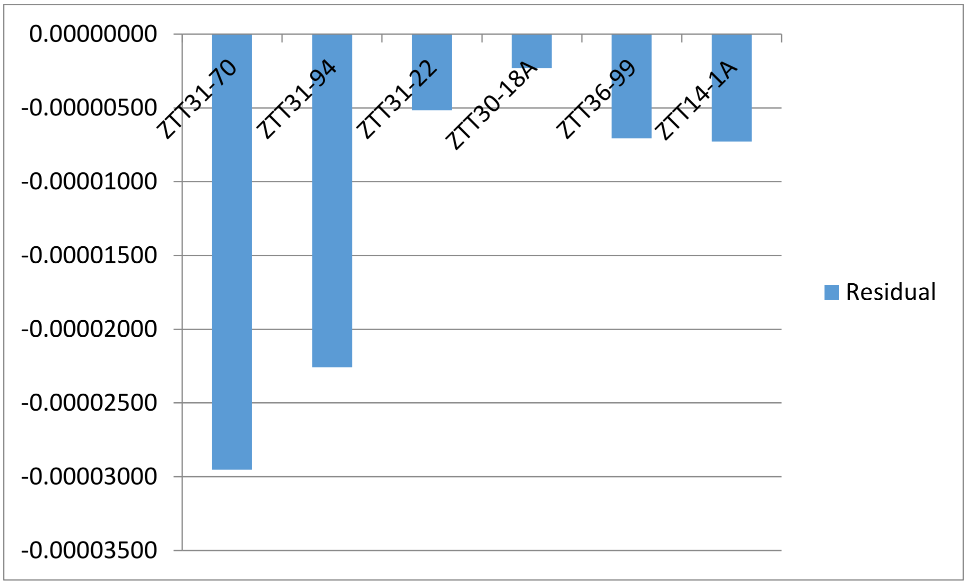

Figure 7: Plot of spatial pattern of the magnitude of residual between Heights of selected control points before and after MSL rise.

The matrix of residuals of the condition equation (V) and the design Matrix (B) are also presented below.

$$V = \begin{pmatrix} -0.00000064 \\ +0.00000247 \\ +0.00000312 \\ -0.00000328 \\ -0.00000016 \\ -0.00000080 \\ -0.00000293 \\ +0.00000528 \\ +0.00000822 \\ +0.00000494 \\ +0.00000281 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

The Variance-Covariance matrix of adjusted observations after application of the gauss markov functional model therefore becomes:

$C_v =$

-2.83E-10	-5.96E-11	2.39E-10	2.98E-11	2.68E-10	-2.98E-11	5.96E-11	5.96E-11	0.00E+00	2.98E-11	1.19E-10
-5.96E-11	-3.88E-10	-3.43E-10	7.45E-11	-2.68E-10	-3.28E-10	-2.24E-10	-1.19E-10	1.04E-10	1.79E-10	2.83E-10
2.39E-10	-3.43E-10	-5.67E-10	4.47E-11	-5.37E-10	-2.98E-10	-2.83E-10	-1.79E-10	1.04E-10	1.49E-10	1.64E-10
2.98E-11	7.45E-11	4.47E-11	-3.73E-10	-3.43E-10	-3.13E-10	-2.09E-10	-1.04E-10	1.04E-10	-2.83E-10	-1.79E-10
2.68E-10	-2.68E-10	-5.37E-10	-3.43E-10	-8.65E-10	-6.11E-10	-4.92E-10	-2.83E-10	2.09E-10	-1.34E-10	-1.49E-11
-2.98E-11	-3.28E-10	-2.98E-10	-3.13E-10	-6.11E-10	-6.26E-10	-4.32E-10	-2.24E-10	2.09E-10	-1.04E-10	1.04E-10
5.96E-11	-2.24E-10	-2.83E-10	-2.09E-10	-4.92E-10	-4.32E-10	-5.37E-10	-2.83E-10	2.68E-10	5.96E-11	-5.96E-11
5.96E-11	-1.19E-10	-1.79E-10	-1.04E-10	-2.83E-10	-2.24E-10	-2.83E-10	-3.28E-10	-5.96E-11	-1.64E-10	-2.24E-10
0.00E+00	1.04E-10	1.04E-10	1.04E-10	2.09E-10	2.09E-10	2.68E-10	-5.96E-11	-3.13E-10	-2.24E-10	-1.64E-10
2.98E-11	1.79E-10	1.49E-10	-2.83E-10	-1.34E-10	-1.04E-10	5.96E-11	-1.64E-10	-2.24E-10	-4.92E-10	-3.43E-10
1.19E-10	2.83E-10	1.64E-10	-1.79E-10	-1.49E-11	1.04E-10	-5.96E-11	-2.24E-10	-1.64E-10	-3.43E-10	-4.92E-10

A summary of the results is presented in table 3 (a & b):

Table 3(a): Summary of Results

Measured Parameter (Among Baselines)	**Value (m)**
Standard Deviation	∓ 0.000000000015
RMSE	0.00000000000299
Variances along bases $\begin{bmatrix} \delta h_1 \\ \delta h_2 \\ \delta h_3 \\ \delta h_4 \\ \delta h_5 \\ \delta h_6 \\ \delta h_7 \\ \delta h_8 \\ \delta h_9 \\ \delta h_{10} \end{bmatrix}$	$\begin{bmatrix} -0.000000000283 \\ -0.000000000388 \\ -0.000000000567 \\ -0.000000000373 \\ -0.000000000865 \\ -0.000000000626 \\ -0.000000000537 \\ -0.000000000328 \\ -0.000000000313 \\ -0.000000000492 \end{bmatrix}$

Table 3(b): Summary of Results (Contd)

Measured Parameter (Among Baselines)	**Values (m)**
Minimum Deviation along base lines (δh_1)	-0.000000000283
Maximum Deviation along base lines (δh_5)	-0.000000000865

DISCUSSION OF RESULTS

From Table 1 and further supported by Figure 7, all residuals in the heights are negative. This trend is expected as a rise in MSL will lead to inundation of land masses. As seen however in Figure 7, the pattern of the inundation is irregular; further proving the non-parallelism of the equipotential surfaces. The control points ZTT36-99 and ZTT 14-1A which both lie close to the Lagoon (about same equipotential surface) experienced similar amount of inundation as a further evidence to the non-parallelism of the equi-potential surfaces and dependence of heights on geo-potential differences.

ZTT31-70 and ZTT31-94 which are located far from the shoreline/coast area however experienced a greater inundation than some coast-line controls; signifying a possibility of greater effects of MSL variation on interland points than in coastal areas.

The statistical analysis performed as summarized in Table 3 shows a standard deviation of ∓ 0.000000000015 along the base lines; while the variances experienced along each baseline is also as expressed in Table 3. The standard deviation implies an unnoticeable change in the vertical reference frame as a result of sea level rise within the study area. It can however be seen from the gauss marcov functional statistical model that the maximum effect of the MSL variation in the VRF is noticeable along the baseline δh_5 and minimum variation along baseline δh_1.

It should be noted that the variation observed in results could have been different if actual gravity observations were taken on the selected control stations rather than resorting to the normal gravity (as used in this research); although the expected differences in both cases should not be large.

CONCLUSION

It therefore can be concluded that the variation of the MSL has a minimal effect (∓ 0.000000000015 m) on the VRF within the study area (Miyahara, 2015). As such, the changing MSL does not pose any danger to the suitability of the existing vertical controls within the study area.

It can also be verified from the gauss marcov functional model analysis that the effect of MSL variation on VRF is non-linear i.e does not dependent on distances between baselines.

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