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Application of wavelet transform in spectrum sensing for cognitive radio: A survey

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ABSTRACT

Spectrum sensing is an important technological requirement in the quest to realize dynamic spectrum access (DSA) in today's wireless world. Cognitive radio (CR) has been identified as an enabling technology that will considerably mitigate the effect of spectrum underutilization and cushion spectrum scarcity. But for this to happen, fast and accurate sensing technique must be developed. Quite a number of spectrum sensing techniques are available in literature, but these are not without inherent short comings. Recently, applications of wavelet techniques for spectrum sensing is receiving attention in the research community, this is attributed to its unique ability to operate both in the time and frequency domains and its suitability for wideband sensing. This paper takes a general look at the applications of wavelets in solving problems in science and engineering and then focused on its recent applications in spectrum sensing. Besides discussing the general spectrum sensing techniques in literature, the paper also discussed wavelet-based spectrum sensing, and its variants; pointing out the merits and limitations of each. It noted that, like any other sensing technique, wavelet-based technique has its strengths and weaknesses, hence, the advantages and disadvantages of this technique are also highlighted. Also, wavelet techniques in spectrum sensing was variously compared with existing wavelet sensing techniques; other spectrum sensing techniques; and existing wideband sensing techniques. Emerging research trends involving wavelets in wireless communications systems design are discussed while some challenges posed by wavelet techniques are mentioned. The paper is intended to provide necessary information and serve as a pointer to relevant literatures for researchers seeking information about wavelets and their applications in science and engineering and particularly in spectrum sensing for CR.

1. Introduction

The electromagnetic radio spectrum is a unique natural resource which can be reused over and over by both transmitters and receivers that are licensed by regulatory bodies and also the unlicensed radios based on a given plan. This property of reusability makes the radio spectrum a high-valued commodity with the capability of allowing a large number of concurrent users derive maximum benefit from it as long as good management and careful planning are observed. However, with increasing wireless applications, there appears to be a scarcity of radio spectrum. Investigation has revealed that this scarcity is caused by poor utilization of allocated radio spectrum by primary users of such spectrum, causing wastage and making the spectrum appear scarce [1,2]. The second cause of the scarcity of radio spectrum is the convergence of wireless communications and computing systems, which include entertainment systems, information systems, and multimedia systems. This convergence has increased the competition for available wireless bandwidth, hence the scarcity [3].

Cognitive radio (CR) is a technological innovation that is envisaged to provide solution to the problem of static spectrum allocation and thus enable dynamic spectrum access and management in wireless communication systems. Its core objective is the provision of spectrum through dynamic and opportunistic access of the primary user spectrum so long as there is no harmful interference to the primary user; in so doing it helps to guarantee efficient utilization of the radio spectrum. CR devices are developed with an in-built capacity to sense and predict the environment in which they operate. Apart from sensing, CR is also characterized by three other distinct operations [4,5], these operations include spectrum decision, spectrum mobility and spectrum sharing. Fig. 1 [5] shows the relationship between these four operations.

The relationship between these four operations [4,5] is a workflow process which a CR must go through in order to achieve its

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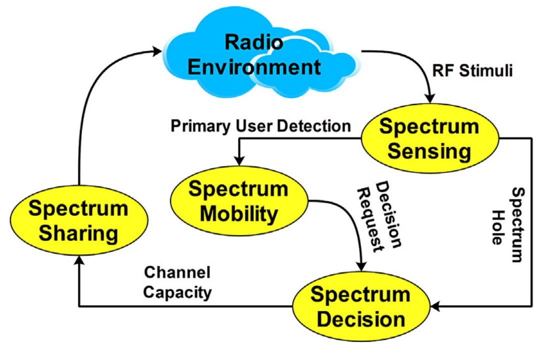

Fig. 1. Cognitive cycle of a cognitive radio.

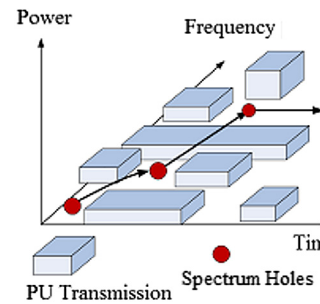

Fig. 2. Spectrum band with spectrum holes or white spaces.

core objective. The four operations are distinct from each other, but they also rely on each other to achieve the desired performance for a CR network. The first operation the CR performs is spectrum sensing in order to detect whether or not a radio spectrum is present; if it is, then the spectrum sensing operation determines how much of the spectrum is available for utilization by estimating the frequencies that are vacant. The spectrum mobility and decision operations both tap from the output of the spectrum sensing operation to make a transition/decision, as the case maybe, to better frequency bands; in the course of this transition, it also ensures there is no interruption in communication. The spectrum decision operation decides on how best to use a vacant spectrum and how long a secondary user (SU) can utilize the vacant frequency. The spectrum sharing operation ensures that the allocation of vacant frequencies is done fairly among secondary users. From Fig. 1, it can be seen that all the four distinct operations are critically important for a successful CR operation; if any of the four operations fails, then CR functionality will not be attained. For example, the failure of the spectrum sensing operation, which is the first operation performed by the CR, would imply the basic objective of dynamic spectrum access will not be achieved.

In the CR cycle, spectrum sensing operation has received a lot of attention from researchers probably owing to the fact that it is the first in the series of functions a CR must perform to achieve dynamic spectrum access. Recent researches into the development of techniques for spectrum sensing includes the application of particle swarm optimization to address the trade-off between sensing time and throughput [6]. Kernel least mean square (KLMS) algorithm has been applied to achieve spectrum sensing in which each SU makes a binary decision on its local sensing using energy detection [7]. Analytical and learning-based spectrum sensing over channels with both fading and shadowing was proposed in [8]. The technique involves the analysis of the performance of an energy detector under local and collaborative scenarios in unreliable environments dominated by multipath fading and shadowing effects.

In a CR, the spectrum sensing operation is achieved through the identification of spectrum holes or white spaces in a given radio spectrum. The spectrum holes [9] are a band of frequencies allocated to a licensed user, but at a particular instance of the space–time continuum, such band is unutilized by the primary user. Fig. 2 [10] depicts a spectrum band with spectrum holes in it.

Quite a number of techniques exist in the identification of spectrum holes or spectrum sensing as will be discussed in the following sections. However, this paper will pay a special attention to wavelet-based spectrum sensing for two major reasons: (a) it can sense a wide range of frequencies without the non-cost-effective deployment of multiple bandpass filters (BPF) and (b) its ability to resolve signal components and features in both time domain and frequency domain. It is anticipated that wideband sensing will become important to CRs of the future, which will need a better opportunity for spectral hole identification. Therefore, the main objective of this paper is to conduct a comprehensive survey on the application of wavelets in spectrum sensing in terms of its strengths, weaknesses, and current research challenges in its application. The paper also highlights the properties of wavelets which makes them particularly suitable for spectrum sensing, and the advantages and disadvantages of the application of the three variants of wavelets in spectrum sensing. Another objective for this survey is to highlight how well wavelet-based spectrum sensing performs when compared with other spectrum sensing techniques especially in wideband sensing.

The rest of the paper is organized as follows. Section 2 explores different spectrum sensing techniques in cognitive radio, highlighting the characteristic equations of each. Section 3 presents the types of wavelets available in literature, and their classification based on the type of properties each possesses. Section 4 throws light on the application of wavelets in different fields of science and engineering. Section 5 explores wavelet based spectrum sensing, highlighting the three variants of wavelet-based spectrum sensing, with their advantages and disadvantages. Section 6 presents an example of the performance of a wavelet system based on the choice made in terms of the mother wavelet. Section 7 presents three comparisons, the first being on previous works on wavelet based spectrum sensing; the second being a comparison of wavelet based spectrum sensing with other spectrum sensing techniques; and the third is a comparison between wavelet and other wideband sensing techniques. Section 8 gives the generalized advantages and disadvantages of wavelet-based spectrum sensing technique. Section 9 reviews current research trends in wavelet-based spectrum sensing. Section 10 highlights research challenges in wavelet based spectrum sensing. Finally, the paper gives a conclusion in Section 11.

2. Spectrum sensing techniques in cognitive radio

There are quite a number of techniques that are used in spectrum sensing, some of which include matched filter detection, energy detection, waveform-based detection, Cyclostationary feature detection, and wavelet-based detection. In this section, we analyze these spectrum sensing techniques.

2.1. Matched filter detection

The matched filter detection is used in the detection of primary user as long as there is prior knowledge of the transmitted signal. This technique achieves maximum signal-to-noise ratio (SNR) in the presence of additive white Gaussian noise (AWGN) [11]. Fig. 3 [11] shows the block diagram of spectrum sensing using matched filter detection technique.

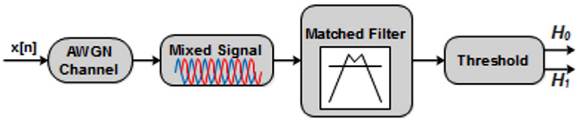

Fig. 3. Spectrum sensing using matched filter detection.

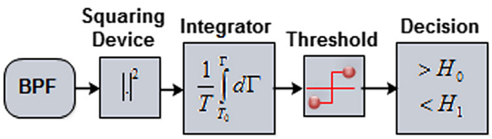

Fig. 4. Energy detector.

As shown in Fig. 3, there are two possible outcomes from the thresholding block, one is H_0 which is the NULL hypothesis, and the other is H_1 which is the alternate hypothesis. These hypotheses are mathematically represented [11], as a distribution of a test statistic T:

$$T \sim \begin{cases} H_0 : x[n] = w[n], n = 0, 1, \ldots, N-1 \\ H_1 : x[n] = s[n] + w[n], n = 0, 1, \ldots, N-1 \end{cases} \tag{1}$$

where $x[n]$ is the received signal, $w[n]$ is AWGN with zero mean and variance σ^2, and $s[n]$ is the source signal which is known.

The probability of false alarm and probability of detection for the matched filter are expressed as:

$$\begin{aligned} P_{fa} &= P\left(T > \gamma'/H_0\right) = Q\left(\frac{\gamma'}{\sqrt{\sigma^2\varepsilon}}\right) \\ P_d &= P\left(T > \gamma'/H_1\right) = Q\left(\frac{\gamma' - \varepsilon}{\sqrt{\sigma^2\varepsilon}}\right) \end{aligned} \tag{2}$$

where γ' is the threshold, and it is equivalent to $Q^{-1}\left(P_{fa}\right)\sqrt{\sigma^2\varepsilon}$; $Q(.)$ is the Gaussian Complementary Density Function (CDF).

The matched filter detection has the following advantages: it requires low sensing time, and it has high processing gain achieved in a small amount of time. The disadvantage of the matched filter detection includes the requirement for perfect knowledge of the primary user signal, and the dedication of a receiver for each primary user signal.

2.2. Energy detection

Energy detection technique requires no prior knowledge of the primary user signal to achieve spectrum sensing [12]. This makes the technique achieve low computational and implementation complexities. The operation of the energy detection method as shown in Fig. 4 is such that a received signal is passed through a bandpass filter which narrows the measurement of information in the received signal to the band of interest. The signal is then passed through a squaring device in which every term in the signal is squared in order to measure the energy by the integrator block [13]. A threshold device is then applied to the computed signal energy to determine whether or not a primary user is present.

Given the binary hypothesis in (1), the energy over N samples is computed as [14]:

$$y = \frac{1}{N}\sum_{n=1}^{N} |x[n]|^2. \tag{3}$$

If y is greater than a predetermined threshold λ, the receiver selects the hypothesis H_1, otherwise hypothesis H_0 is selected i.e.

$$\begin{aligned} P_{fa} &= P\left(y < \lambda/H_0\right) \\ P_d &= P\left(y \geq \lambda/H_1\right). \end{aligned} \tag{4}$$

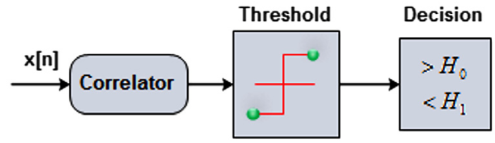

Fig. 5. Waveform-based detection.

The two major advantages of the energy detection methods are: low computational as well as implementation cost, and non-requirement of prior knowledge of the primary user signal. The disadvantages of this technique include: not being useful for direct sequence and frequency hopping signals, and poor performance at low signal-to-noise ratio.

2.3. Waveform-based detection

Waveform-based detection is usually applied when the system knows the pattern of the incoming signal, which is usually used for synchronization purposes. The patterns which are also called preambles, are known sequences transmitted before each burst [15]. As shown in Fig. 5, sensing is achieved by performing a correlation between the received signal and a known copy of itself [16].

For a received signal $x[n]$ defined in (1), the correlation block in Fig. 5 involves the use of waveform-based sensing metric which is given as [16]:

$$M = \Re e\left[\sum_{n=1}^{N} x(n)\, s^*(n)\right] \tag{5}$$

where $*$ is conjugation operation. When a primary user is absent, the metric becomes:

$$M = \Re e\left[\sum_{n=1}^{N} w(n)\, s^*(n)\right]. \tag{6}$$

In the presence of a primary user, the metric becomes:

$$M = \sum_{n=1}^{N} |s(n)|^2 + \Re e\left[\sum_{n=1}^{N} w(n) s^*(n)\right]. \tag{7}$$

The decision on whether a primary user signal is present or not is made by comparing the decision metric against a fixed threshold.

The advantages of waveform-based detection include low power consumption, and medium complexity in implementation. The disadvantages of this technique include the requirement of prior knowledge of the primary user signal, and low sensing accuracy.

2.4. Cyclostationary detection

Cyclostationary detection deals with the periodicity of periodic signals. This technique uses spectral correlation function to detect the periodicity of the signal from the primary user [17]. Sinusoidal carriers, pulse trains, spreading code, and other features are exploited by Cyclostationary detection technique in the detection of periodicity [18]. The procedure by which Cyclostationary detection achieves spectrum sensing is shown in Fig. 6.

For the received signal $x[n]$, the cyclic spectral density (CSD) function is [16]:

$$S(f, \alpha) = \sum_{\tau=-\infty}^{\infty} R_x^{\alpha}(\tau)\, e^{-j2\pi f \tau} \tag{8}$$

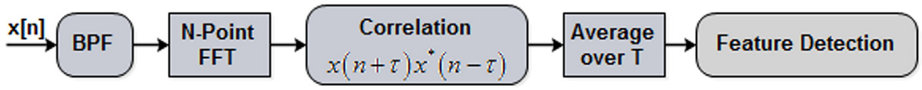

Fig. 6. Cyclostationary detection.

where $R_x^{\alpha}(\tau) = E\left[x(n+\tau)\, x^*(n-\tau)\, e^{j2\pi\alpha n}\right]$ is the autocorrelation function, and α is the cyclic frequency.

For a given white Gaussian noise present at the input of the detector defined as $\psi = \left|\hat{\rho}x^{\alpha}\right|^2$, the detection statistic will have a cumulative distribution function expressed as [19]:

$$F_{\psi}(\psi \mid H_0) = 1 - (1-\psi)^{N-1} \tag{9}$$

where $\hat{\rho}x^{\alpha}$ is the spectral autocoherence function, N is number of samples, H_0 is the NULL hypothesis i.e. when the primary user is absent. Using the cumulative distribution function, the probability of false alarm P_{FA} can be determined for the H_0 case. If the given threshold is d, then:

$$P_{FA} = \Pr(\psi > d \mid H_0) = (1-d)^{N-1}. \tag{10}$$

Cyclostationary detection has the advantage of performing well at low signal-to-noise ratio, and being very robust to noise. However, the disadvantages of this technique include high complexity in computation, and long observation time in order to achieve desired level of performance.

2.5. Wavelet-based detection

The wavelet-based detection technique is unique from other techniques in the sense that while the other techniques operate only in the frequency domain, the wavelet-based technique operates in both the frequency and time domains [20]. The time-frequency characteristics makes a wavelet peculiar in the sense that it is able to localize a signal in terms of frequency and time through the use of its scaling and wavelet functions. Wavelets are implemented using filter banks with highly desirable properties for spectrum sensing. These properties include orthonormality, paraunitary condition, and k-regularity.

2.5.1. Orthonormality

This is a property in which the bases of a wavelet in a particular application are all mutually orthogonal to each other, and are all of unit lengths. This is a very attractive property for spectrum sensing based on wavelets because the signal being sensed can be decomposed into separate, independent, and non-interacting parts which can be processed individually [21]. Any wireless communication system built on wavelets enjoys great spectral efficiency because of orthonormality of wavelets, which eliminates the need for guard bands and cyclic prefixes as in Fourier transforms. The orthonormal property of a wavelet has the form [21–23]:

$$\psi_{j,k} = |\det A|^{j/2}\psi\left(A^j x - k\right), j \in \mathbb{Z}, k \in \mathbb{Z}^n \tag{11}$$

where A is an expansive matrix, $\psi_{j,k}^{i} : i = 1, \ldots, l, j \in \mathbb{Z}, k \in \mathbb{Z}^n$ is an orthonormal basis for $L^2(\mathbb{R}^n)$.

2.5.2. Paraunitary condition

The paraunitary condition guarantees the generation of orthonormal bases in a wavelet application. Its most attractive property is that it enables perfect reconstruction in a wavelet-based transmission system, which is very important in developing highly accurate spectrum sensing techniques based on wavelets [24].

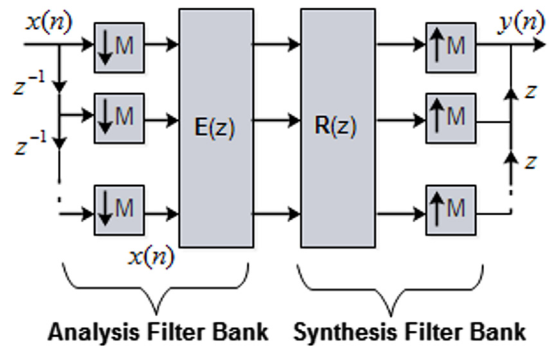

Fig. 7. Filter bank implementation of a wavelet system.

Given the filter bank implementation of a wavelet shown in Fig. 7 [24], paraunitary condition exists when [24,25]:

$$\bar{\mathbf{E}}(z)\mathbf{E}(z) = \mathbf{I} \tag{12}$$

where $\mathbf{E}(z) = \mathbf{E}\left({}^{1}/{}_{z^*}\right)$.

2.5.3. K-regularity

Regularity gives a measure of the smoothness of wavelet function $\psi(t)$ and scaling function $\phi(t)$ in both time and frequency. The regularity of the scaling function $\phi(t)$ is the maximum value of r such that [26,27]:

$$|\Phi(\Omega)| \le \frac{c}{(1+|\Omega|)^{r+1}}, \Omega \in R \tag{13}$$

which implies that $\phi(t)$ is said to be m-times continuously differentiable with $r \ge m$. In (13), the decay of $\Phi(\Omega)$ determines the regularity or smoothness of $\phi(t)$ and $\psi(t)$.

Regularity also provides the means by which compactly supported orthonormal wavelet bases can be generated. Regularity of wavelets is very attractive in spectrum sensing because it guarantees the accuracy of a spectrum sensing technique built with wavelets because of the smoothness factor.

3. Types and classification of wavelets

Wavelets, as shown in Fig. 8, are categorized into two groups [28]. The first category are wavelets defined by mathematical expressions which are continuous and infinite in nature [28]. They are also called crude wavelets, and for them to be useful in any signal processing system, they have to be converted to wavelet filters with a finite number of discrete points. A typical example of this type of wavelet is the Mexican hat wavelet shown in Fig. 8.

The second category of wavelets are wavelets that start out as filters having two points of definition in the initial state [28]. Through the interpolation and extrapolation of more points from the initial two points, these wavelets generate an approximation of a continuous wavelet. A typical example of this type of wavelet is the Daubechies 4 (Db4) wavelet shown in Fig. 8 [28,29].

Wavelets have an extensive application in different fields of science and engineering. They have different properties, which make them most suitable for a particular area of application. The biorthogonal wavelets have wavelet and scaling functions that are

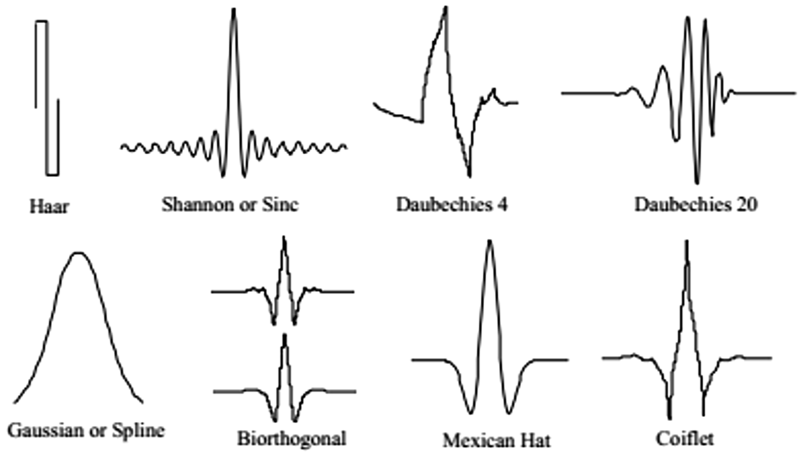

Fig. 8. Types of wavelets.

symmetric. This property makes the biorthogonal wavelets ideal for human vision perception since the human vision system is more tolerant to symmetric errors than asymmetric error [30]. The asymmetric errors are usually found in wavelet transforms like Haar, symlet, and Daubechies.

The Shannon wavelets have sharp localization and infinite support in frequency. This makes them ideal in the identification of events with specific frequencies [31,32]. The Haar wavelets have two important properties that make them ideal for edge detection. The first is that Haar wavelets conserve the energies of signals [33]. The second is that Haar wavelets are exactly reversible without edge effects [34]; this property is very important in edge detection.

A high number of vanishing moments coupled with the almost interpolating and linear phase low-pass within a given passband makes Coiflet wavelets ideal in numerical analysis and is hence able to deal with fractals in signal processing [35].

The Daubechies wavelets have good regularity, multiple vanishing moments, and approximate symmetry. These features are highly desirable in signal processing and data compression applications because the wavelets increase in smoothness as the vanishing moments increase [36,37]. An example illustrating Daubechies suitability for spectrum sensing is provided in Section 6.

The Morlet wavelet is often applied in environmentally-oriented signals like earthquake vibrations. The ability of the Morlet wavelet to capture both magnitude and phase characteristics of a signal while retaining the temporal nature of the signal makes it attractive in this area of application [38–40].

The Mexican hat wavelets have good localization of patch and gap events. They also have good resemblance of MUAPs (motor unit action potential) which makes them widely applicable in EMG (electromyography) [41,42]. The foregoing classification of wavelets based on their functions is summarized in Table 1.

4. Application of wavelets

Wavelets have found use in different fields of human endeavor owing to their flexibility and ability to localize events. It functions by detecting the edges of the object of interest. In the study of earthquakes, wavelets have been extensively applied [62–64] to analyze the impact of earthquake on buildings, and also analyze the behavior of volcanic mountains; the analysis is usually done by measuring the energy contents of decomposition levels for every monitoring points in a building, based on wavelet energy content analysis. In the field of medical sciences, wavelets are used extensively in classification of tissues, and production of medical images [65–67]. In the detection of faults in machines, wavelets are used to identify and localize faults in rotating machine components [68,69]. Wavelets have also found use in nutrition, especially in the study of consumption patterns by humans [70]. In signal processing and analysis, wavelets are the natural tool of choice for the study of signals with abrupt and localized changes; this is because wavelets can be shifted in time and stretched in frequency. As a result of this dual property, wavelets are able to detect edges in both frequency and time for any given signal by identifying the time and frequency location of various levels of energy in the signal [71,72]. Wavelets have also found use in stock market prediction, especially in the study and prediction of stock market trading [73].

Of recent, wavelets have been applied in spectrum sensing, with a promising prospect of bringing on a new and better approach to spectrum sensing especially when wide band signals are involved [74–76]. The next section will discuss wavelet application in this area.

5. Wavelet-based spectrum sensing

Wavelets are signals of finite duration. They differ theoretically from sinusoids in the fact that while sinusoids stretch from $-\infty$ to ∞, wavelets have a finite starting and terminal points. In wavelet-based spectrum sensing, the assumption is that the CR system receives a signal which is occupying N spectrum bands, and the CR has to detect the PSD (power spectrum density) levels and the frequency positions of each band. Fig. 9 [77] shows a spectrum band located between f_0 and f_N, with their respective sub-band frequencies located at $f_0 < f_1 < \cdots < f_n$. The nth band shown in Fig. 9 is defined by [78] as:

$$B_n : \{f \in B_n : f_{n-1} \leq f < f_n\}, n = 1, 2, \ldots, N. \tag{14}$$

A CR system receiving an input signal, computes its PSD by:

$$S_r(f) = \sum_{n=1}^{N} \alpha_n^2 S_n(f) + S_w(f), f \in [f_0, f_N]. \tag{15}$$

Application of wavelet technique to spectrum sensing currently has three variants. These include the continuous wavelet technique, discrete wavelet technique and discrete wavelet packet technique. These are briefly discussed as follows.

5.1. Continuous wavelet transform-based spectrum sensing

The continuous wavelet (CWT) based spectrum sensing [77] measures the similarity between a signal and an analyzing function

Table 1
Classification of wavelets.

Area(s) of application	Type of wavelet
Human vision perception	Biorthogonal wavelets [43,44]
Identification of events with specific frequencies	Shannon wavelets [45,46]
Edge detection and reconstruction of binary pulses	Haar wavelets [47–49]
Fractals—data with self-similarities	Coiflets wavelets [50–53]
Signal processing and data compression	Daubechies wavelets [54–57]
Vibration and sound	Gaussian and Morlet wavelets [58,59]
Biomedical signal analysis	Mexican hat wavelets [60,61]

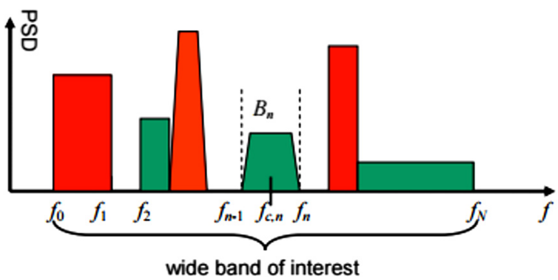

Fig. 9. *N* frequency bands with piecewise smooth PSD.

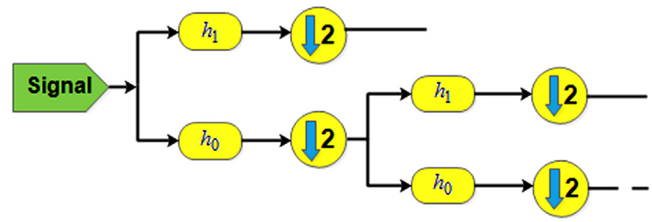

Fig. 10. DWT filter bank.

by using inner coefficients. There are two approaches to achieving spectrum sensing using continuous wavelet: multi-scale product, and multi-scale sum. The multi-scale product technique involves the determination of discontinuities in the PSD of a signal by taking the multi-scale and wavelet transforms, and then estimating the edges given in [77,79] as:

$$p(f) = \prod_{j=1}^{J} W'_{s=2^j} S_r(f) \tag{16}$$

where J is the upper limit for scale j, the index of the scaling function; $S_r(f)$ the PSD of the received signal, $W'_{s=2^j}$ the first order derivative at scale $s = 2^j$, and $p(f)$ is the multiscale product. This computation is done under an assumption that discontinuities that exist in the PSD represent the spectral boundaries. For each sub-band, the energy is calculated to determine the spectrum occupancy.

The multi-scale sum technique [80] relies on the fact that different signals have different cross scales information at dissimilar scales. This implies that the wavelet transforms at different scales contain information about the Lipschitz exponent at sharp variation points. Hence multi-scale sum technique is used to preserve information about the signals at all scales, and also avoid attenuation. For a CWT, the multi-scale sum at the jth dyadic scale is given as:

$$X_j S_r(f) = \sum_{j=1}^{J} W_{2^j} S_r(f). \tag{17}$$

Table 2 shows some of the advantages and disadvantages of the continuous wavelet transform-based spectrum sensing technique.

5.2. Discrete wavelet transform (DWT) based spectrum sensing

The discrete wavelet transform (DWT) decomposes an input signal $x[m]$ to obtain a coarse and detail information. The decomposition which enables the DWT to analyze a signal at different frequency bands and resolution, is achieved through successive highpass and lowpass filtering of the time-frequency domain signal [81,82]. Mathematically, this can be expressed as:

$$y_{high}[k] = \sum_m x[m]h_1[2k - m] \tag{18}$$

$$y_{low}[k] = \sum_m x[m]h_0[2k - m] \tag{19}$$

$y_{high}[k]$ is the highpass filter output and $y_{low}[k]$ is the lowpass filter output; h_0 is the lowpass filter coefficient, and h_1 is the highpass filter coefficient.

The outputs are obtained after filtering and down sampling by 2. To achieve spectrum sensing, the DWT upon decomposition of the signal, obtains the scaling (coarse) coefficients, and wavelet (detail) coefficients of the signal. The signal power in these coefficients is calculated and compared against a predetermined threshold. Mathematically, the scaling (c_j) and wavelet (d_j) coefficients are represented in [83,84] as:

$$c_j(k) = \sum_m h_0(m - 2k)c_{j+1}(m) \tag{20}$$

$$d_j(k) = \sum_m h_1(m - 2k)c_{j+1}(m). \tag{21}$$

Fig. 10 [85] shows the DWT filter bank used in the implementation of the DWT.

The discrete wavelet transform-based spectrum sensing technique has its merits and demerits, these are summarized in Table 3.

5.3. Discrete wavelet packet transform (DWPT) based spectrum sensing

The discrete wavelet packet transform (DWPT) operates like the discrete wavelet transform, but, with a difference which lies in the fact that the DWPT transform decomposes both the approximate space and detail space of a signal [86,87]. Fig. 11 [88] shows the structure of the DWPT. From Fig. 11, it can be seen that DWPT decomposes a signal $x[n]$ into 2^L sub-bands, where L is the level of decomposition.

To achieve spectrum sensing, the energy in each of the sub-bands is calculated [88] and compared against a threshold to determine if the sub-band is either occupied or not occupied by the primary user. The calculation of the energy in each sub-band is

Table 2
Advantages and disadvantages of continuous wavelet transform-based spectrum sensing technique.

Advantages	Disadvantages
CWT makes better localization of transients in a signal, also better characterization of oscillatory behavior.	CWT has excessive redundancy and it is computationally intensive, so it is often used in offline analysis.
CWT has fine grained resolution, which is why it is usually chosen for singularity detection.	CWT does not provide phase information for an analyzed signal.
CWT has high fidelity in signal analysis due to its fine sampling scales.	An original signal cannot be perfectly reconstructed from CWT coefficients.

Table 3
Advantages and disadvantages of discrete wavelet transform-based spectrum sensing technique.

Advantages	Disadvantages
DWT provides sparse representation of many natural signals by using a subset of coefficients to capture important features of signals.	DWT suffers from shift sensitivity whereby a shift in an input signal causes an unpredictable change in coefficients of the transforms.
DWT provides a high quality approximation of a signal thereby achieving signal compression. This is done by discarding many of its coefficients that are close to zero.	DWT has poor directionality which compromises the optimality of the DWT representation of signals in image processing.
DWT have perfect reconstruction and are computationally efficient because they have non-redundant orthonormal bases.	DWT lacks phase information which is important in the description of the amplitude and local behaviour of a function.

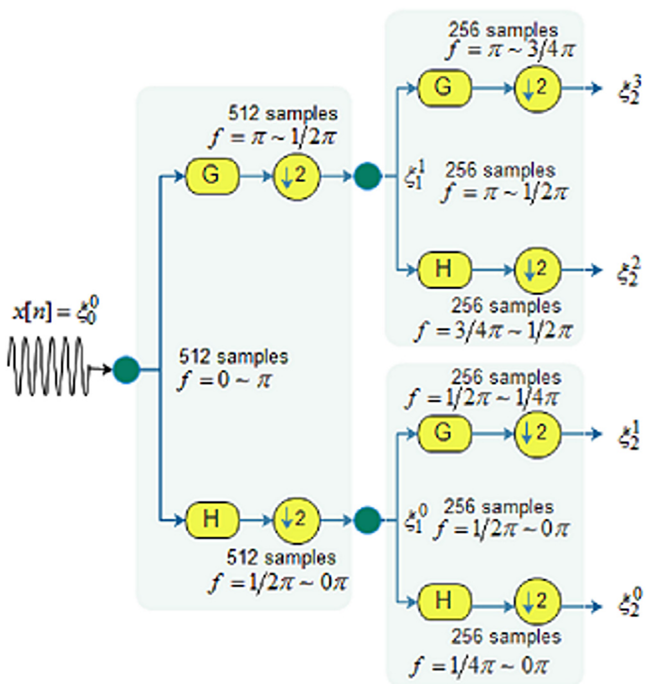

Fig. 11. DWPT structure.

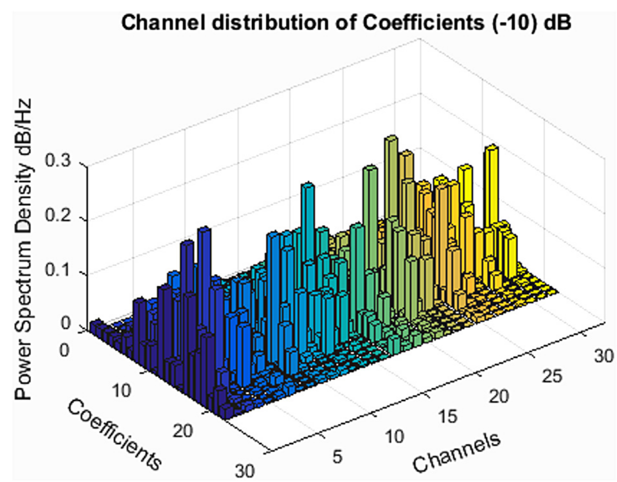

Fig. 12. Coefficients in sub-band channels at level-5 DWPT Decomposition.

given by [89,90] as:

$$E = \frac{1}{T}\int_0^T \left[\sum_{j\geq j_0}\sum_k c_{j,k}\varphi_{j,k}(t) + d_{j,k}\psi_{j,k}(t)\right]^2 dt \tag{22}$$

$$E = \frac{1}{T}\sum_{j\geq j_0}\sum_k \left(c_{j,k}^2 + d_{j,k}^2\right). \tag{23}$$

It can be seen from (23) that for a DWPT, the sum of the square of the coefficients of the approximate space and detailed space is used in the calculation of the energy in each sub-band. The discrete wavelet transform-based spectrum sensing technique also has some benefits and limitations, which are summarized in Table 4.

6. An example of performance of wavelet systems in terms of choice of mother wavelet

The choice of a mother wavelet in any design depends on the application area as shown in Table 1. For spectrum sensing in cognitive radio, the choice of a mother wavelet would be Daubechies wavelet because spectrum sensing could be viewed as signal processing. As an example, using Daubechies wavelet as a mother wavelet because of its desirable properties in signal processing and data compression in communications systems, a cognitive radio system implemented as discrete wavelet packet transform based orthogonal frequency division multiplexing (OFDM) with five levels of decomposition would yield 32 sub-band channels as shown in Fig. 12, with each channel having its own distribution of coefficients [91].

The sub-band channels in Fig. 12 with higher energy value have coefficients with higher values. This assertion is true because the energy in each sub-band channel is a function of the coefficients in that sub-band channel, as indicated by (23). Using an appropriate thresholding scheme, each sub-band channel in Fig. 12 can be determined as a vacant channel or occupied channel, thereby achieving spectrum sensing; this is shown in Fig. 13.

As seen in Fig. 13, six sub-band channels i.e. 6, 9, 18, 25, 26, and 29 were identified as vacant channels at 10 dB because the energy

Table 4
Advantages and disadvantages of discrete wavelet packet transform-based spectrum sensing technique.

Advantages	Disadvantages
DWPT has higher fidelity than DWT because it decomposes both the high and low frequency component of an input signal.	The high-pass coefficients in a DWPT oscillate around singularities of a signal.
DWPT has good universality in the adaptation of its transform to a signal without assuming any statistical property of the signal.	In speech recognition, DWPT does not respect the non-linear frequency perception phenomena.
DWPT has a high number of ways to encode a signal, by providing $2^{2^{n-1}}$ possible ways of achieving it.	

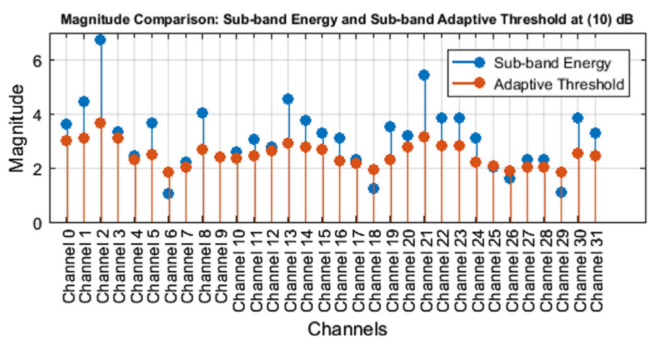

Fig. 13. Spectrum sensing using DWPT at 10 dB.

in those channel is less than the corresponding adaptive threshold for the channel. Fig. 13 also implies that the frequencies of signals in channel 6, 9, 18, 25, 26, and 29 are vacant frequencies.

7. General comparisons on wavelet based spectrum sensing

In this section we present three categories of comparisons on wavelet based spectrum sensing.

7.1. Comparative analysis of previous works on wavelet based spectrum sensing technique

There are quite a number of research in literature that has been done regarding the application of wavelets in spectrum sensing. We present in Table 5, a summary of some these research in terms of their performance metrics, strengths, and weaknesses.

7.2. Comparison of wavelet based spectrum sensing with other spectrum sensing techniques

Wavelet based spectrum sensing competes favorably with other spectrum sensing techniques. We present in Table 6, a comparison between wavelet based spectrum with other spectrum sensing techniques. The metrics for performance measurement include: speed, accuracy, preprocessing signal information, complexity, and spectral efficiency.

7.3. Comparison of wavelet based spectrum sensing with other wideband spectrum sensing techniques

Wideband spectrum sensing techniques are implemented as either Nyquist wideband sensing or sub-Nyquist wideband sensing. Both of these approaches have their advantages and disadvantages. We present in Table 7, these advantages and disadvantages.

Based on the information provided in Table 7, we make a comparison in Table 8 between wavelet based wideband sensing and other wideband sensing techniques, categorizing as either Nyquist wideband sensing or sub-Nyquist wideband sensing.

8. Advantages and disadvantages of wavelet-based spectrum sensing technique

Like any other spectrum sensing technique, wavelets have their merits and limitations. However, considering the stringent need for dynamic frequency management, the merits of wavelet sensing techniques outweighs its limitations. These advantages of wavelet-based spectrum sensing are as itemized below:

i. Significant data compression is achieved through the implementation of wavelet based algorithms. Such compression reduces the number of sensing measurements required thereby increasing the speed of estimation and reducing the communication power required for transmission. Reduction in required transmission power is a very important benefit for mobile communication devices in terms of longevity of power source.
ii. Wavelet-based estimates have the attractive property of fewer side-lobes and therefore reduced leakages than most traditional methods for sharp-featured sources.
iii. Wavelets have an excellent ability to tune the time-frequency window in such a manner as to maintain orthogonality, thereby detecting the dynamic variation of statistical parameters of any spectrum.
iv. The effect of noise and interference on signal can be minimized through the flexible design of the time-frequency window.
v. The mitigation of channel effects like inter-symbol interference (ISI) and inter-carrier interference (ICI) is achieved through the tuning capability and flexibility of wavelet bases.
vi. In OFDM applications, wavelets do not require cyclic prefix and guard bands, thus making it more efficient than Fourier transform in spectrum utilization.

The limitations of wavelet-based spectrum sensing are as itemized below:

i. Not naturally frequency selective.
ii. Requires modification to carry phase information of a signal.
iii. Requires very stringent conditions in the design of filters that implement the desired wavelet.
iv. For discrete wavelet transform, and the discrete wavelet packet transform, the higher the level of decomposition, the more complex the system becomes.

9. Current research trends in wavelet-based spectrum sensing

Another recent research trend in the application of wavelets in spectrum sensing include the development of wavelet orthogonal frequency division multiplexing (WOFDM), which is based on wavelets [115]. The research which is based on the discrete wavelet transform, showed that WOFDM has higher data rate than the traditional OFDM based on the Fourier transform because wavelet is a faster transform. The same research showed that WOFDM has better spectral efficiency than the traditional OFDM.

Table 5
Comparative analysis of previous works on wavelet based spectrum sensing.

S/N	Author(s)	Year	Title of paper	Metrics	Strength	Weakness
1	Nam-Seog, K., & Jan, M, R.	(2017)	A dual-resolution wavelet-based energy detection spectrum sensing for UWB-based cognitive radios	Measured detected power against input power	Has good energy efficiency and reduced PLL division ratio because of the dual resolution spectrum sensing approach.	The implementation of the technique in the analog domain makes it susceptible to offset errors over time. Secondly, like any other analog system, reconfigurability can be quite a challenge
2	Kang, A, S., Sharma, V., & Singh, J. S.	(2017)	Efficient spectrum sensing using discrete wavelet packet transform energy in cognitive radio.	Measured the power in sub-band channels using wpdec function.	It has low complexity because the technique is based on energy detection	The use of MAC for second stage sensing makes the technique undesirable when a high degree of decomposition is involved and at the same time meeting a tight constraint of speed.
3	Said, E., Mina, B. Abd-el-Malek., & Sara, K.	(2016)	A stationary wavelet transform approach to compressed spectrum sensing in cognitive radio	Measured subcarrier occupancy error rate against SNR, PFD against SNR, and PT against SNR	Improved performance and more accuracy in the estimation of wideband channel boundaries	Lack of down sampling of output signal results in high data rate thereby making this technique expensive to implement in hardware.
4	Arti, G. & Savitri, K.	2014	Performance evaluation of energy detection using different wavelet family for spectrum sensing in cognitive radio	Measured P_d for varying SNR	Classification of performance of different wavelet families in energy detection	Low frequency selectivity between transition bands.
5	Said El-Khamy, et al.	2013	Multiscale Hilbert transform approach to widespread sensing for CR networks	Measured P_d against P_{fa}	High immunity to increase in noise power	Loss of localization caused by the Hilbert transform
6	Omar A, et al.	2013	Noise immune spectrum sensing algorithm for CR	Measured P_d with varying SNR	Combines WPT and higher order statistics to achieve noise immune spectrum sensing.	Additional complexity is introduced by the higher order statistics
7	Raghave S, et al.	2013	Wavelet and S-transform based spectrum sensing in CR	Detection of primary users for varying continuous wavelet signal given as an input for S-transform.	Achieved measurement of phase data about a signal	Response time is increased due to combination of S-transform and wavelet transform
8	Varadharajan, E & Rajkumar, M	2012	Discrete wavelet transform based spectrum sensing in CR using Eigen Filter	Variation of signal against Eigen filter	High resolution than conventional methods.	Different approach must be used for CWT and DWT
9	Shrutika, S & Kumbhar, M.	2012	Wavelet Packet Transform based Energy Detector for Spectrum Sensing	Measured P_d for varying SNR.	Improved energy detector.	Low frequency selectivity between transition bands.
10	Shiann-Shiun et al.	2011	Wavelet-based spectrum sensing for cognitive Radios using Hilbert transform	Power spectral density measurement	Better edge detection	Used continuous wavelet transform which is computationally very intensive and adds redundancy in design.

Another current research trend is the combination of wavelet transform and the s-transform which is used to calculate the exact occupation of primary user signal and noise signal [116]. In this research, continuous wavelet technique is used to sense vacant spectrum and sub band edges, while the s-transform is used to detect frequency boundaries at low signal-to-noise ratio.

Stationary Wavelet Transform (SWT) which is derived from the discrete wavelet transform (DWT) to overcome the lack of translation-invariance of the DWT, is another emerging area of research in wavelet-based spectrum sensing. It involves the utilization of SWT to achieve improved compressed wideband spectrum sensing. This technique has the major advantage of allowing a CR to operate at sub-Nyquist rates, and thus, has fast and better edge estimation for the location of channel edges in a wideband signal [117].

The application of genetic algorithm to achieve improved reconstruction for a wavelet (DWT and DWPT) filter bank is another recent research area in wavelet-based compressive spectrum sensing. In this technique, a population of initial guesses is created and applied to a genetic algorithm in order to find the sparsest solution. This approach has been shown to yield good performance in the framework of CR spectrum sensing [118].

The application of wavelet networks in the reduction of envelope fluctuations in Wireless MAN-OFDM systems is an emerging area of research. It is shown in this area of research that the mitigation of high peak-to-average power ratio (PAPR) in OFDM is achieved by using wavelet networks in fixed WiMAX systems [119].

Statistically matched wavelet is another emerging field of research in wavelet based spectrum sensing [120]. It involves the design of wavelets based on the characteristics of the power quality event using fractional Brownian motion. The technique has shown promising results as it outperforms well-known wavelets in the detection of power quality events.

In addition to the current research trends above, a promising area of research is the application of Hilbert transform to the discrete wavelet packet transform [121]. This area of research seeks to enhance the subband frequency edge detection capability of the

Table 6
Comparison of wavelet based spectrum sensing with other spectrum sensing techniques.

	Wavelet based detection	Cyclostationary detection	Matched filter	Waveform based detection	Eigenvalue/eigenvector based detection	Energy based detection
Speed	Has high speed because no domain transformation is required [92,93].	Requires long observation before making a decision [19].	Has high latency due to the amount of time required to obtain perfect knowledge of PU signal [94].	The requirement of knowledge of PU user signal implies the speed of this technique may be undesirable when high sensing speed is required [95].	Requires long sensing time to compute Eigenvalues when rank of signal matrix representation is large [96].	It has good speed because it does not require knowledge of the PU signal [95]
Accuracy	Leverages on the time-frequency resolution capability of wavelets to yield high accuracy [81].	Performs well at low signal-to-noise ratio [19].	Accuracy depends on perfect knowledge of PU signal, which is not practical in all cases [94].	Accuracy depends on the quality of preamble patterns used for synchronization; this can be a challenge at low SNR [95].	It outperforms wavelet based detection because it is insensitive to noise uncertainty [96].	The accuracy is poor especially at low signal-to-noise ratio [95]
Preprocessing signal information	Requires no prior information of the PU signal [81,92,93].	Requires prior information of the type of carrier, and cyclic prefixes [97].	Requires prior information of PU signal like modulation technique and spreading codes [97].	Requires preamble patterns for synchronization purposes [16].	Requires no prior information of PU signal [98].	Requires no knowledge of the PU signal [96]
Complexity	The complexity is low because the technique only makes decisions on the magnitude of each coefficient in a sub-band channel [92,93].	Requires very high complexity in computation [19].	Requires very high complexity in computation [99].	Has low complexity in implementation [96].	The complexity depends on the rank of the matrix used in representing the signal [96].	It has low complexity [95]
Spectral efficiency	It is spectrally very efficient in OFDM systems because it does not require the creation of guard bands in its implementation [100].	Not as efficient as wavelet based detection because it relies on Fourier transform for its implementation, thereby requiring the use of guard bands in OFDM implementation [101].	Not as efficient as wavelet based detection because it requires guard bands to handle frequency offset sensitivity in OFDM applications [102].	The addition of preambles, mid-ambles, and pilot carrier reduces the spectral efficiency of waveform based detection [103].	If the eigenvalue based detection is based on FFT-OFDM, the need for guard bands also reduces spectral efficiency [104].	Exhibits spectral efficiency if implemented using wavelet technique [100].

Table 7
Advantages of Nyquist wideband sensing and sub-Nyquist wideband sensing.

	Nyquist wideband sensing	Sub-Nyquist wideband sensing
Advantages	Generally has a simple structure [105] Has a high dynamic range [107].	Low sampling rate [106]. The low sampling rate makes it possible to achieve data acquisition at a lower cost than Nyquist wideband sensing.
Disadvantages	Energy cost is high due to high sampling rate [108]. Development of optimization technique can be very complex due to complexity of hardware requirement [109].	It requires multiple sampling channels due to sensing at sub-Nyquist rate. It is sensitive to design imperfections [110].

Table 8
Comparison between wavelet and other wideband sensing techniques.

Technique	Sampling scheme	Implementation complexity
Wavelet detection	Nyquist [107]	Low
Multiband joint detection	Nyquist [105]	High
Filter bank detection	Nyquist [111]	High
Distributed compressive sensing	Sub-Nyquist [106,112]	High
Eigenvector	Sub-Nyquist [113]	Depends on rank of matrix used for signal representation.
Sparse fast Fourier transform	Sub-Nyquist [114]	Low

discrete wavelet packet transform through the instantaneous frequency spread derived from the Hilbert transform. The application of the Hilbert transform to the discrete packet wavelet transform does not add any significant complexity for the application at hand because the Hilbert transform does not require a change of domain for its operation.

10. Research challenges in wavelet-based spectrum sensing

Wavelets, like other spectrum sensing techniques has its own unique challenges in its application to spectrum sensing. In [122], edge detection in spectrum sensing using wavelet multiscale sums have been shown to have difficulty in the precise location of the points of singularity.

Another research challenge in wavelet based spectrum sensing is the choice of filters used in the generation of the wavelet bases in a transmission. This is a very delicate task as there are no clear guidelines, and stringent constraints must be applied for the application at hand [100].

The use of continuous wavelet transform in spectrum sensing is another area of research challenge. This is because continuous wavelet adds excessive redundancy in communication system design, and it is also computationally intensive thereby making its implementation prohibitive due to cost [123].

11. Conclusion

In this paper, a survey was carried out on the wavelet transform and its applications. Different fields of endeavor in which the wavelet transform is applied were highlighted. Properties of different mother wavelets were briefly described, they were also classified according to areas of application. Specific applications of the wavelet transform in spectrum sensing was discussed in some detail, which included the continuous wavelet transform, discrete wavelet transform, and the discrete wavelet packet transform. The mathematical formulations for different applications of wavelets in spectrum sensing were highlighted. Comparative analysis of wavelet application in spectrum sensing and other spectrum sensing techniques was presented. Wavelet as a wideband sensing technique was also compared with other wideband sensing techniques. Current research trends and challenges in wavelet-based spectrum sensing were also discussed. This survey paper provides some necessary information and serve as a pointer to relevant references for researchers seeking information about wavelets and their applications in various areas of research and specifically in spectrum sensing for cognitive radio.

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