Application of Homotopy Perturbation Method on Bank Asset and Liability Portfolio System

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Abstract

The research presents the dynamic nature of decision making support for asset and liability management using Ordinary Differential Equation.The model was tested with the use of maple17 software for analysis, which shows banking industry in Nigeria can manage their asset and liability through cash flow of deposits and loans. Setting the bank's initial position and different deposit flow situations, the model allows to present simulations with the use of Homotopy Perturbation Method for analytical solution and can be used for measurement of liquidity risk to examine loan decisions to choose a realistic result. The result shows that people are encouraged to save their money when the interest rate of deposit is high. To the contrary, people are discouraged to take loan when the interest rate is so high. The result of stress-testing shows the kind of dynamic processes taking place in banking sectors. In this case, it is possible for managers to adjust their bank liabilities to earn assets as much as possible.

Keywords: Asset, Liability, Ordinary Differential Equation, Liquidity Risk, Loan, Deposit

1.0 Introduction

Bank can be defined as a financial body that provides banking and other financial services to their clients. A banking sector also referred to as a structure that offers cash management for customers, reporting the dealings of their balance sheet all through the day. A banking firm is a complex system within the context of management administrative policies [1]. In addition, asset and liability management are what constitute the core banking system. The banks hold in asset and liability management to reach three major goals: credit rate risk, liquidity rate risk and security [2]. The policy of maintaining stock asset and liability allows banks to achieve agreements that will reflects in sound routine that actualizes maximum returns and required liquidity rate [3]. A proper management of financial statement leads to maximizing income and taking into account interest rate and liquidity rate risk [4].Asset and Liability Management (ALM) can be described as a means of managing the risk that may arise from the differences occurring between assets and liabilities. ALM was pioneered by banking industries in the 1970s as the interest rates became more and more volatile. This volatility has risk implications on financial institutions.

With vast experiences gathered, most financial industries focuses mostly on ALM, where they sought to manage balance sheets so as to maintain a mixture of loans and deposits consistent with the firm's target for long-term growth and risk management. ALM boards are been set up to manage the ALM procedure. ALM is the capacity to control interest rate risk, credit rate risk and liquidity risk, which alludes to the threat that a given security can't be exchanged rapidly as much as fundamental in the market to keep a misfortune. ALM likewise looks to handle different risks, for example, remote exchange risks and significant risks (covering territories, for example, trick and legitimate risks, and additionally natural risks). Securitization has supported firms to specifically manage resource and obligation risk by expelling assets or liabilities from their financial report [5]. Ajibola *et al.[6]* formulated a model of liability decrease and asset increase, using goal programming technique to analyze the fiscal report of United Bank of Africa (UBA) Nigeria for the period of 2007 to 2011. They used POM-QM Version 3 software for numerical solution and their result is in close agreement with the works of [3] and [4]. Voloshyn [7] developed a deterministic model for analytical simulation of impact of the change in yield curve on bank's interest income from a fixed rate loans portfolio. The model allows integrating the balance sheet, income statement and asset-liability management.Amirmehdi *et al.*, [8] formulated two basic mathematical models that can forecast credit rate

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risk based on two years records. Their result showed that the mean deviations between output model and genuine results are around 9% and 2.46%, in first and second year separately. Their outcome demonstrated that the proposed strategy can be utilized to gauge the stock liquidity utilizing diverse bank factors.

2.0 Materials and Method

The Homotopy Perturbation Method (HPM) developed in [9] was used to solve the Ordinary Differential Equations representing the dynamics of bank asset and liability portfolio system extended from the work of [7] and [1]. The HPM provides solution to various linear and non-linear differential equations [10].

The basic idea of the method is defined in the following non-linear differential equation:

$$A(U)-f(r)=0,\ r\in\Omega \tag{1}$$

with the boundary condition

$$B\left(U,\frac{\partial U}{\partial n}\right)=0,r\in\Gamma \tag{2}$$

Where A is the general differential operator, B the boundary operator, $f(r)$ is the analytical function and Γ is the boundary of the domain Ω. The A operator can be divided into two major parts L and N been the linear and nonlinear component respectively. Equation (1) can be written as follows:

$$L(U)+N(U)-f(r)=0,r\in\Omega \tag{3}$$

The HPM is composed as follows:

$$H(V,h)=(1-h)\left[L(V)-L(U_0)\right]+h\left[A(V)-f(r)\right]=0 \tag{4}$$

Where $V(r,P):\Omega\in[0,1]\to R$

$P\in[0,1]$ is an embedded parameter and U_0 is the approximation that satisfies the boundary condition. The solution to equation (4) can be assumed as power series in h as follows:

$$V=V_0+hV_1+h^2V_2+\ldots \tag{5}$$

With approximation best obtain when:

$$U=\lim_{h\to 1}=v_0+hv_1+h^2v_2+\ldots \tag{6}$$

The rate of convergence majorly depends on the nonlinear operator A (V).

The dynamics of bank asset and liability portfolio can mathematically be represented by the following ordinary differential equations:

$$\frac{dL_r}{dt}-u_r(t)-R_rL_r-u_r(t-r)e^{-R_rr}=0 \tag{7}$$

$$\frac{dC_r}{dt}-\mu C_r+\sigma C_r+\lambda=0 \tag{8}$$

$$\frac{dS_r}{dt}-\frac{dD_r}{dt}-\frac{dL_r}{dt}+\rho_rL_r-\eta_rD_r-\lambda+\gamma=0 \tag{9}$$

$$\frac{dD_r}{dt}-v_r(t)-\omega_rD_r-v_r(t-r)e^{-\omega_rr}=0 \tag{10}$$

$$\frac{dE}{dt}-L\rho_r\alpha_r-D\eta_r\beta_r+\gamma-\lambda=0 \tag{11}$$

In the model, L_r represent loans, D_r is deposits, u_r is the cash outflow, μ denote as average portfolio return on trading securities, σ is volatility on securities portfolio, R_r is the repayment of loans in maturity date, γ is purchase (+) or sale (-), η_r interest on deposits, ρ_r is interest on loans, λ is operation cost bank activities, v_r inflow on fixed deposits, ω_r inflow on cash deposits, S_r denote as shares, C_r cash reserves, E represent equity, α , β represents loans and deposits structures respectively.

With the following initial conditions $L_r(0)=L_r o,\ S_r(0)=S_r o,\ D_r(0)=D_r o,\ E(0)=E_0,\ C_r(0)=C_r 0$

3.0 The Analytic Solution

Let

$$L_r = a_0 + ha_1 + h^2 a_2 + ... \tag{12}$$

$$S_r = b_0 + hb_1 + h^2 b_2 + ... \tag{13}$$

$$D_r = c_0 + hc_1 + h^2 c_2 + ... \tag{14}$$

$$E = e_0 + he_1 + h^2 e_2 + ... \tag{15}$$

$$C_r = f_0 + hf_1 + h^2 f_2 + ... \tag{16}$$

Applying (HPM) to (7), we get

$$(1-h)\frac{dL_r}{dt} + h(\frac{dL_r}{dt} - u_r(t) - R_r L_r - u(-r)e^{-R_r r}) = 0 \tag{17}$$

Substituting equation (12) in (17) we have

$$\begin{aligned}&(1-h)(a_0^1 + ha_1^1 + h^2 a_2^1 + ...) + h(a_0^1 + ha_1^1 + h^2 a_2^1 + ...)\\&+R_r(a_0 + ha_1 + h^2 a_2 + ...) + u_r(-r)e^{-R_r r} - u_r(t) = 0\end{aligned} \tag{18}$$

$$\begin{aligned}&a_0^1 + ha_1^1 + h^2 a_2^1 + ... + h(R_r(a_0 + ha_1 + h^2 a_2 + ...)\\&+u_r(-r)e^{-R_r r} - u_r(t)) = 0\end{aligned} \tag{19}$$

Collect the coefficients of powers of h, which gives

$$h^0 : a_0^1 = 0 \tag{20}$$

$$h^1 : a_1^1 = R_r a_0 + u_r(-r)e^{-R_r r} - u_r(t) \tag{21}$$

$$h^2 : a_2^1 = R_r a_1 \tag{22}$$

Applying (HPM) to equation (8) we get

$$(1-h)\frac{dC_r}{dt} + h(\frac{dC_r}{dt} - \mu C_r + \sigma C_r + \lambda) = 0 \tag{23}$$

Substituting equation (16) in (23) we have

$$\begin{aligned}&(1-h)(f^1{}_0 + hf^1{}_1 + h^2 f^1{}_2 + ...) + h((f^1{}_0 + hf^1{}_1 + h^2 f^1{}_2 + ...)\\&-\mu(f_0 + hf_1 + h^2 f_2 + ...) - \sigma(f_0 + hf_1 + h^2 f_2 + ...) - \lambda) = 0\end{aligned} \tag{24}$$

$$\begin{aligned}&f^1{}_0 + hf^1{}_1 + h^2 f^1{}_2 + ... + h(-\mu(f_0 + hf_1 + h^2 f_2 + ...)\\&-\sigma(f_0 + hf_1 + h^2 f_2 + ...) - \lambda) = 0\end{aligned} \tag{25}$$

Collecting the coefficients of powers of h, we get

$$h^0 : f_0^1 = 0 \tag{26}$$

$$h^1 : f_1^1 = -\mu f_0 + \lambda \tag{27}$$

$$h^2 : f_2^1 = -\sigma f_1 \tag{28}$$

Applying (HPM) to equation (9) we have

$$(1-h)\frac{dS_r}{dt} + h(\frac{dS_r}{dt} - \frac{dD_r}{dt} - \frac{dL_r}{dt} + \rho_r L_r - \eta_r D_r - \lambda + \gamma = 0 \tag{29}$$

Substituting equations (12), (13) and(14) into equation (29) gives

$$(1-h)b_0^1 + hb_1^1 + h^2b_2^1 + ... + h((b_0^1 + hb_1^1 + h^2b_2^1 + ...)$$
$$-c_0^1 + hc_1^1 + h^2c_2^1 + ... + (a_0^1 + ha_1^1 + h^2a_2^1 + ...)$$
$$+\rho_r(a_0 + ha_1 + h^2a_2 + ...)$$
$$+\eta_r(c_0 + hc_1 + h^2c_2 + ...) + \lambda + \gamma) = 0 \tag{30}$$

Collecting the coefficients of powers of h, we get

$$h^0 : b_0^1 = 0 \tag{31}$$
$$h^1 : b_1^1 = c_0^1 + a_0^1 - \rho_r a_0 + \eta_r c_0 + \lambda + \gamma \tag{32}$$
$$h^2 : b_2^1 = b_1^1 + c_1^1 + a_1^1 - \rho_r a_1 + \eta_r c_1 = 0 \tag{33}$$

Applying (HPM) to equation (10) gives

$$(1-h)\frac{dD_r}{dt} + h(\frac{dD_r}{dt} - v_r(t) + \omega_r D_r + v_r(-r)e^{-\omega_r r}) = 0 \tag{34}$$

Substituting equation (14) in to equation (34) gives

$$(1-h)c_0^1 + hc_1^1 + h^2c_2^1 + ... + h((c_0^1 + hc_1^1 + h^2c_2^1 + ...)$$
$$-v_r(t) + \omega_r(c_0 + hc_1 + h^2c_2 + ...) + v_r(-r)e^{-\omega_r r}) = 0 \tag{35}$$
$$c_0^1 + hc_1^1 + h^2c_2^1 + ... + h(-v_r(t)$$
$$+\omega_r(c_0 + hc_1 + h^2c_2 + ...) + v_r(-r)e^{-\omega_r r}) = 0 \tag{36}$$

Collecting the coefficients of the powers of h

$$h^0 : c_0^1 = 0 \tag{37}$$
$$h^1 : c_1^1 = -v_r(t) + \omega_r c_0 + v_r(-r)e^{-\omega_r r} \tag{38}$$
$$h^2 : c_2^1 = \omega_r c_1 \tag{39}$$

Applying (HPM) to equation (11) gives

$$(1-h)\frac{dE}{dt} + h(\frac{dE}{dt} - L\rho_r\alpha_r - D\eta_r\beta_r + \gamma - \lambda) = 0 \tag{40}$$

Putting equation (15) we have

$$(1-h)e_0^1 + he_1^1 + h^2e_2^1 + ... + h((e_0^1 + he_1^1 + h^2e_2^1 + ...)$$
$$-L\rho_r\alpha_r + D\eta_r\beta_r - \gamma + \lambda) = 0 \tag{41}$$

Collecting the coefficients of the powers of h, we have

$$h^0 : e_0^1 = 0 \tag{42}$$
$$h^1 : e_1^1 = e_0^1 - L\rho_r\alpha_r + D\eta_r\beta_r - \gamma + \lambda \tag{43}$$
$$h^2 : e_2^1 = e_1 \tag{44}$$

From equation (20)

$$a_0^1 = 0 \tag{45}$$

Integrate both sides with respect to time (t) and applying initial condition

$$a_0 = A_3 \tag{46}$$

$$L_r(0) = L_r 0 = a_0 \tag{47}$$
$$L_r(0) = a_0 \tag{48}$$

From equation (25)

$$f_0^1 = 0 \tag{49}$$

Integrating both sides with respect to time (t) and applying the initial condition

$$f_0 = B_3 \tag{50}$$

$$C_r(0) = C_r o = f_0 \tag{51}$$

$$f_0 = C_r(0) \tag{52}$$

From equation (31)

$$b_0^1 = 0 \tag{53}$$

Integrating both sides with respect to time (t) and applying the initial condition

$$b_o = C_3 \tag{54}$$

$$S_r(0) = S_r 0 = b_0 \tag{55}$$

$$b_0 = S_r(0) \tag{56}$$

From equation (37)

$$C_0^1 = 0 \tag{57}$$

Integrate both sides with respect to time (t) and applying the initial condition

$$C_0 = D_3 \tag{58}$$

$$C_r(0) = C_r 0 = C_0 \tag{59}$$

$$C_0 = C_r(0) \tag{60}$$

From equation (42)

$$e_0^1 = 0 \tag{61}$$

Integrating both sides with respect to time (t) and applying the initial condition

$$e_0 = F_3 \tag{62}$$

$$E(0) = E0 = e_0 \tag{63}$$

$$e_0 = E(0) \tag{64}$$

From equation (21)

$$a_1^1 = R_r a_0 + u_r(-r)e^{-R_r r} - u_r \tag{65}$$

Integrating both sides with respect to time (t) and applying the initial condition

$$a_1 = R_r a_0 + u_r(-r)e^{-R_r r} - u_r \tag{66}$$

Substituting equation (48) in (66) gives

$$a_1 = (R_r L_r(0) + u_r(-r)e^{-R_r r} - u_r)t \tag{67}$$

From equation (27)

$$f_1^1 = -\mu f_0 + \lambda \tag{68}$$

Substituting equation (52) in (68) gives

$$f_1^1 = -\mu C_r(0) + \lambda \tag{69}$$

Integrating both sides with respect to time (t) and applying initial condition $f_1(0) = 0$

$$f_1 = (-\mu C_r(0) +)\lambda \tag{70}$$

From equation (32)

$$b_1^1 = C_0 + a_0 - \rho_r a_0 + \eta_r C_o + \lambda + \gamma \tag{71}$$

Putting equations (45), (57) and(48) in (71) gives

$$b^1{}_1 = -\rho_r L_r(0) + \eta_r C_r(0) + \lambda + \gamma \tag{72}$$

Integrating both sides with respect to time (t) and applying the initial condition

$$b_1 = \left(-\rho_r L_r(0) + \eta_r C_r(0) + \lambda + \gamma\right)t \tag{73}$$

From equation (38)

$$C_1^1 = -v_r + R_r C0 + v_r(-r)e^{-R_r r} \tag{74}$$

Putting equations (60) in (74)

$$C_1^1 = -v_r + R_r C0 + v_r(-r)e^{-R_r r} \tag{75}$$

Integrating both side respect to time (t) and applying initial condition $C_1(0) = 0$

$$C_1 = (-v_r + R_r C0 + v_r(-r)e^{-R_r r})t \tag{76}$$

From equation (43)

$$e_1^1 = e_0^1 - L\rho_r\alpha_r + D\eta_r\beta_r - \gamma + \lambda \tag{77}$$

Putting equations (61) into (77) we get

$$e_1^1 = -L\rho_r\alpha_r + D\eta_r\beta_r - \gamma + \lambda \tag{78}$$

Integrating both sides with respect to time (t) and applying the initial condition $e_1(0) = 0$

$$e_1 = (-L\rho_r\alpha_r + D\eta_r\beta_r - \gamma + \lambda)t \tag{79}$$

From equation (22)

$$a_1^1 = R_r a_1 \tag{.80}$$

Putting equation (67) into (80) gives

$$a_2^1 = R_r(R_r L_r(0) + u_r(-r)e^{-R_r r} - u_r)t \tag{81}$$

Integrating both sides with respect to time (t) and applying initial condition $a_0(0) = 0$

$$a_2 = R_r(R_r L_r(0) + u_r(-r)e^{-R_r r} - u_r)\frac{t^2}{2} \tag{82}$$

Putting equations (48) and(82) into (12) gives

$$\begin{aligned} L_r = \lim_{h \to 1} = L_r(0) + h(R_r L_r(0) + u_r(-r)e^{-R_r r} - u_r) \\ + h^2(R_r L_r(0) + u_r(-r)e^{-R_r r} - u_r)\frac{t^2}{2} \end{aligned} \tag{83}$$

$$\begin{aligned} L_r = L_r(0) + (R_r L_r(0) + u_r(-r)e^{-R_r r} - u_r) \\ + (R_r L_r(0) + u_r(-r)e^{-R_r r} - u_r)\frac{t^2}{2} \end{aligned} \tag{84}$$

From equation (28)

$$f_2^1 = \sigma f_1 \tag{85}$$

Putting equation (70) into (85) gives

$$f_2^1 = +\sigma(-\mu C_r(0) + \lambda)t \tag{86}$$

Integrating both sides with respect to time (t) and applying initial condition $f_2(0) = 0$

$$f_2 = +\sigma(-\mu C_r(0) + \lambda)\frac{t^2}{2} \tag{87}$$

Substituting equations (52), (70), and(87) in (16) gives

$$\begin{aligned} C_r = \lim_{h \to 1} = C_r(0) + h(-\mu C_r(0) + \lambda)t \\ + h^2(\sigma - \mu C_r(0) + \lambda)\frac{t^2}{2} \end{aligned} \tag{88}$$

$$C_r = C_r(0) + (-\mu C_r(0) + \lambda)t \\ + (\sigma - \mu C_r(0) + \lambda)\frac{t^2}{2} \tag{89}$$

From equation (33)

$$b_2^1 = b_1^1 + c_1^1 + a_1^1 - \rho_r a_1 + \eta_r c_1 \tag{90}$$

Substituting equations (72), (75), (65), (67) and(76) into (90) gives

$$b_2^1 = (-\rho_r L_r(0) + \eta_r C_r(0) + \lambda + \gamma) + (-v_r + R_r C_r(0) + v_r(-r)e^{-R_r r}) \\ -R_r L_r(0) + u_r(-r)e^{-R_r r} - u_r - \rho_r(R_r L_r(0) + u_r(-r)e^{-R_r r} - u_r)t \\ -\eta_r(-v_r + R_r C_r(0) + v_r(-r)e^{-R_r r})t \tag{91}$$

Integrating both sides with respect to time (t) and applying the initial condition $b_2(0) = 0$

$$b_2 = (-\rho_r L_r(0) + \eta_r C_r(0) + \lambda + \gamma) + (-v_r + R_r C_r(0) + v_r(-r)e^{-R_r r}) \\ -R_r L_r(0) + u_r(-r)e^{-R_r r} - u_r)t - \rho_r(R_r L_r(0) + u_r(-r)e^{-R_r r} - u_r) \\ -\eta_r(-v_r + R_r C_r(0) + v_r(-r)e^{-R_r r})\frac{t^2}{2} \tag{92}$$

From equation (13)

$$S_r = b_0 + hb_1 + h^2 b_2$$

Putting equations (56), (73) and(92) in (13) gives

$$S_r = \lim_{h \to 1} = S_r(0) + h(-\rho_r L_r(0) + \eta_r C_r(0) + \lambda + \gamma)t + h^2((-\rho_r L_r(0) \\ + \eta_r C_r(0) + \lambda + \gamma) + (-v_r + R_r C_r(0) + v_r(-r)e^{-R_r r}) \\ + R_r L_r(0) + u_r(-r)e^{-R_r r} - u_r)t - (\rho_r(R_r L_r(0) + u_r(-r)e^{-R_r r} - u_r) \\ + \eta_r(-v_r(t) + R_r C_r(0) + v_r(-r)e^{-R_r r}))\frac{t^2}{2} \tag{93}$$

$$S_r = S_r(0) + h(-\rho_r L_r(0) + \eta_r C_r(0) + \lambda + \gamma)t + ((-\rho_r L_r(0) \\ + \eta_r C_r(0) + \lambda + \gamma) + (-v_r + R_r C_r(0) + v_r(-r)e^{-R_r r}) \\ + R_r L_r(0) + u_r(-r)e^{-R_r r} - u_r)t - (\rho_r(R_r L_r(0) + u_r(-r)e^{-R_r r} - u_r) \\ + \eta_r(-v_r(t) + R_r C_r(0) + v_r(-r)e^{-R_r r}))\frac{t^2}{2} \tag{94}$$

From equation (39)

$$c_2^1 = R_r c_1$$

Putting equation (76) in (39) gives

$$C_2^1 = R_r(-v_r + R_r C_r(0) + v_r(-r)e^{-R_r r})t \tag{95}$$

Integrating both sides with respect to time (t) and applying the initial condition $C_2(0) = 0$

$$C_2 = R_r\left(-v_r + R_r C_r(0) + v_r(t-r)e^{-R_r r}\right)\frac{t^2}{2} \tag{96}$$

From equation (14)

$$D_r = c_0 + hc_1 + h^2 c_2 + \ldots$$

Substituting equations (60), (76), and(96) in (14) gives

$$D_r = \lim = C_r(0) + h(-v_r + R_r C_r(0) + v_r(-r)e^{-R_r r})t$$
$$h \to 1 \qquad (97)$$
$$+ h^2 (R_r(-v_r + R_r C_r(0) + v_r(-r)e^{-R_r r}))\frac{t^2}{2}$$

$$D_r = C_r(0) + (-v_r + R_r C_r(0) + v_r(-r)e^{-R_r r})t$$
$$+ (R_r(-v_r + R_r C_r(0) + v_r(-r)e^{-R_r r}))\frac{t^2}{2} \qquad (98)$$

From equation (44)

$e_2^1 = e_1$

Substituting equation (79) into (44) gives

$$e_2^{\,1} = \left(-L\rho_r \alpha_r + D\eta_r \beta_r - \gamma + \lambda\right)t \qquad (99)$$

Integrating both sides with respect to time (t) and applying the initial condition $e_2(0) = 0$

$$e_2 = \left(-L\rho_r \alpha_r + D\eta_r \beta_r - \gamma + \lambda\right)\frac{t^2}{2} \qquad (100)$$

From equation (15)

$E = e_0 + he_1 + h^2 e_2 + \ldots$

Substituting the values of equations (64), (79) and(100) in (15) gives

$$E(t) = \lim = E(0) + h(-L\rho_r \alpha_r + D\eta_r \beta_r - \gamma + \lambda)t$$
$$h \to 1 \qquad (101)$$
$$+ (-L\rho_r \alpha_r + D\eta_r \beta_r - \gamma + \lambda)\frac{t^2}{2}$$

$$E(t) = E(0) + h(-L\rho_r \alpha_r + D\eta_r \beta_r - \gamma + \lambda)t$$
$$+ (-L\rho_r \alpha_r + D\eta_r \beta_r - \gamma + \lambda)\frac{t^2}{2} \qquad (102)$$

The following are analytical solution of the model:

$$L_r = L_r(0) + (R_r L_r(0) + u_r(-r)e^{-R_r r} - u_r)$$
$$+ (R_r L_r(0) + u_r(-r)e^{-R_r r} - u_r)\frac{t^2}{2}$$

$$C_r = C_r(0) + (-\mu C_r(0) + \lambda)t$$
$$+ (\sigma - \mu C_r(0) + \lambda)\frac{t^2}{2}$$

$$S_r = S_r(0) + h(-\rho_r L_r(0) + \eta_r C_r(0) + \lambda + \gamma)t + ((-\rho_r L_r(0)$$
$$+ \eta_r C_r(0) + \lambda + \gamma) + (-v_r + R_r C_r(0) + v_r(-r)e^{-R_r r})$$
$$+ R_r L_r(0) + u_r(-r)e^{-R_r r} - u_r)t - (\rho_r(R_r L_r(0) + u_r(-r)e^{-R_r r} - u_r)$$
$$+ \eta_r(-v_r(t) + R_r C_r(0) + v_r(-r)e^{-R_r r}))\frac{t^2}{2}$$

$$D_r = C_r(0) + (-v_r + R_r C_r(0) + v_r(-r)e^{-R_r r})t$$
$$+ (R_r(-v_r + R_r C_r(0) + v_r(-r)e^{-R_r r}))\frac{t^2}{2}$$

$$E(t) = E(0) + h(-L\rho_r\alpha_r + D\eta_r\beta_r - \gamma + \lambda)t + (-L\rho_r\alpha_r + D\eta_r\beta_r - \gamma + \lambda)\frac{t^2}{2}$$

4.0 Numerical Simulations

The stress-testing was carried out using the following variables and parameters for the initial conditions, computations were run in maple17 software.

Table 1: Initial conditions for each plot for the variables and parameters

Variables and Parameters	Value	Source
L_0	12,350,849	UBA Nigeria Financial Report (2015)
D_0	11,174,379	"
S_0	633,215	"
E_0	1,176,470	"
C_0	516,651	"
λ	0.75	"
γ	0.81	Assumed
ω	0.81	Assumed
μ	0.75	
σ	1.01	Assumed
r	1.01	Assumed
v_r	0.5	Assumed
β	0.5	Assumed
α	0.20	Assumed
η_r	0.10	Assumed
ρ_r	0.15	Assumed
	0.75	Amirmehdi, *et., al* (2014)
	0.75	"

Results of Numerical simulations are presented on Fig. 1-6

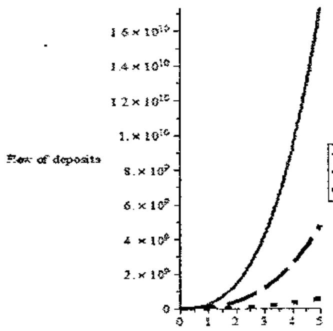

Figure 1 Shows graph of flows on deposit against time

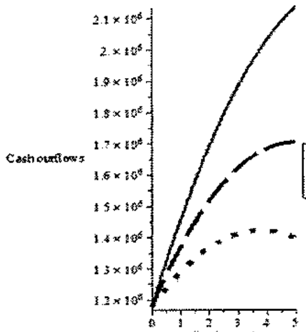

Figure 2 Graph of interest (cash outflows) against time

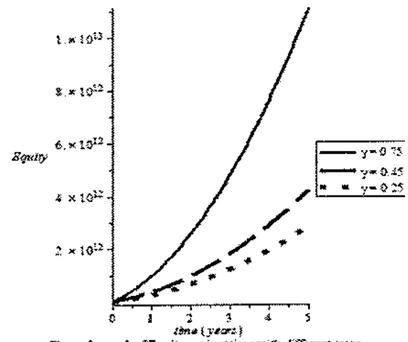

Figure 3: graph of Equity against time with different rates

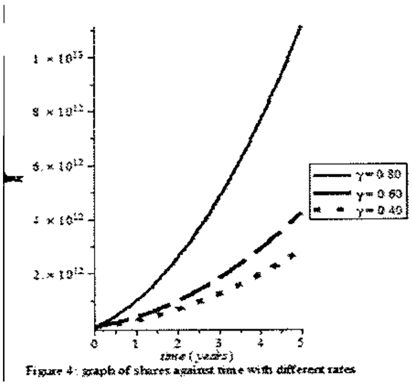

Figure 4: graph of shares against time with different rates

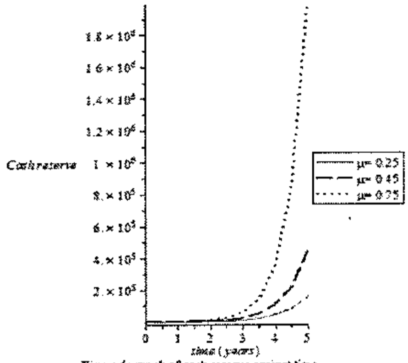

Figure 5: graph of cash reserve against time

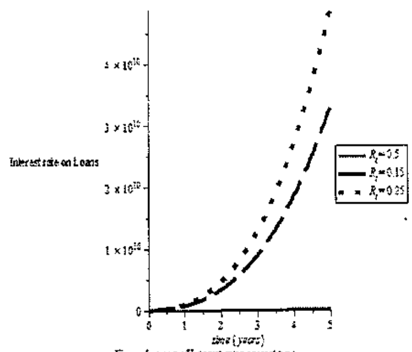

Figure 6: graph of Interest rates against time

5.0 Discussion of Results

Figure 1: Plot shows the dynamic of cash inflow on deposits against time with different rate. It can be deduced from the graph that people are encouraged to save their money in the bank when the interest rate of deposit is high. In this case a reduced the rate of liquidity risk.

Figure 2: Plot of dynamics cash outflows $u_r(t)$ on loans against time with different rates at $(t)=5$. It is observed that the loan issued decreases when interest rates on loans increases. In this case, people are discouraged to take loan when the interest rate of loan is so high.

Figure 3: Plot shows the graph of equity against time with different rates of purchase or sale (γ) at $t=5$. It is noted that the owner's interest increases as that of purchase and sale increases. In this case, the graph shows the owning supply in a corporation over time will if possible yield principal gains for shareholders and dividends as well. This is maximized.

Figure 4: Plot display a graph of shares against time for different rates of securities investment on trading (γ). It is seen that when the rates of $\gamma_1=80\%$, $\gamma_2=60\%$ and $\gamma_3=40\%$, the company earned profit as periodic dividends based on the number of stock.

Figure 5: Plot shows cash reserve against time with different rates of security portfolio return. It can be deduced from the graph when the rates are (μ=25%, 45% and μ=0.75%), the banks can obtain much profit on their capital by lending our cash to borrowers instead of holding it in their vaults or depositing it in similar institutions.

Figure 6: Plot of interest rates against time with different rates of repayment of loans (R_t). This shows the bank enjoyed interest income on loan portfolio. In this case the duration of the loans issued over time end in maturity age.

6.0 Conclusion

In this research work, efforts were made to develop a mathematical model for asset and liability portfolio system on bank using Homotopy Perturbation Method (HPM) for numerical simulation. We used ordinary differential equations on asset and liability portfolio to model cash flows in asset and liability accountsof the bank based on the dynamics. The model were tested with the use of maple 14 software for analysis, which shows indigenous bank can manage their asset and liability portfolio through increase on cash flows or financial flows. Our graph of cash flows shows desired behavior and better when compared with the graph of [7] and [1] which considered only interest cash flows and dynamics of interest rate on loan portfolio rather than both cash flows and investment security trade on portfolio return..

7.0 References

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