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MATHEMATICAL RECIPE FOR CONTROLLING THE SPREAD OF LASSA FEVER

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Abstract

In this study, a Mathematical Recipe for Controlling the Spread of Lassa Fever was presented. The positive invariant region of the model was established. The model has a solution set that remain positive for all t>0. The Disease Free and Endemic Equilibrium States of the model were evaluated. Hence there is possibility of total eradication of the Disease from the population.

KEYWORDS: *Lassa Fever, Positive Invariant Region, Disease free, Endemic Equilibrium State and Stability.*

INTRODUCTION

Lassa fever is a viral hemorrhagic fever whose symptoms include fever, sore throat, headache, facial swelling, muscle fatigue, vomiting, muscles pain, cough, bleeding, meningitis, and hypertension (Omilabu *et al*, 2005). In some cases neurological problems such hearing loss may be transient or permanent and tremors have been described (Omilabu *et al*, 2005).

Diagnosis of Lassa fever is usually difficult because of the variation in its symptoms and difficulty in distinguishing it from other viral haemorrhagic fever (World health Organization, 2017).

Currently, there is no vaccine that inoculates against the infection of Lassa fever. The only drug that is used for the treatment of Lassa fever is Ribavirin. The antiviral drug Ribavirin appears to be an effectual treatment for Lassa fever if given early on in the course of clinical illness.

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There is no evidence to support the function of Ribavirin as post-exposure prophylactic treatment of Lassa fever (World Health organization, 2017).

Main transmission of the Lassa virus from its host to humans can be prevented by avoiding contact with *Mastomys* rodents, specifically in places where epidemics occur. Covering food in rodent-proof containers and keeping the home clean help to discourage rodents from entering. Ingestion of these rodents as a delicacy is highly discouraged. Setting traps in and around homes can help reduce rodent populations; however, the wide distribution of *Mastomys* rats in Africa makes complete control of this rodents reservoir impractical (Center for Disease Control and Prevention, 2015)

Bawa et al.,(2014) formulaed a mathematical model which incorporated vital dynamics, standard incidence, disease induced death due to human infection, reservoirs R and aerosol (airborne) transmissions . Their analysis revealed that the disease can be control if the basic reproduction number R0 is strictly less than unity. Their work didn't take into account treated and recovered humans.

Mohammed et al., (2014) carried out sensitivity analysis on a Lassa fever deterministic mathematical model. This was done to ascertain the most sensitive parameters in the model and they discovered that the most sensitive parameters are; the human immigration, human recovery rate and then person to person contact rate. They concluded that control strategies should be focused on human immigration, effective drugs for treatment and education to reduce person to person contact. But their work didn't include treatment class

James et al., (2015) formulated an SIR model of Lassa Fever disease dynamics. The disease free equilibrium and the endemic equilibrium states were calculated and analyzed for stability. The result of their analysis show that the disease free equilibrium will be stable any time the birth rate of the human population is smaller than the death rate and also when the birth rate of the mastomys-natalensis is smaller than the whole population. In their work, they didn't consider the rodents population.

Onuorah et al., (2016) formulated a sex-structured mathematical model which subdivided the human population into males and females, and the animal reservoirs into active and inactive reservoirs. They considered

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sexual transmission of the virus among sexually active humans as one of the means of transmitting the virus in humans. Sensitivity analysis on the parameters of their model showed that, the basic reproduction number is most sensitive to parameters representing human birth, condom efficacy and compliance rates.

Akanni et al., (2018) ran sensitivity analysis of the dynamical transmission of Lassa fever virus. This was done to discover the most sensitive parameters on the transmission of the disease. Their findings indicated that the most sensitive parameter is the progression rate to active Lassa fever (γ), followed by the force of infection of the susceptible individuals with the infected individuals (λ). They also discovered that the least sensitive parameter is the treatment rate of infective class (θ).they concluded that the parameters (γ) and (λ) that have great sensitivity to the transmission of Lassa fever be put in check. But they didn't consider asymptomatic infected compartment.

Suleiman et al., (2018) formulated a mathematical model for the transmission dynamics of the Lassa fever virus infection by splitting the infectious human population into symptomatic and asymptomatic infectious and also assumed that the rodents do not recover from the infection. They obtained the equilibrium states and analyzed them for stability. The also obtained the basic reproduction number of the humans' population and carried out sensitivity analysis on the basic reproduction number of which they ascertained that are most sensitive to the transmission rates, recovery rates and the natural mortality rates of the humans.

In this study, we analyse and investigate the effect of treatment on the Lassa fever transmission dynamics.

METHODOLOGY

Model Formulation

The model subdivides the human population into six (6) mutually exclusive compartments, which are; susceptible humans ($\mathbf{S_H}$), exposed humans ($\mathbf{E_H}$), asymptomatic infected humans ($\mathbf{A_H}$), symptomatic infected humans ($\mathbf{I_H}$), Treated humans ($\mathbf{T_H}$) and recovered humans ($\mathbf{R_H}$). Similarly, the reservoir population is subdivided into two (2) mutually

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exclusive compartments, which are; susceptible reservoirs (S_R) and infected reservoirs (I_R).

The population of the susceptible human (S_H) increases through constant recruitment $\wedge_1$ of individuals into the population by birth or immigration. The population decreases as susceptible human move to the Exposed compartment (E_H) through interaction of the susceptible humans (S_H) with either infected reservoirs (I_R), asymptomatic infected humans, symptomatic infected human or humans undergoing treatment at the rate β, and further decreases through natural death of human at a rate μ_1. Infection rate is reduced due to treatment at the rateδ, where $\delta \epsilon [0,1]$.

The population of the exposed humans compartment (E_H) decreases due to natural death at the rate μ_1 and also due to movement to infected classes after incubation period at the rate α. A Proportion of α move to the symptomatic infected compartment (I_H) at the rate $\rho\alpha$, while the remaining of the proportion move to the asymptomatic infected compartment (A_H) at the rate $(1-\rho)\alpha$, where $\rho \epsilon [0,1]$. The population of the asymptomatic infected compartment decreases due to treatment at the rate η, also due to disease-induced death at the rate (ϕ), and also due to natural death at the rate (μ_1).

The population of the symptomatic infected compartment decreases due to treatment at the rate γ, also due to disease-induced death at the rate (ϕ), and also due to natural death at the rate (μ_1).

The population of the treatment compartment decreases due to recovery at the rate κ, also due to disease-induced death at the rate (ϕ), and also due to natural death at the rate (μ_1).

The population of the recovered compartment decreases due to natural death at the rate (μ_1). There is permanent immunity after recovery.

The population of the susceptible reservoir (S_R) increases through constant recruitment $\wedge_2$ of reservoir into the population by birth or immigration. The population decreases as susceptible reservoir move to

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the infected reservoirs compartment (I_R) through interaction of the susceptible reservoirs (S_R) with infected reservoirs (I_R) at the rate λ, also decreases due to hunting at the rate ν and due to natural death at the rate μ_2.

The population of the infected reservoirs decreases due to hunting at the rate ν, and due to natural death at the rate μ_2.

This leads to the following system of ordinary differential equations.

$$\frac{dS_H}{dt} = \wedge_1 - \beta(I_R + I_H + A_H + \delta T_H)S_H - \mu_1 S_H \quad (3.1)$$

$$\frac{dE_H}{dt} = \beta(I_R + I_H + A_H + \delta T_H)S_H - (\alpha + \mu_1)E_H \quad (3.2)$$

$$\frac{dA_H}{dt} = (1-\rho)\alpha E_H - (\eta + \phi + \mu_1)A_H \quad (3.3)$$

$$\frac{dI_H}{dt} = \rho\alpha E_H - (\gamma + \phi + \mu_1)I_H \quad (3.4)$$

$$\frac{dT_H}{dt} = \gamma I_H + \eta A_H - (\kappa + \phi + \mu_1)T_H \quad (3.5)$$

$$\frac{dR_H}{dt} = \kappa T_H - \mu_1 R_H \quad (3.6)$$

$$\frac{dS_R}{dt} = \wedge_2 - \lambda S_R I_R - (\nu + \mu_2)S_R \quad (3.7)$$

$$\frac{dI_R}{dt} = \lambda S_R I_R - (\nu + \mu_2)I_R \quad (3.8)$$

The Positive Invariant Region

The positive invariant region can be established by using the following theorem.

Theorem 3.1

The solutions of the system of equations (3.1) through (3.8) are feasible for $t > 0$ if they enter the invariant region D.

Proof

The total human population is $N_H = S_{H+}E_H + A_H + I_H + T_H + R_H$ (3.9)

And the total reservoir population is $N_R = S_R + I_R$

(3.10)

Where,

$$\frac{dN_H}{dt} = \frac{dS_H}{dt} + \frac{dE_H}{dt} + \frac{dA_H}{dt} + \frac{dI_H}{dt} + \frac{dT_H}{dt} + \frac{dR_H}{dt}$$

$$\frac{dN_H}{dt} = \wedge_1 - \mu_1 N_H - \phi(A_H + I_H + T_H) \quad (3.11)$$

Similarly,

$$\frac{dN_R}{dt} = \frac{dS_R}{dt} + \frac{dI_R}{dt}$$

$$\frac{dN_R}{dt} = \wedge_2 - (\nu + \mu_2) N_R \quad (3.12)$$

Let $D = \left(S_H, E_H, A_H, I_H, T_H, R_H, S_R, I_R\right) \in R^8_+$

Be any solution of the system of equations (3.1) to (3.8) with positive initial conditions.

Suppose there are no disease-induced deaths, equation (3.11) becomes

$$\frac{dN_H}{dt} \leq \wedge_1 - \mu_1 N_H \quad (3.13)$$

i.e

$$\frac{dN_H}{dt} + \mu_1 N_H \leq \wedge_1 \quad (3.14)$$

The integrating factor (IF) of (3.14) is $e^{\mu_1 t}$

Multiplying both sides of (3.14) by $e^{\mu_1 t}$ gives

$$e^{\mu_1 t} \frac{dN_H}{dt} + \mu_1 e^{\mu_1 t} N_H \leq e^{\mu_1 t} \wedge_1 \quad (3.15)$$

$$d(N_H e^{\mu_1 t}) \leq \wedge_1 e^{\mu_1 t} dt \quad (3.16)$$

Integrating both sides of (3.16) gives

$$N_H e^{\mu_1 t} \leq \frac{\wedge_1}{\mu_1} e^{\mu_1 t} + c \quad (3.17)$$

$$N_H \leq \frac{\wedge_1}{\mu_1} + c e^{-\mu_1 t} \quad (3.18)$$

Applying the initial condition t = 0, $N_H(0) = N_{H0}$ in equation (3.18) gives

$$\frac{\wedge_1}{\mu_1} + c = N_{H0} \quad (3.19)$$

$$\Rightarrow c = N_{H0} - \frac{\wedge_1}{\mu_1} \quad (3.20)$$

Substituting equation (3.20) into (3.18) gives

$$N_H \leq \frac{\wedge_1}{\mu_1} + (N_{H0} - \frac{\wedge_1}{\mu_1})e^{-\mu_1 t} \quad (3.21)$$

Equation (12) can be re-written as

$$\frac{dN_R}{dt} + (\nu + \mu_2)N_R \leq \wedge_2 \quad (3.22)$$

The integrating factor of (3.22) is $e^{(\nu+\mu_2)t}$

Multiplying through (3.22) by $e^{(\nu+\mu_2)t}$ gives

$$e^{(\nu+\mu_2)t}\frac{dN_R}{dt} + e^{(\nu+\mu_2)t}(\nu + \mu_2)N_R \leq e^{(\nu+\mu_2)t} \wedge_2 \quad (3.23)$$

$$d(N_R e^{(\nu+\mu_2)t}) \leq \wedge_2 e^{(\nu+\mu_2)t} dt \quad (3.34)$$

Integrating both sides of (3.34) gives

$$N_R e^{(\nu+\mu_2)t} \leq \frac{\wedge_2}{(\nu + \mu_2)} e^{(\nu+\mu_2)t} + c_2 \quad (3.35)$$

$$\Rightarrow N_R \leq \frac{\wedge_2}{(\nu + \mu_2)} + c_2 e^{-(\nu+\mu_2)t} \quad (3.36)$$

Applying the initial condition t = 0, $N_R(0) = N_{R0}$ in equation (3.36) gives

$$N_{R0} = \frac{\wedge_2}{(\nu + \mu_2)} + c_2$$

$$c_2 = N_{R0} - \frac{\wedge_2}{(\nu + \mu_2)} \quad (3.37)$$

Substituting equation (3.37) into (3.36) gives

$$\Rightarrow N_R \leq \frac{\wedge_2}{(\nu + \mu_2)} + (N_{R0} - \frac{\wedge_2}{(\nu + \mu_2)})e^{-(\nu+\mu_2)t} \quad (3.38)$$

Therefore, as $t \to \infty$ in (3.21) the human population N_H approaches $\frac{\Lambda_1}{\mu_1} = K_1$ (That is, $N_H \to K_1 = \frac{\Lambda_1}{\mu_1}$) . the parameter $K_1 = \frac{\Lambda_1}{\mu_1}$ is called the carrying capacity of the human population.

Similarly, as $t \to \infty$ in (3.38) the reservoir population N_R approaches $\frac{\Lambda_2}{(\nu + \mu_2)} = K_2$ (That is, $N_R \to K_2 = \frac{\Lambda_2}{(\nu + \mu_2)}$) . the parameter $K_2 = \frac{\Lambda_2}{(\nu + \mu_2)}$ is called the carrying capacity of the reservoir population.

Hence all feasible solution set of the human population and the reservoir population of the model equations (3.1) to (3.8) enters the region,

$$D=\left\{\left(S_H,E_H,A_H,I_H,T_H,R_H,S_R,I_R\right)\in R^8+:S_H,E_H,A_H,I_H,T_H,R_H,S_R,I_R\geq 0;N_H\leq\frac{\Lambda_1}{\mu_1},N_R\leq\frac{\wedge_2}{(\nu+\mu_2)}\right\}$$

Therefore, the region D is positively-invariant (A region is positively-invariant if the solution that starts in it remains in it $\forall t>0$). And if $N_H(0)\leq\frac{\Lambda_1}{\mu_1}$, then $N_H(t)\leq\frac{\Lambda_1}{\mu_1}$ since $N_H(t)\leq\frac{\Lambda_1}{\mu_1}$ if $N_H(0)\leq\frac{\Lambda_1}{\mu_1}$, Also, if $N_R(0)\leq\frac{\wedge_2}{(\nu+\mu_2)}$, then, $N_R\leq\frac{\wedge_2}{(\nu+\mu_2)}$ since $N_R\leq\frac{\wedge_2}{(\nu+\mu_2)}$ if $N_R(0)\leq\frac{\wedge_2}{(\nu+\mu_2)}$ therefore the region D is positively invariant) and equations (3.1) through (3.8) are epidemiologically meaningful and mathematically well-posed in the domain D. Therefore, in this region it is appropriate to consider the dynamics of flow generated by the model (3.1) through (3.8). In addition, the usual existence, uniqueness, and continuation of the results hold for the system.

Positivity of Solutions

Theorem 3.2

Let the initial data be $\left\{S_H(0),E_H(0),A_H(0),I_H(0),T_H(0),R_H(0),S_R(0),I_R(0)\geq 0\right\}\in D$

Then the solution set $\left\{S_H,E_H,A_H,I_H,T_H,R_H,S_R,I_R\right\}(t)$ of the system of equations (3.1) to (3.8) is positive for all $t>0$

Proof

From equation (3.1), we have

$$\frac{dS_H}{dt}=\wedge_1-\beta(I_R+I_H+A_H+\delta T_H)S_H-\mu_1S_H\geq-\beta(I_R+I_H+A_H+\delta T_H+\frac{\mu_1}{\beta})S_H \quad (3.39)$$

$$\frac{dS_H}{dt}\geq-(\psi_R+\psi_H+\frac{\mu_1}{\beta})S_H \quad (3.40)$$

Where,

$$\psi_R=\beta I_R,\psi_H=\beta(I_H+A_H+\delta T_H) \quad (3.41)$$

Separating variables and integrating both sides of (40) gives

$$\frac{dS_H}{S_H} \geq -(\psi_R + \psi_H + \frac{\mu_1}{\beta})dt \quad (3.42)$$

$$\ln(S_H) \geq -(\psi_R + \psi_H + \frac{\mu_1}{\beta})t + c \quad (3.42)$$

$$S_H \geq Ae^{-(\psi_R + \psi_H + \frac{\mu_1}{\beta})t} \quad (3.43)$$

Where A=e^c

Applying the initial condition $t = 0, S_H(0) = S_{H0}$ on (3.43) gives

$$S_{H0} \geq A \quad (3.44)$$

Therefore,

$$S_H \geq S_{H0}e^{-(\psi_R + \psi_H + \frac{\mu_1}{\beta})t} \geq 0 \quad (3.45)$$

Similarly, from equation (3.2), we have

$$\frac{dE_H}{dt} = \beta(I_R + I_H + A_H + \delta T_H)S_H - (\alpha + \mu_1)E_H \geq -(\alpha + \mu_1)E_H \quad (3.46)$$

$$\frac{dE_H}{dt} \geq -(\alpha + \mu_1)E_H \quad (3.47)$$

Separating variables and integrating both sides of equation (3.47) gives

$$\frac{dE_H}{E_H} \geq -(\alpha + \mu_1)dt \quad (3.48)$$

$$\ln(E_H) \geq -(\alpha + \mu_1)t + c_1 \quad (3.49)$$

$$E_H \geq Be^{-(\alpha+\mu_1)t} \quad (3.50)$$

Applying the initial condition, $t = 0, E_H(0) = E_{H0}$ on (3.50) gives

$$E_{H0} \geq B \quad (3.51)$$

Therefore,

$$E_H \geq E_{H0}e^{-(\alpha+\mu_1)t} \quad (3.52)$$

Similarly, it can be verified that the rest of the equations are positive for all $t \succ 0$ since $e^r > 0$
For all $r \in R$

Existence of Equilibrium States of the Model

Disease free Equilibrium States of the Model

Theorem 1: A disease free equilibrium state of the model exists at the point

$$[S_H, E_H, A_H, I_H, T_H, R_H, S_R, I_R] = \left[\frac{\wedge_1}{\mu_1}, 0, 0, 0, 0, 0, \frac{\wedge_2}{\nu+\mu_2}, 0\right]$$

Proof:

At equilibrium, $\frac{dS_H}{dt} = \frac{dE_H}{dt} = \frac{dA_H}{dt} = \frac{dI_H}{dt} = \frac{dT_H}{dt} = \frac{dR_H}{dt} = \frac{dS_R}{dt} = \frac{dI_R}{dt} = 0$ (3.53)

This implies,

$$\wedge_1 - \beta(I_R + I_H + A_H + \delta T_H)S_H - \mu_1 S_H = 0 \quad (3.54)$$

$$\beta(I_R + I_H + A_H + \delta T_H)S_H - (\alpha + \mu_1)E_H = 0 \quad (3.55)$$

$$(1-\rho)\alpha E_H - (\eta + \phi + \mu_1)A_H = 0 \quad (3.56)$$

$$\rho\alpha E_H - (\gamma + \phi + \mu_1)I_H = 0 \quad (3.57)$$

$$\gamma I_H + \eta A_H - (\kappa + \phi + \mu_1)T_H = 0 \quad (3.58)$$

$$\kappa T_H - \mu_1 R_H = 0 \quad (3.59)$$

$$\wedge_2 - \lambda S_R I_R - (\nu + \mu_2)S_R = 0 \quad (3.60)$$

$$\lambda S_R I_R - (\nu + \mu_2)I_R = 0 \quad (3.61)$$

From equation (3.56), we have

$$A_H = \frac{(1-\rho)\alpha E_H}{(\eta + \phi + \mu_1)} \quad (3.62)$$

From equation (3.57), we have

$$I_H = \frac{\rho\alpha E_H}{(\gamma + \phi + \mu_1)} \quad (3.63)$$

Substituting equations (3.62) and (3.63) into equation (3.58) gives

$$\frac{(1-\rho)\alpha E_H}{(\eta + \phi + \mu_1)} + \frac{\rho\alpha E_H}{(\gamma + \phi + \mu_1)} = (\kappa + \phi + \mu_1)T_H \quad (3.64)$$

$$\Rightarrow T_H = \frac{[\eta\alpha\gamma + \eta\alpha\phi + \eta\alpha\mu_1 - \eta\rho\alpha\phi - \eta\rho\alpha\mu_1 + \gamma\rho\alpha\phi + \gamma\rho\alpha\mu_1]E_H}{(\eta + \phi + \mu_1)(\gamma + \phi + \mu_1)(\kappa + \phi + \mu_1)} \quad (3.65)$$

From equation (3.61), we have

$$I_R(\lambda S_R - (\nu + \mu_2)) = 0 \quad (3.66)$$

$$\Rightarrow I_R = 0 \quad (3.67)$$

$$S_R = \frac{\nu + \mu_2}{\lambda} \tag{3.68}$$

Substituting equations (3.62), (3.63) ,(3.65) and (3.67) into equation (3.55) gives

$$\begin{bmatrix} \left[\frac{(1-\rho)\alpha}{(\eta+\phi+\mu_1)} + \frac{\rho\alpha E_H}{(\gamma+\phi+\mu_1)} + \frac{\delta[\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1]}{(\eta+\phi+\mu_1)(\gamma+\phi+\mu_1)(\kappa+\phi+\mu_1)} \right] S_H \\ -(\alpha+\mu_1) \end{bmatrix} E_H = 0$$

$$\Rightarrow E_H = 0 \tag{3.69}$$

$$S_H = \frac{(\alpha+\mu_1)(\eta+\phi+\mu_1)(\gamma+\phi+\mu_1)(\kappa+\phi+\mu_1)}{\beta(\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1)(\kappa+\phi+\mu_1+\delta)} \tag{3.70}$$

Substituting (3.69) into (3.62), (3.63) and (3.65) gives

$$A_H = 0 \tag{3.71}$$

$$I_H = 0 \tag{3.72}$$

$$T_H = 0 \tag{3.73}$$

Substituting (3.73) into (3.59) gives

$$R_H = 0 \tag{3.74}$$

Substituting (3.67) into (3.60) gives

$$S_R = \frac{\wedge_2}{\nu + \mu_2} \tag{3.75}$$

Substituting (3.67), (3.71), (3.72) and (3.73) into (3.54) gives

$$S_H \frac{\wedge_1}{\mu_1} \tag{3.76}$$

Equations (3.67), (3.69), (3.71), (3.72), (3.73),(3.74), (3.75), (3.76) give the disease-free equilibrium state.

That is,

$$[S_H, E_H, A_H, I_H, T_H, R_H, S_R, I_R] = \left[\frac{\wedge_1}{\mu_1}, 0, 0, 0, 0, 0, \frac{\wedge_2}{\nu+\mu_2}, 0 \right] \tag{3.77}$$

Existence of Endemic equilibrium state

Theorem : An Endemic Equilibrium state exist at $(S_H, E_H, A_H, I_H, T_H, R_H, S_R, I_R)$=

$$\left(\begin{array}{l}\dfrac{(\alpha+\mu_1)(\eta+\phi+\mu_1)(\gamma+\phi+\mu_1)(\kappa+\phi+\mu_1)}{\beta(\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1)(\kappa+\phi+\mu_1+\delta)},\\ \dfrac{A}{B},\dfrac{(1-\rho)\alpha A}{(\eta+\phi+\mu_1)B},\dfrac{\rho\alpha A}{(\gamma+\phi+\mu_1)B},\dfrac{[\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1]A}{(\eta+\phi+\mu_1)(\gamma+\phi+\mu_1)(\kappa+\phi+\mu_1)B},\\ \dfrac{\kappa[\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1]A}{\mu_1(\eta+\phi+\mu_1)(\gamma+\phi+\mu_1)(\kappa+\phi+\mu_1)B},\dfrac{\nu+\mu_2}{\lambda},\dfrac{\wedge_2\lambda-(\nu+\mu_2)^2}{\lambda(\nu+\mu_2)}\end{array}\right)$$

Where,

$$A=\begin{pmatrix}\wedge_1\beta\lambda(\nu+\mu_2)(\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1)(\kappa+\phi+\mu_1+\delta)\\ -(\alpha+\mu_1)(\eta+\phi+\mu_1)(\gamma+\phi+\mu_1)(\kappa+\phi+\mu_1)[\beta(\wedge_2-(\nu+\mu_2)^2)+\mu_1\lambda(\nu+\mu_2)]\end{pmatrix}$$

and

$$B=\beta\lambda(\nu+\mu_2)(\alpha+\mu_1)[\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1](\kappa+\phi+\mu_1+\delta)$$

Proof:

Substituting (3.68) into (3.60) gives

$$I_R=\frac{\wedge_2\lambda-(\nu+\mu_2)^2}{\lambda(\nu+\mu_2)} \qquad (3.78)$$

Substituting equations (3.62), (3.63), (3.65), (3.70) and (3.78) into equation (3.54) gives

$$\left[\begin{array}{l}\wedge_1-\left[\left(\begin{array}{l}\beta\left(\dfrac{\wedge_2\lambda-(\nu+\mu_2)^2}{\lambda(\nu+\mu_2)}\right)+\beta\dfrac{(1-\rho)\alpha E_H}{(\eta+\phi+\mu_1)}+\beta\dfrac{\rho\alpha E_H}{(\gamma+\phi+\mu_1)}\\ \beta\delta\dfrac{[\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1]E_H}{(\eta+\phi+\mu_1)(\gamma+\phi+\mu_1)(\kappa+\phi+\mu_1)}\end{array}\right)\right]\\ \left(\dfrac{(\alpha+\mu_1)(\eta+\phi+\mu_1)(\gamma+\phi+\mu_1)(\kappa+\phi+\mu_1)}{\beta(\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1)(\kappa+\phi+\mu_1+\delta)}\right)\\ =\dfrac{\mu_1(\alpha+\mu_1)(\eta+\phi+\mu_1)(\gamma+\phi+\mu_1)(\kappa+\phi+\mu_1)}{\beta(\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1)(\kappa+\phi+\mu_1+\delta)}\end{array}\right] \qquad (3.79)$$

$$\left[\begin{array}{l}\frac{\wedge 1\beta(\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1)(\kappa+\phi+\mu_1+\delta)}{(\alpha+\mu_1)(\eta+\phi+\mu_1)(\gamma+\phi+\mu_1)(\kappa+\phi+\mu_1)}- \\ \beta\left(\frac{\wedge_2\lambda-(\nu+\mu_2)^2}{\lambda(\nu+\mu_2)}\right)-\beta\left(\begin{array}{l}\frac{(1-\rho)\alpha}{(\eta+\phi+\mu_1)}+\frac{\rho\alpha}{(\gamma+\phi+\mu_1)}+ \\ \frac{[\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1]}{(\eta+\phi+\mu_1)(\gamma+\phi+\mu_1)(\kappa+\phi+\mu_1)}\end{array}\right)E_H= \\ \mu_1\end{array}\right] \quad (3.80)$$

$$\left[\begin{array}{l}\frac{\wedge 1\beta(\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1)(\kappa+\phi+\mu_1+\delta)}{(\alpha+\mu_1)(\eta+\phi+\mu_1)(\gamma+\phi+\mu_1)(\kappa+\phi+\mu_1)} \\ -\beta\left(\frac{\wedge_2\lambda-(\nu+\mu_2)^2}{\lambda(\nu+\mu_2)}\right)-\mu_1=\beta\left(\begin{array}{l}\frac{(1-\rho)\alpha}{(\eta+\phi+\mu_1)}+\frac{\rho\alpha}{(\gamma+\phi+\mu_1)}+ \\ \frac{[\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1]}{(\eta+\phi+\mu_1)(\gamma+\phi+\mu_1)(\kappa+\phi+\mu_1)}\end{array}\right)E_H\end{array}\right] \quad (3.81)$$

$$\left[\begin{array}{l}\frac{\wedge_1\beta\lambda(\nu+\mu_2)(\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1)(\kappa+\phi+\mu_1+\delta)}{\lambda(\nu+\mu_2)(\alpha+\mu_1)(\eta+\phi+\mu_1)(\gamma+\phi+\mu_1)(\kappa+\phi+\mu_1)}- \\ \frac{(\alpha+\mu_1)(\eta+\phi+\mu_1)(\gamma+\phi+\mu_1)(\kappa+\phi+\mu_1)[\beta(\wedge_2-(\nu+\mu_2)^2)+\mu_1\lambda(\nu+\mu_2)]}{\lambda(\nu+\mu_2)(\alpha+\mu_1)(\eta+\phi+\mu_1)(\gamma+\phi+\mu_1)(\kappa+\phi+\mu_1)}= \\ \frac{\beta[\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1](\kappa+\phi+\mu_1+\delta)E_H}{(\eta+\phi+\mu_1)(\gamma+\phi+\mu_1)(\kappa+\phi+\mu_1)}\end{array}\right] \quad (3.82)$$

$$E_H=\frac{\left(\begin{array}{l}\wedge_1\beta\lambda(\nu+\mu_2)(\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1)(\kappa+\phi+\mu_1+\delta) \\ -(\alpha+\mu_1)(\eta+\phi+\mu_1)(\gamma+\phi+\mu_1)(\kappa+\phi+\mu_1)[\beta(\wedge_2-(\nu+\mu_2)^2)+\mu_1\lambda(\nu+\mu_2)]\end{array}\right)}{\beta\lambda(\nu+\mu_2)(\alpha+\mu_1)[\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1](\kappa+\phi+\mu_1+\delta)} \quad (3.83)$$

Substituting (3.83) into (3.62) gives

$$A_H=\frac{(1-\rho)\alpha}{(\eta+\phi+\mu_1)}\left(\begin{array}{l}\frac{\left(\begin{array}{l}\wedge_1\beta\lambda(\nu+\mu_2)(\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1) \\ (\kappa+\phi+\mu_1+\delta)-(\alpha+\mu_1)(\eta+\phi+\mu_1)(\gamma+\phi+\mu_1)(\kappa+\phi+\mu_1) \\ [\beta(\wedge_2-(\nu+\mu_2)^2)+\mu_1\lambda(\nu+\mu_2)]\end{array}\right)}{\beta\lambda(\nu+\mu_2)(\alpha+\mu_1)[\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1]} \\ (\kappa+\phi+\mu_1+\delta)\end{array}\right) \quad (3.84)$$

Substituting (3.83) into (3.63) gives

$$I_H = \frac{\rho\alpha}{(\gamma+\phi+\mu_1)}\left(\frac{\begin{pmatrix}\wedge_1\beta\lambda(\nu+\mu_2)(\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1)\\(\kappa+\phi+\mu_1+\delta)-(\alpha+\mu_1)(\eta+\phi+\mu_1)(\gamma+\phi+\mu_1)(\kappa+\phi+\mu_1)\\[\beta(\wedge_2-(\nu+\mu_2)^2)+\mu_1\lambda(\nu+\mu_2)]\end{pmatrix}}{\beta\lambda(\nu+\mu_2)(\alpha+\mu_1)[\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1](\kappa+\phi+\mu_1+\delta)}\right) \quad (3.85)$$

Substituting (3.83) into (3.65) gives

$$T_H = \left[\frac{\begin{pmatrix}[\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1]\\(\wedge_1\beta\lambda(\nu+\mu_2)(\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1)(\kappa+\phi+\mu_1+\delta)\\-(\alpha+\mu_1)(\eta+\phi+\mu_1)(\gamma+\phi+\mu_1)(\kappa+\phi+\mu_1)[\beta(\wedge_2-(\nu+\mu_2)^2)+\mu_1\lambda(\nu+\mu_2)]\end{pmatrix}}{(\eta+\phi+\mu_1)(\gamma+\phi+\mu_1)(\kappa+\phi+\mu_1)}}{[\beta\lambda(\nu+\mu_2)(\alpha+\mu_1)[\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1](\kappa+\phi+\mu_1+\delta)]}\right] \quad (3.86)$$

Substituting (3.86) into (3.60) gives

$$R_H = \left[\frac{\begin{pmatrix}\kappa[\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1]\\(\wedge_1\beta\lambda(\nu+\mu_2)(\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1)(\kappa+\phi+\mu_1+\delta)\\-(\alpha+\mu_1)(\eta+\phi+\mu_1)(\gamma+\phi+\mu_1)(\kappa+\phi+\mu_1)[\beta(\wedge_2-(\nu+\mu_2)^2)+\mu_1\lambda(\nu+\mu_2)]\end{pmatrix}}{\mu_1(\eta+\phi+\mu_1)(\gamma+\phi+\mu_1)(\kappa+\phi+\mu_1)}}{[\beta\lambda(\nu+\mu_2)(\alpha+\mu_1)[\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1](\kappa+\phi+\mu_1+\delta)]}\right] \quad (3.87)$$

Equations (3.68), (3.70), (3.78), (3.83), (3.84), (3.85), (3.86), (3.87) give the endemic equilibrium state.

That is;

(S_H, E_H, A_H, I_H, T_H, R_H, S_R, I_R)=

$$\left(\begin{matrix}\frac{(\alpha+\mu_1)(\eta+\phi+\mu_1)(\gamma+\phi+\mu_1)(\kappa+\phi+\mu_1)}{\beta(\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1)(\kappa+\phi+\mu_1+\delta)},\\ \frac{A}{B},\frac{(1-\rho)\alpha A}{(\eta+\phi+\mu_1)B},\frac{\rho\alpha A}{(\gamma+\phi+\mu_1)B},\frac{[\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1]A}{(\eta+\phi+\mu_1)(\gamma+\phi+\mu_1)(\kappa+\phi+\mu_1)B},\\ \frac{\kappa[\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1]A}{\mu_1(\eta+\phi+\mu_1)(\gamma+\phi+\mu_1)(\kappa+\phi+\mu_1)B},\frac{\nu+\mu_2}{\lambda},\frac{\wedge_2\lambda-(\nu+\mu_2)^2}{\lambda(\nu+\mu_2)}\end{matrix}\right)$$

Where,

$$A = \begin{pmatrix}\wedge_1\beta\lambda(\nu+\mu_2)(\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1)(\kappa+\phi+\mu_1+\delta)\\-(\alpha+\mu_1)(\eta+\phi+\mu_1)(\gamma+\phi+\mu_1)(\kappa+\phi+\mu_1)[\beta(\wedge_2-(\nu+\mu_2)^2)+\mu_1\lambda(\nu+\mu_2)]\end{pmatrix} \quad (3.88)$$

$$B = \beta\lambda(\nu+\mu_2)(\alpha+\mu_1)[\eta\alpha\gamma+\eta\alpha\phi+\eta\alpha\mu_1-\eta\rho\alpha\phi-\eta\rho\alpha\mu_1+\gamma\rho\alpha\phi+\gamma\rho\alpha\mu_1](\kappa+\phi+\mu_1+\delta) \quad (3.89)$$

CONCLUSION

We have studied a Mathematical Recipe for Controlling the Spread of Lassa Fever, The solution set of the model remain positive for all t>0. There exists a Disease Free and Endemic Equilibrium States of the model signifying the possibility of total eradication of the Disease from the population.

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