Harvard Research and Publications International

December 31, 2019 Vol. 11 No. 2 ISSN: 0081 – 3998

JOURNAL OF AFRICAN SUSTAINABLE DEVELOPMENT

NUMERICAL SIMULATIONS OF A DETERMINISTIC COMPARTMENTAL MODEL OF ZIKAVIRUS DISEASE DYNAMICS WITH CONTROLS.

[1]ABDULLAH IDRIS ENAGI, [2] MOHAMMED OLANREWAJU IBRAHIM [1]ISHAQ ZAINAB.

[1]Department of Mathematics, Federal University of Technology, Minna, Nigeria. [2]Department of Mathematics, University of Illorin, Nigeria.

Abstract

In this study, a deterministic model of Zika virus disease dynamics with controls was presented. Parameters for treatment and use of insecticides were incorporated. The model equations were solved using Homotopy perubation method. Graphical profiles of each compartment were generated from the results using maple software. The result of the numerical

Keywords: *Zika virus, Control Parameters, Homotopy Perturbation, Insecticides and Treatment*

INTRODUCTION

Zika virus disease is a member of the family called Flaviviridae and the genus Flavivirus. (Musso *et al*,2015). World health organization WHO has concluded that Zika virus infection during pregnancy is a cause of congenital brain abnormalities, including microcephaly; and that infection is a trigger of Guillain-Barre syndrome. Intense efforts are continuing to investigate the link between infectious virus and a range of neurological disorders, within a rigorous research framework (WHO, 2016). According to Kucharski *et al*., (2016) several lines of proof agree that the infection transmission is causing symptoms such as fever and rash. The infection has also been linked to increased incidence of neurological disorder, including Guillain-Barre Syndrome and microcephaly in infants

simulation indicates that treatment and the use of insecticides is the best strategy to control the disease.

born to mothers who were infected with infectious virus during pregnancy.

About 80% of human infectivity develops mild disease consisting of mild fever, skin rashes, arthralgia, particularly swelling, lymphadenopathy, malaise or headache, conjunctivitis (Red eyes), muscle and joint pain, which last for 2-7 days normally ,with no documented fatalities seen in a recent large outbreak. The microcephaly morbidity rate of the new baby is very high if the pregnancy was infected by the disease. And some evidence suggests that it causes Guillain-Barre syndrome (GBS) as well. According to the US Centre for Disease Control (CDC).

Infectivity of the disease is through mosquito bites, it is transmitted to people primarily through the bite of an infected *Aedes* species mosquito. It can be transmitted from mother to child; a pregnant woman can pass the virus to her fetus during pregnancy (Centre for Disease Control, 2016). The infection is the cause of microcephaly and other severe brain defects. The infectious virus can remain in semen longer than in other body fluids, including vaginal fluids, urine, and blood. However, the transmission through blood transfusion, were reports of laboratory acquired Zika virus infections, although the route of transmission was not clearly established in all cases,(Musso *et al*,2015)

Infection with Zika virus may be suspected based on symptoms and recent history of travel. A diagnosis of the virus infection can only be confirmed through laboratory tests on blood or other body fluids, such as urine, saliva or semen (CDC, 2016).

For now, there is no vaccine for prevention of the disease. However, it can be prevented by avoiding mosquito bites through the use of bed nets and the use of insecticide spray and proper sanitation. The use of Condoms can reduce the chance of getting the virus from sex. In the case of blood transfusion, bloods should be screen before it is been transmitted to humans (CDC, 2016).

Unfortunately, no vaccine, specific treatment, or fast diagnostic test is available to treat, prevent, or diagnose ZIKV infection at this time. There is no specific Treatment for the disease yet but apparent symptoms are normally mild and can be treated with common pain and fever medicines, bed rest and taking plenty of water and healthy fluids (Khalid & Khan 2016)

LITERATURE REVIEW

Ding *et al* (2016) formulated a compartmental mathematical model according to the transmission rules in order to investigate the interior mechanism of ZIKA virus transmission. Model results illustrate the use of basic reproduction number, the optimal control strategies are suggested to provide and control the disease. In their model, recruitment rate of mosquitoes, probability of both human and mosquitoes getting infected, natural death of both mosquitoes and human, death due to infection and the recovered class was not also considered.

Bonyah and Okosun (2016) formulated a model of ZIKA, virus with its optimal control strategies. In their models, they proposed optimal control to examine the effect of mass treatment and insecticide. The purpose of this control is to minimize the number of infected host and vectors with optimal cost of mass treatment and insecticide. The global stability of disease-free equilibrium was found to be asymptotically stable. Optimal control strategies of prevention, treatment and insecticide are incorporated into the model. However, the model does not consider death due to the infection of ZIKA virus.

The model:

Colombia (Korzeniewski *et al*., 2016). Developed a mathematical model of transmission dynamics of Zika virus. The model examined 2013-14 outbreak of the virus infection in French Polynesia, the results suggested that the infection transmission in island population may follow similar pattern to dengue fever, generating large, sporadic outbreak with high degree of under- reporting. However, in their model recruitment rate of mosquitoes, natural death of human, natural death of mosquitoes and also death due to the infection of ZIKA virus infection was not considered.

Momoh and Fugenschuh (2017) developed an optimal control strategy and a cost effectiveness analysis for the Zika virus disease, they considered four preventive measures as control strategy: the use of treated bed nets, the use of condoms, a medical treatment of infected persons, and the use of indoor residual spray. They obtained the reproduction number for the model and carried out a stability analysis, they also examine the implementation of various combinations of the controls in order to determine the most cost-effective strategy among all the control strategies considered, based on the computational results obtained, they concluded that a strategy based on treated bed nets, treatment of infected individuals, and indoor residual spray is the most cost effective of all their examined strategies for a control of the Zika virus. However in their models death due to Zika virus infection was not considered.

Onuorah *et al*., (2016) developed a deterministic model describing the dynamics of ZIKA virus; a set of ordinary differential equation was developed. The basic reproductive number (R_0) of the model which is an important threshold for disease control was obtained. Disease free equilibrium (E_0) which is stable whenever $R_0 < 1$, this means that one infected individual introduced in to an environment where everyone is susceptible is effecting less than one person, in which case, the virus will die out. Further analysis of the model shows that the endemic equilibrium exists and the conditions for its stability are given. However death due to Zika virus infection was not considered.

MATERIALS AND METHODS

We formulate a mathematical model of Zika virus infection. The model incorporated the use of control strategies (Treatment and insecticides) where Treatment is represented by u_1 and insectcides by u_2. The model contains two populations, which are human and Aedes mosquitoes with three and two compartments respectively. We examine the susceptible-infected-recovered model to explore the dynamics of transmission of Zika to human. The three compartments, namely: susceptible human (S_H), infected human (I_H) and the recovered human (R_H). The other two compartments for aedes mosquitoes are susceptible aedes mosquitoes (S_A) and infected aedes (I_A).
This leads to the following SIR-SI model.

MODEL EQUATIONS

$$\frac{dS_H}{dt} = \Lambda_H - (\beta \varepsilon \phi I_A + \phi_1 I_H + \mu_H) S_H \tag{3.1}$$

$$\frac{dI_H}{dt} = (\beta \varepsilon \phi I_A + \phi_1 I_H) S_H - (\mu_H + \mu_0 + \gamma_H + u_1) I_H \tag{3.2}$$

$$\frac{dR_H}{dt} = (\gamma_H + u_1) I_H - \mu_H R_H \tag{3.3}$$

$$\frac{dS_A}{dt} = \Lambda_A - (\beta_1 \varepsilon \phi I_H + \mu_A + u_2) S_A \tag{3.4}$$

$$\frac{dI_A}{dt} = \beta_1 \varepsilon \phi I_H S_A - (\mu_A + u_2) I_A \tag{3.5}$$

With variables and parameters defined as follows

Λ_H Recruitment rate of human.

Λ_A Recruitment rate of Aedes mosquitoes.

S_H Susceptible individuals.

I_H Infected individuals.

R_H Recovered individual.

S_A Susceptible Aedes mosquitoes.

I_A Infected Aedes mosquitoes.

β Transmission probability of human getting infected

β_1 Transmission probability of Aedes mosquitoes getting infected

ε Per capital biting rate.

ϕ Contact rate of vector per human.

ϕ_1 Transmission probability after sexual interaction between a susceptible human and infected human.

γ_H Natural recovery.

μ_H Natural death rate of human

μ_A Natural death of Aedes mosquitoes

μ_0 Death due to Zika virus infection

ANALYTICAL SOLUTION OF THE MODEL USING HOMOTOPY PERTURBATION METHOD (HPM)

The fundamental of Homotopy Perturbation Method (HPM) was first proposed by Ji-Haun (2000). The Homotopy Perturbation Method (HPM), which provides analytical approximate solution, is applied to various linear and non-linear equations. The homotopy perturbation method (HPM) is a series expansion method used in the solution of nonlinear partial differential equations Jiya (2010).

To show the simple concepts of this method, he considered the following non-linear differential equation:

$$A_3(U)-f(r)=0, \qquad r\in\Omega \tag{3.6}$$

Subject to the boundary condition

$$B_3\left(U,\frac{\partial U}{\partial n}\right)=0, \qquad r\in\Gamma \tag{3.7}$$

where A_3 is a general differential operator, B_3 a boundary operator, $f(r)$ is a known analytical function and Γ is the boundary of the domain Ω. The operator

A3 can be divided into two parts L and N, where L is the linear part, and N is the nonlinear part. Equation (3.6) can be written as:

$$L(U)+N(U)-f(r)=0, \quad r \in \Omega \tag{3.8}$$

The Homotopy Perturbation structure is shown as follows

$$H(V,h)=(1-h)[L(V)-L(U_0)]+h[A(V)-f(r)]=0 \tag{3.9}$$

where $V(r,P):\Omega \in [0,1] \to R$

In equation (3.8) $P \in [0,1]$ is an embedding parameter and U_0 is the approximation that satisfies the boundary condition. It can be assumed that the solution of the equation (3.8) can be written as power series in h as follows:

$$V=V_0+hV_1+h^2V_2+\ldots \tag{3.10}$$

And the best approximation for the solution is:

$$U=\lim_{h\to 1} v=v_0+hv_1+h^2v_2+\ldots \tag{3.11}$$

The series (3.11) is convergent for most cases. However, the convergent rate depends on the nonlinear operator A (V)

3.3 SOLUTION OF THE MODEL EQUATIONS

$$\frac{dS_H}{dt}+(\beta\varepsilon\phi I_A+\phi_1 I_H+\mu_H)S_H-\Lambda_H=0 \tag{3.12}$$

$$\frac{dI_H}{dt}+(\mu_H+\mu_0+\gamma_H+u_1)I_H-(\beta\varepsilon\phi I_A+\phi_1 I_H)S_H=0 \tag{3. 13}$$

$$\frac{dR_H}{dt}+\mu_H+R_H-(\gamma_H+u_1)I_H=0 \tag{3.14}$$

$$\frac{dS_A}{dt}+(\beta_1\varepsilon\phi I_H+\mu_A+u_2)S_A-\Lambda_A=0 \tag{3.15}$$

$$\frac{dI_A}{dt}+(\mu_A+u_2)I_A-\beta_1\varepsilon\phi I_H S_A=0 \tag{3.16}$$

With the following initial conditions $S_H(0)=S_{H(0)}$, $I_H(0)=I_{H(0)}$, $R_H(0)=R_{H(0)}$, $S_A(0)=S_{A(0)}$,

And $I_A(0)=I_{A(0)}$

Let

$$S_H = a_o + ha_1 + h^2 a_2 + ... \quad (3.17)$$

$$I_H = b_o + hb_1 + h^2 b_2 + ... \quad (3.18)$$

$$R_H = c_O + hc_1 + h^2 c_2 + ... \quad (3.19)$$

$$S_A = d_o + hd_1 + h^2 d_2 + ... \quad (3.20)$$

$$I_A = e_o + he_1 + h^2 e_2 + ... \quad (3.21)$$

Applying HPM to (3.12)

$$(1-h)\frac{dS_H}{dt} + h\left[\frac{dS_H}{dt} + (\beta\varepsilon\phi I_A + \phi_1 I_H + \mu_H)S_H - \Lambda_H\right] = 0 \quad (3.22)$$

Substituting equation (3.17),(3.18) and (3.21) in to equation (3.12)

$$(1-h)(a_o^' + ha_1^' + h^2 a_2^' + ...) + h\left[\begin{array}{l} a_o^' + ha_1^' + ha_2^' + ... \\ (\beta\varepsilon\varphi(b_o + hb_1 + h^2 b_2 + ...) + \varphi(b_o + hb_1 + h^2 b_2 + ...) \end{array}\right.$$

$$+\mu_H)(a_o + ha_1 + h^2 a_2 + ...) - \Lambda_H] \quad (3.23)$$

Expanding and collecting the coefficient of the powers of h,we have

$$h^o : a_o^' = 0 \quad (3.24)$$

$$h^1 : a_1^' + \beta\varepsilon\phi b_o + \phi_1 b_o + \mu_H + a_o - \Lambda_H = 0 \quad (3.25)$$

$$h^2 : a_2^' + b_1 \beta\varepsilon\phi + \phi_1 b_1 + a_1 = 0 \quad (3.26)$$

Applying (H P M) to (3.13)

$$(1-h)\frac{dI_H}{dt} + h\left[\frac{dI_H}{dt} + (\mu_H + \mu_O + \gamma_H + u_1)I_H - (\beta\varepsilon\phi I_A + \phi_1 I_H)\right]S_H = 0 \quad (3.27)$$

Substituting equation (3.18),(3.21) and (3.17) in to equation (3.27)

$$(1-h)(b_o^' + hb_1^' + h^2 b_2^' + ...) + h\left[\begin{array}{l} b_o^' + hb_1^' + h^2 b_2^1 + ...(\mu_H + \mu_O + \gamma_H + u_1) \\ (hb_1 + h^2 b_2 + ...) - (\beta\varepsilon\varphi(e^o + he_1 + h^2 e_2) \end{array}\right.$$

$$+\varphi_1(b_o + bh_1 + h^2 b_2 + ...)(a_o + ha_1 + h^2 a_2 + ...)]$$

(3.28)

Expanding and collecting the coefficients of the powers of h,we have

$$h^o : b_o^' = 0 \quad (3.29)$$

$$h^1 : b_1^' + (\mu_H + \mu_o + \gamma_H + u_1)b_0 - \beta\varepsilon\phi e_0 + \phi_1 b_o + a_o = 0 \quad (3.30)$$

$$h^2 : b_2^' + (b_1(\mu_H + \mu_O + \gamma_H + u_1)b_1 - (e_1 \beta\varepsilon\phi + \phi_1 b_1 + a_1) = 0 \quad (3.31)$$

Applying HPM to (3.14)

$$(1-h)\frac{dR_H}{dt}+h\left[\frac{dR_H}{dt}+\mu_H+R_H-(\gamma_H+u_1)I_H\right]=0 \quad (3.32)$$

Substituting equation (3.19) and (3.18) in to equation (3.32)

$$(1-h)\begin{bmatrix}\left(c_o^1+hc_1^{'}+h^2c_2^{'}+.....\right)+\left(c_o^{'}+hc_1^{'}+h^2c_2^{'}+....\right)\\ \left(\mu_H\left(c_o+hc_1+h^2c_2+......\right)\gamma_H+u_1\left(b_o+hb_1+h^2b_2+....\right)\right)\end{bmatrix}=0 \quad (3.33)$$

Expanding and collecting the coefficient of the powers of h,we have

$$h^o:c_o^{'}=0 \quad (3.34)$$

$$h^1:c_1^{'}+\mu_H c_o-(\gamma_H+u_1)b_o=0 \quad (3.35)$$

$$h^2:c_2^{'}+\mu_H c_1+(\gamma_H+u_1)b_1=0 \quad (3.36),$$

Applying HMP to (3.15)

$$(1-h)\frac{dS_A}{dt}+h\left[\frac{dS_A}{dt}+(\beta_1\varepsilon\phi I_A+(\mu_A+u_2))S_A-\Lambda_A\right]=0 \quad (3.37)$$

Substituting equation (3.20) and (3.21) in to equation (3.37)
Expanding and collecting of powers of h, we have

$$h^o:d_o^1=0 \quad (3.38)$$

$$h^1:d_1^{'}+\beta_1\varepsilon\phi e_o+\mu_A+u_2+d_o=0 \quad (3.39)$$

$$h^2:d_2^1+e_1\beta_1\phi+\mu_A+u_2+d_1=0 \quad (3.40)$$

Applying HMP to (3.16)

$$(1-h)\frac{dI_A}{dt}+h\left[\frac{dI_A}{dt}+(\mu_A+u_2)I_A-\beta_1\varepsilon\phi I_H S_A)\right]=0 \quad (3.41)$$

Substituting equation (3.21),(3.18) and (3.20) in to equation (3.41)

$$(1-h)\left(e_o^{'}+he_1^{'}+h^2e_2^{'}+...\right)+h\left(e_o^{'}+he_1^{'}+h^2a_2^{'}+....\right)$$

$$\left(\mu_A+u_2\left(e_o+he_1+h^2e_2+.....\right)\left(\beta_1\varepsilon\varphi\left(b_0+hb_1+h^2b_2+...\right)\right.\right.$$

$$\left(a_o+ha_1+h^2a_2+.....\right)=0 \quad (3.42)$$

Expanding and collecting the coefficient of the powers of h,we have

$$h^o:e_o^{'}=0 \quad (3.43)$$

$$h^1:e_1^{'}+(\mu_A+u_2)e_o-\beta_1\varepsilon\phi b_o+a_o=0 \quad (3.44)$$

$$h^2:e_2^{'}+e_1(\mu_A+u_2)-\beta_1\varepsilon\phi b_1+a_1=0 \quad (3.45)$$

From equation (3.24)

$$a_o^{'} = 0 \tag{3.46}$$

Integrating both sides and then applying the initial condition

$$a_o(o) = S_{H(O)} = a_0 \tag{3.47}$$

we have

$$a_o = S_{H(O)} \tag{3.48}$$

From (3.29)

$$b_o^{'} = 0 \tag{3.49}$$

Integrating and applying the initial condition

$$b_o(0) = I_{H(0)} = b_o \tag{3.50}$$

We have

$$b_0 = I_{H(0)} \tag{3.51}$$

From (3.34)

$$h^o : c_o^{'} = 0 \tag{3.52}$$

Integrating (3.126) and applying the initial condition

$$c_0(0) = R_{H(0)} = c_0 \tag{3.53}$$

We have

$$c_0 = R_{H(0)} \tag{3.54}$$

From equation (3.38)

$$h^o : d_o^1 = 0 \tag{3.55}$$

Integrating equation (3.129) and applying the initial condition

$$d_o(0) = S_{A(0)} = d_o \tag{3.56}$$

i.e

$$d_o = S_{A(0)} \tag{3.57}$$

From (3.43)

$$h^o : e_o^{'} = 0 \tag{3.58}$$

Integrating equation and applying the initial condition

$$e_o(0) = I_{A(0)} = e_o \tag{3.59}$$

i.e

$$e_o = I_{A(0)} \tag{3.60}$$

From equation (3.25)

$$h^1 : a_1^{'} + \beta \varepsilon \phi b_o + \phi_1 b_o + \mu_H + a_o - \Lambda_H = 0 \quad (3.61)$$

$$a_1^{'} = \Lambda_H - \beta \varepsilon \phi b_o - \phi_1 b_o - \mu_H \quad (3.62)$$

Integrating (3.62) and applying the initial condition $a_1(0) = 0$

$$a_1 = (\Lambda_H - \beta \varepsilon \phi b_o - \phi_1 b_o - \mu_H)t \quad (3.63)$$

Put equation (3.51) in to equation (3.63)

$$a_1 = (\Lambda_H - \beta \varepsilon \phi I_{H(0)} - \phi I_{H(0)} - \mu_H)t \quad (3.64)$$

From equation (3.30)

$$h^1 : b_1^{'} + (\mu_H + \mu_o + \gamma_H + u_1)b_0 - \beta \varepsilon \phi e_o + \phi_1 b_o + a_o = 0 \quad (3.65)$$

$$b_1^{'} = \beta \varepsilon \phi e_0 - a_0 - \phi_0 b_0 - (\mu_H + \mu_0 + \gamma_H + u_1)b_0 \quad (3.66)$$

Integrating equation (3.66) and applying the initial condition $b_1(0) = 0$

$$b_1 = ((\beta \varepsilon \phi e_0 - a_o - \phi_1 b_0 - (\mu_H + \mu_0 + \gamma_H + u_1)b_0)\, t \quad (3.67)$$

Substituting equation (3.60),(3.48)and (3.52) in to (3.67)

$$b_1 = ((\beta \varepsilon \phi R_{H(0)} - S_{H(0)} - \phi I_{H(0)} - (\mu_H + \mu_0 + \gamma_H + u_1))I_{H(0)})t \quad (3.68)$$

From equation (3.35)

$$h^1 : c_1^{'} + \mu_H c_o - (\gamma_H + u_1)b_o = 0 \quad (3.69)$$

$$c_1^{'} = (\gamma_H + u_1)b_0 - \mu_H c_0 = 0 \quad (3.70)$$

Integrating equation (3.70) and applying the initial condition $c_1(0) = 0$

$$c_1 = ((\gamma_H + u_1)b_0 - \mu_H c_0)t \quad (3.71)$$

Substituting equation (3.51) and (3.54) in to equation (3.71)

$$c_1 = ((\gamma_H + u_1)I_{H(0)} - \mu_H R_{H(0)})\, t \quad (3.72)$$

From equation (3.39)

$$h^1 : d_1^{'} + \beta_1 \varepsilon \phi e_o + (\mu_A + u_2) + d_o = 0 \quad (3.73)$$

$$d_1^{'} = (\mu_A + u_2) - d_o - \beta_1 \varepsilon \phi e_0 = 0 \quad (3.74)$$

Integrating equation (3.74) and applying the initial condition $d_1(0) = 0$

$$d_1 = ((\mu_A + u_2) - d_0 - \beta_1 \varepsilon \phi e_0)t \quad (3.75)$$

Substituting equation (3.57) and (3.60) in to (3.75)

$$d_1 = ((\mu_A + u_2) - S_{A(0)} - \beta_1 \varepsilon \phi I_{A(0)})\, t \quad (3.76)$$

From equation (3.44)

$$h^1 : e_1^{'} + (\mu_A + u_2)e_o - \beta_1 \varepsilon \phi b_o + a_o = 0 \quad (3.77)$$

$$e_1' = (a_0 - (\mu_A + u_2)e_0 + \beta_1 \varepsilon \phi b_0) = 0 \tag{3.78}$$

Integrating equation (3.78) and applying the initial condition $e_1(0) = 0$

$$e_1 = (a_0 - (\mu_A + u_2)e_0 + \beta_1 \varepsilon \phi b_0)\, t \tag{3.79}$$

Substituting equation (3.48),(3.60) and (3.51) in to equation (3.79)

$$e_1 = (S_{H(0)} - (\mu_A + u_2) I_{A(0)} + \beta_1 \varepsilon \phi I_{H(0)})\, t \tag{3.80}$$

From equation (3.26)

$$h^2 : a_2' + b_1 \beta \varepsilon \phi + \phi_1 b_1 + a_1 = 0 \tag{3.81}$$

$$a_2' = -b_1 \beta \varepsilon \phi - \phi_1 b_1 - a_1 \tag{3.82}$$

Put equation (3.68) and (3.63) in to equation

$$a_2' = \begin{bmatrix} -(\beta \varepsilon \varphi + \varphi_1)\left(\left(\beta \varepsilon \varphi R_{H(0)} - S_{H(0)} - \varphi I_{H(0)} - (\mu_H + \mu_0 + \gamma_H + u_1) I_{H(0)}\right)\right. \\ -\left(\Lambda_H - \beta \varepsilon \varphi I_{H(0)} - \varphi I_{H(0)} - \mu_H\right) \end{bmatrix} t \tag{3.83}$$

Integrating (3.83) and applying the initial condition $a_2(0) = 0$,we have

$$a_2 = \begin{bmatrix} (-\beta \varepsilon \varphi + \varphi_1)\left(\left(\beta \varepsilon \varphi R_{H(0)} - S_{H(0)} - \varphi I_{H(0)} - (\mu_H + \mu_0 + \gamma_H + u_1) I_{H(0)}\right)\right. \\ -\left(\Lambda_H - \beta \varepsilon \varphi I_{H(0)} - \varphi I_{H(0)} - \mu_H\right) \end{bmatrix} \frac{t^2}{2} \tag{3.84}$$

Substitute equation (3.48), (3.64) and (3.84) in to equation (3.17) and then take these limit as $h \to 1$

$$S_H = S_{H(O)} + \left(\Lambda_H - \beta \varepsilon \varphi I_{H(0)} - \varphi I_{H(0)} - \mu_H\right) t + [(-\beta \varepsilon \varphi + \varphi_1)$$
$$\left(\left(\beta \varepsilon \varphi R_{H(0)} - S_{H(0)} - \varphi I_{H(0)} - (\mu_H + \mu_0 + \gamma_H + u_1) I_{H(0)}\right)\right.$$
$$-\left(\Lambda_H - \beta \varepsilon \varphi I_{H(0)} - \varphi I_{H(0)} - \mu_H\right)] \frac{t^2}{2} \tag{3.85}$$

From equation (3.31),

$$h^2 : b_2' + (b_1(\mu_H + \mu_O + \gamma_H + u_1) b_1 - (e_1 \beta \varepsilon \phi + \phi_1 b_1 + a_1) = 0 \tag{3.86}$$

$$b_2' = (e_1 \beta \varepsilon \phi + \phi_1 b_1 + a_1) + (b_1(\mu_H + \mu_O + \gamma_H + u_1) = 0 \tag{3.87}$$

$$b_2' = \begin{bmatrix} \left(S_{H(0)} - (\mu_A + u_2) I_{A(0)} + \beta_1 \varepsilon \varphi I_{H(0)}\right) \beta \varepsilon \varphi + \\ \varphi_1 \left(\left(\beta \varepsilon \varphi R_{H(0)} - S_{H(0)} - \varphi I_{H(0)} - (\mu_H + \mu_0 + \gamma_H + u_1) I_{H(0)}\right) + \right. \\ \left(\Lambda_H - \beta \varepsilon \varphi I_{H(0)} - \varphi I_{H(0)} - \mu_H\right) - \left(\begin{pmatrix} \beta \varepsilon \varphi R_{H(0)} - S_{H(0)} - \varphi I_{H(0)} \\ -(\mu_H + \mu_0 + \gamma_H + u_1) \end{pmatrix} I_{H(0)}\right) \\ (\mu_H + \mu_0 + \gamma_H + u_1) \end{bmatrix} t \tag{3.88}$$

Integrating equation (3.88) and applying the initial condition $b_2(0)=0$,we have

$$b_2 = \begin{bmatrix} \left(S_{H(0)} - (\mu_A + u_2) I_{A(0)} + \beta_1 \varepsilon \varphi I_{H(0)}\right) \beta \varepsilon \varphi \\ +\varphi_1 \left(\left(\beta \varepsilon \varphi R_{H(0)} - S_{H(0)} - \varphi I_{H(0)} - (\mu_H + \mu_0 + \gamma_H + u_1) I_{H(0)}\right) + \right. \\ \left(\Lambda_H - \beta \varepsilon \varphi I_{H(0)} - \varphi I_{H(0)} - \mu_H\right) - \left(\begin{pmatrix} \beta \varepsilon \varphi R_{H(0)} - S_{H(0)} - \varphi I_{H(0)} \\ -(\mu_H + \mu_0 + \gamma_H + u_1) \end{pmatrix} I_{H(0)} \right) \\ (\mu_H + \mu_0 + \gamma_H + u_1) \end{bmatrix} \frac{t^2}{2} \tag{3.89}$$

Substitute equation (3.51),(3.68)and (3.89)in to (3.18),and then take the limit as $h \to 1$

$$\begin{aligned} I_H &= I_{H(0)} + \left(\left(\beta \varepsilon \varphi R_{H(0)} - S_{H(0)} - \varphi I_{H(0)} - (\mu_H + \mu_0 + \gamma_H + u_1) I_{H(0)}\right) t \right. + \\ &[\left(S_{H(0)} - (\mu_A + u_2) I_{A(0)} + \beta_1 \varepsilon \varphi I_{H(0)}\right) \beta \varepsilon \varphi + \varphi_1 \left(\begin{pmatrix} \beta \varepsilon \varphi R_{H(0)} - S_{H(0)} - \varphi I_{H(0)} \\ -(\mu_H + \mu_0 + \gamma_H + u_1) \end{pmatrix} I_{H(0)} \right) + \\ &\left(\Lambda_H - \beta \varepsilon \varphi I_{H(0)} - \varphi I_{H(0)} - \mu_H\right) - \left(\begin{pmatrix} \beta \varepsilon \varphi R_{H(0)} - S_{H(0)} - \varphi I_{H(0)} \\ -(\mu_H + \mu_0 + \gamma_H + u_1) \end{pmatrix} I_{H(0)} \right) \\ &(\mu_H + \mu_0 + \gamma_H + u_1) \,] \frac{t^2}{2} \end{aligned} \tag{3.90}$$

From equation (3.36),

$$h^2 : c_2' + \mu_H c_1 + (\gamma_H + u_1) b_1 = 0 \tag{3.91}$$

$$c_2' = (\gamma_H + u_1) b_1 - \mu_H c_1 \tag{3.92}$$

Put equation (3.68) and (3.72) in to equation (3.92)

$$c_2' = \begin{bmatrix} \gamma_H + u_1 \left(\left(\beta \varepsilon \varphi R_{H(0)} - S_{H(0)} - \varphi I_{H(0)} - (\mu_H + \mu_0 + \gamma_H + u_1) I_{H(0)}\right) \right. \\ -\mu_H \left((\gamma_H + u_1) I_{H(0)} - \mu_H R_{H(0)} \right) \end{bmatrix} t \tag{3.93}$$

Integrating equation (3.93) and applying the initial condition $c_2(0)=0$

$$c_2 = \begin{bmatrix} \gamma_H + u_1 \left(\left(\beta \varepsilon \varphi R_{H(0)} - S_{H(0)} - \varphi I_{H(0)} - (\mu_H + \mu_0 + \gamma_H + u_1) I_{H(0)}\right) \right. \\ -\mu_H \left((\gamma_H + u_1) I_{H(0)} - \mu_H R_{H(0)} \right) \end{bmatrix} \frac{t^2}{t} \tag{3.94}$$

Substituting equation (3.54),(3.72) and (3.94) in to equation (3.19) and then take the limit as $h \to 1$

$$R_H = R_{H(0)} + \left(\left(\gamma_H + u_1\right) I_{H(0)} - \mu_H R_{H(0)}\right) t +$$
$$[\gamma_H + u_1 \left(\left(\beta \varepsilon \varphi R_{H(0)} - S_{H(0)} - \varphi I_{H(0)} - \left(\mu_H + \mu_0 + \gamma_H + u_1\right) I_{H(0)}\right)\right.$$
$$-\mu_H \left(\left(\gamma_H + u_1\right) I_{H(0)} - \mu_H R_{H(0)}\right)]\frac{t^2}{2} \quad (3.95)$$

From (3.40)

$$h^2 : d_2^1 + e_1 \beta_1 \phi + \left(\mu_A + u_2\right) + d_1 = 0 \quad (3.96)$$

$$d_2^{'} = \left(\mu_A + u_2\right) + d_1 - e_1 \beta_1 \varepsilon \phi \quad (3.97)$$

Substituting equation (3.76) and (3.79) into equation (3.97),

$$d_2^{'} = \begin{bmatrix} \left(\mu_A + u_2\right) + \left(\mu_A + u_2 - S_{A(0)} - \beta_1 \varepsilon \varphi I_{A(0)}\right) \\ -\left(S_{H(0)} - \left(\mu_A + u_2\right) I_{A(0)} + \beta_1 \varepsilon \varphi I_{H(0)}\right) \beta_1 \varepsilon \varphi \end{bmatrix} t \quad (3.98)$$

Integrating equation (3.98) and applying the initial condition $d_2(0) = 0$

$$d_2 = \left[\mu_A + u_2 + \left(\left(\mu_A + u_2\right) - S_{A(0)} - \beta_1 \varepsilon \varphi I_{A(0)}\right) - \begin{pmatrix} S_{H(0)} - \left(\mu_A + u_2\right) I_{A(0)} \\ + \beta_1 \varepsilon \varphi I_{H(0)} \end{pmatrix} \beta_1 \varepsilon \varphi\right] \frac{t^2}{2} \quad (3.99)$$

Substituting equation (3.57),(3.76) and (3.99) in to equation (3.20)

$$S_A = S_{A(0)} + \left(\left(\mu_A + u_2\right) - S_{A(0)} - \beta_1 \varepsilon \varphi I_{A(0)}\right) t +$$
$$[\mu_A + u_2 + \left(\left(\mu_A + u_2\right) - S_{A(0)} - \beta_1 \varepsilon \varphi I_{A(0)}\right) -$$
$$\left(S_{H(0)} - \left(\mu_A + u_2\right) I_{A(0)} + \beta_1 \varepsilon \varphi I_{H(0)}\right) \beta_1 \varepsilon \varphi] \frac{t^2}{2} \quad (3.100)$$

From equation (3.45),

$$h^2 : e_2^{'} + e_1 \left(\mu_A + u_2\right) - \beta_1 \varepsilon \phi b_1 + a_1 = 0 \quad (3.101)$$

$$e_2^1 = \beta_1 \varepsilon \phi b_1 + a_1 - e_1 \left(\left(\mu_A + u_2\right)\right) \quad (3.102)$$

Substituting equation (3.68),(3.64) and (3.80) in to equation (3.102)

$$e_2^{'} = \begin{bmatrix} \beta_1 \varepsilon \varphi \left(\left(\beta \varepsilon \varphi R_{H(0)} - S_{H(0)} - \varphi I_{H(0)} - \left(\mu_H + \mu_0 + \gamma_H + u_1\right) I_{H(0)}\right)\right. \\ + \left(\Lambda_H - \beta \varepsilon \varphi I_{H(0)} - \varphi I_{H(0)} - \mu_H\right) - \\ \left(S_{H(0)} - \left(\mu_A + u_2\right) I_{A(0)} + \beta_1 \varepsilon \varphi I_{H(0)}\right) \mu_A + u_2 \end{bmatrix} t \quad (3.103)$$

Integrating equation (3.103) and applying the initial condition $e_2(0) = 0$

$$e_2 = \begin{bmatrix} \beta_1 \varepsilon \varphi\left(\left(\beta \varepsilon \varphi R_{H(0)} - S_{H(0)} - \varphi I_{H(0)} - (\mu_H + \mu_0 + \gamma_H + u_1) I_{H(0)}\right) + \right. \\ \left(\Lambda_H - \beta \varepsilon \varphi I_{H(0)} - \varphi I_{H(0)} - \mu_H\right) - \\ \left(S_{H(0)} - (\mu_A + u_2) I_{A(0)} + \beta_1 \varepsilon \varphi I_{H(0)}\right) \mu_A + u_2 \end{bmatrix} \frac{t^2}{2} \qquad (3.104)$$

Substituting equation (3.60),(3.80) and (3.104) in to (3.21) and then take the limit as

$$I_A = I_{A(0)} + \left(S_{H(0)} - (\mu_A + u_2) I_{A(0)} + \beta_1 \varepsilon \varphi I_{H(0)}\right) t +$$

$$[\beta_1 \varepsilon \varphi\left(\left(\beta \varepsilon \varphi R_{H(0)} - S_{H(0)} - \varphi I_{H(0)} - (\mu_H + \mu_0 + \gamma_H + u_1) I_{H(0)}\right) +\right.$$

$$h \to 1 \left(\Lambda_H - \beta \varepsilon \varphi I_{H(0)} - \varphi I_{H(0)} - \mu_H\right) - \left(S_{H(0)} - (\mu_A + u_2) I_{A(0)} + \beta_1 \varepsilon \varphi I_{H(0)}\right) \mu_A + u_2] \frac{t^2}{2} \qquad (3.105)$$

Hence, (3.85), (3.90), (3.95), (3.100) and (3.105) are the general solution of the model.

$$S_H = S_{H(O)} + \left(\Lambda_H - \beta \varepsilon \varphi I_{H(0)} - \varphi I_{H(0)} - \mu_H\right) t + [(-\beta \varepsilon \varphi + \varphi_1)$$

$$\left(\left(\beta \varepsilon \varphi R_{H(0)} - S_{H(0)} - \varphi I_{H(0)} - (\mu_H + \mu_0 + \gamma_H + u_1) I_{H(0)}\right) - \left(\Lambda_H - \beta \varepsilon \varphi I_{H(0)} - \varphi I_{H(0)} - \mu_H\right)\right] \frac{t^2}{2}$$

$$I_H = I_{H(0)} + \left(\left(\beta \varepsilon \varphi R_{H(0)} - S_{H(0)} - \varphi I_{H(0)} - (\mu_H + \mu_0 + \gamma_H + u_1) I_{H(0)}\right) t \; +\right.$$

$$[\left(S_{H(0)} - (\mu_A + u_2) I_{A(0)} + \beta_1 \varepsilon \varphi I_{H(0)}\right) \beta \varepsilon \varphi + \varphi_1 \left(\left(\beta \varepsilon \varphi R_{H(0)} - S_{H(0)} - \varphi I_{H(0)} - (\mu_H + \mu_0 + \gamma_H + u_1) I_{H(0)}\right) +\right.$$

$$\left(\Lambda_H - \beta \varepsilon \varphi I_{H(0)} - \varphi I_{H(0)} - \mu_H\right) - \left(\left(\beta \varepsilon \varphi R_{H(0)} - S_{H(0)} - \varphi I_{H(0)} - (\mu_H + \mu_0 + \gamma_H + u_1) I_{H(0)}\right)\right.$$

$$(\mu_H + \mu_0 + \gamma_H + u_1)\,] \frac{t^2}{2}$$

$$R_H = R_{H(0)} + \left((\gamma_H + u_1) I_{H(0)} - \mu_H R_{H(0)}\right) t +$$

$$[\gamma_H + u_1 \left(\left(\beta \varepsilon \varphi R_{H(0)} - S_{H(0)} - \varphi I_{H(0)} - (\mu_H + \mu_0 + \gamma_H + u_1) I_{H(0)}\right) - \mu_H \left((\gamma_H + u_1) I_{H(0)} - \mu_H R_{H(0)}\right)\right] \frac{t^2}{2}$$

$$S_A = S_{A(0)} + \left((\mu_A + u_2) - S_{A(0)} - \beta_1 \varepsilon \varphi I_{A(0)}\right) t +$$

$$[\mu_A + u_2 + \left((\mu_A + u_2) - S_{A(0)} - \beta_1 \varepsilon \varphi I_{A(0)}\right) - \left(S_{H(0)} - (\mu_A + u_2) I_{A(0)} + \beta_1 \varepsilon \varphi I_{H(0)}\right) \beta_1 \varepsilon \varphi] \frac{t^2}{2}$$

NUMERICAL SIMULATIONS OF THE MODEL

With Mapple software we obtained graphical profiles for each compartment using the results obtained above with the following values of variables and parameters.

Variables/parameters	Value	Source
S_H	1000	Assumed
I_H	700	Assumed
R_H	500	Assumed
	1500	Assumed
I_A	1000	Assumed
Λ_H	100/day	Bonya and Okosun(2016)

Λ_A	100/day	Bonyah and Okosun(2016)
β	0.2/day	Mojumder et al. (2016)
β_1	0.09	Okosun et al. -2010
μ_A	14-Jan	Mojumder et al. (2016)
μ_H	1/(365.600)/day	Nishura et al (2016)
μ_0	100	Assumed
γ_H	0.01	Mojumder et al. (2016)

Table 4.1: Variable and Parameters Values of the model

GRAPHICAL PROFILES OF EACH COMPARTMENT

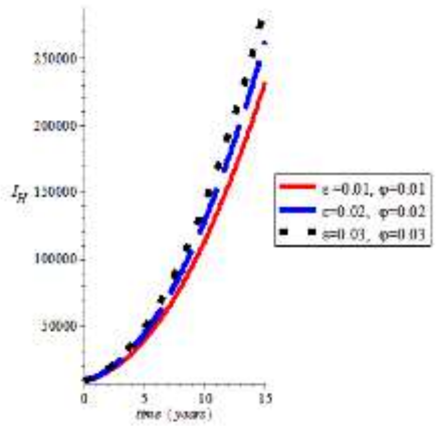

Figure 4.1: Showing the graph of infected individuals against time for different values of biting and contact

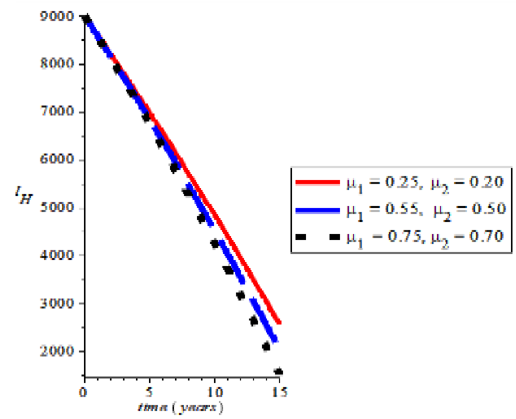

Figure 4.2: Show the graph of infected human against time for different treatment rates.

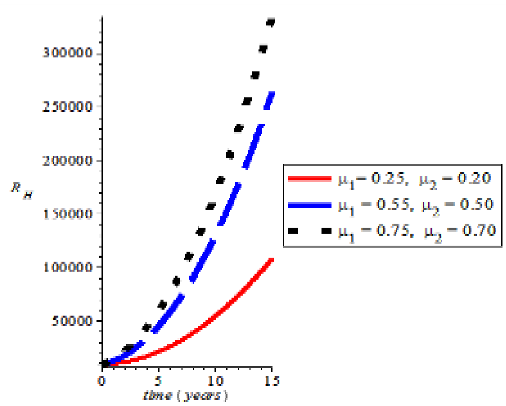

Figure 4.3: Displays the graph of Recovered human against time for varied treatment rate and the use of insecticide.

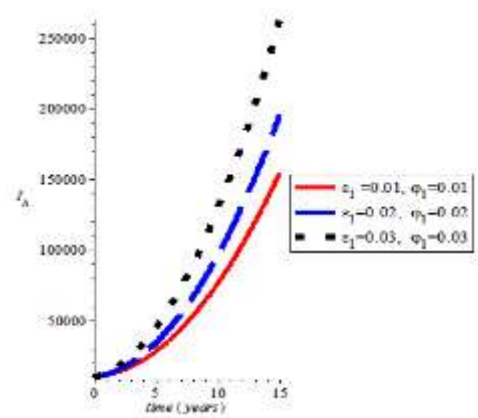

Figure 4.4: Shows the graph of infected Aedes Mosquitors against time for different values of biting and contact rates.

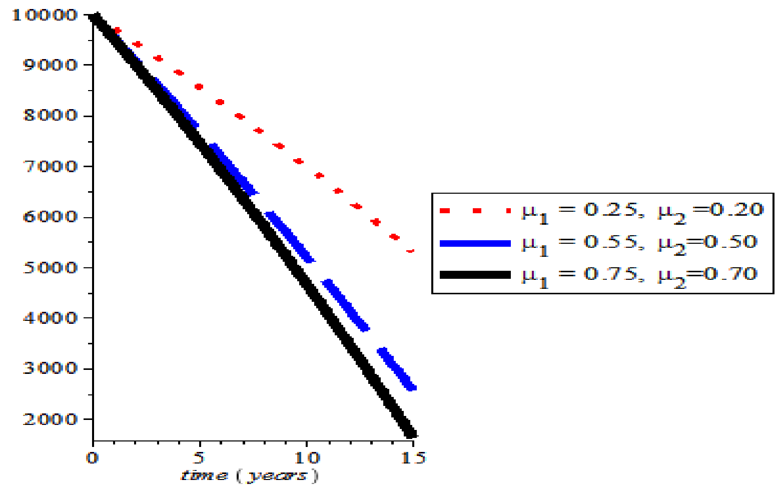

Figure 4.5: Show the graph of Infected Aedes against time for varied treatment rate and the use of insecticide.

Discussion of Results

Figure 4.1 shows the graph of individual infected with Zika virus infection against time for different values of biting and contact rate. The graph shows that as the biting and contact rate increases the number of infected humans also increases (i.e $\varepsilon = 0.01, \varphi = 0.01$, $\varepsilon = 0.02$, $\varphi = 0.02$ *and* $\varepsilon = 0.03, \varphi = 0.03$). It is observed that the population of Zika virus infected individuals' increases as the Zika virus contact parameter also increases. With this in mind, the use of physical barriers such as regular or mesh screens or insecticide treated netting materials on doors and windows should be observed, pregnant women to postpone non-critical travel to areas with ongoing Zika virus transmission and travelers from areas with ongoing Zika virus transmission to practice safer sex and not to donate blood for at least one month after return to Nigeria, to reduce the potential risk of onwards transmission.

Figure 4.2 is the graph of infected human against time for different treatment rate. The graph shows that as the control strategy increases the infected human population decreases (i.e $\mu_1 = 0.25, \mu_2 = 0.20, \mu_1 = 0.55, \mu_1 = 0.50, \mu_1 = 0.75, \mu_2 = 0.70$) .

Figure 4.3 displays the graph of individuals recovered human from Zika virus infection R (t) against time for varied treatment rate and the use of insecticide. The graph shows that as the control strategy increases, the number of recovered individuals also increases (i.e $\mu_1 = 0.25, \mu_2 = 0.20, \mu_1 = 0.55, \mu_1 = 0.50, \mu_1 = 0.75, \mu_2 = 0.70$). Due to the deadly nature of severe Zika virus infection, prevention and early treatment of Zika virus should be encouraged to increase the number of recovered individuals from Zika virus infection.

Figure 4.4 shows the graph of infected aedes mosquitoes against time for different values of biting and contact rate. The graph shows that as the biting and contact rate increases the no of infected aedes mosquitoes increases in population. (i.e $\varepsilon = 0.01, \varphi = 0.01$, $\varepsilon = 0.02$, $\varphi = 0.02$ *and* $\varepsilon = 0.03, \varphi = 0.03$).We observed that as the biting and contact rate increases the number of aedes mosquitoes increases. It is encouraged that insecticides that cover larger area should be used at faster time so as to stop the vectors from reproducing due to the proximity amongst themselves.

Figure 4.5 shows the graph of infected aedes mosquito against time for varied treatment rate and the use of insecticide (i.e $\mu_1 = 0.25, \mu_2 = 0.20, \mu_1 = 0.55, \mu_1 = 0.50, \mu_1 = 0.75, \mu_2 = 0.70$) .It is observed that as the control strategies increases the infected aedes mosquitoes decreases. Therefore it is encouraged to use insecticide and treatment in order to reduce and eradicate the virus completely in Zika endemic areas.

Conclusion

In this study, the impact of treatment and insecticide on the burden of Zika Virus infection was investigated. Appropriate control strategy for the infection eradication and control, was also examined. The result of the numerical simulation indicates that treatment and the use of insecticides is the best strategy to control the disease.

The model shows that the spread of zika virus infection depends largely on the biting and contact rates; hence the National zika virus control program should emphasize on the improvement in early detection of zika virus cases so that transmission can be minimized.

Infectious individuals should be isolated and treated immediately.

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