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GLOBAL STABILITY OF A GANG-FREE EQUILIBRIUM MODEL: THE CASE OF JUVENILE CRIMES

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ABSTRACT

The paper proposes a deterministic (S-G-D-L-R) model to understand the transmission dynamics and control of Juvenile crimes incorporating standard incidence rate, effect of counter-gang strategies which are enhanced by sensitization coverage, Reality Therapy, and Aggression Replacement Training (ART). The effective reproduction number, (R_C) is obtained and thus, established the conditions for local and global stability of the gang-free equilibrium.

Keywords: Gang, Crime, Delinquency, Gang-free equilibrium state, Stability.

Introduction:

According to Weerman *et al.* (2009), a gang, or troublesome youth group, is any durable, street-oriented youth group whose involvement in illegal activities is part of their group identity. Youth gang activities are fast becoming alarming globally as gang members engage in diverse forms of crimes (Hagedorn, 2008). Robbery, felony theft, assault, breaking and entering, illegal drug use

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is considered a cognitive-behavioural approach to therapy; it focuses on facilitating the client to become aware of, and if necessary, change, his/her thoughts and actions (Grant, 2003). Aggression Replacement Training (ART) is a cognitive bahavioural intervention program to help children and adolescents improve social skill competence and moral reasoning, better manage anger, and reduce aggressive behavior. The program specifically targets chronically aggressive children and adolescents ages 12 – 17 years. ART was developed in the US in 1981 and is now used in human services systems including, but not limited to juvenile justice systems, human services schools and adult corrections throughout North America, as well as Europe, South America, and Australia (Goldstein *et al.*, 1998).

Many sociologists have discovered that peer pressure is the major predictor of delinquent behavior in early adolescence (Sullivan, 2006). In a research (Brown, 1993) found that susceptibility to peer pressure reaches its peak in the younger generation and in people with low confidence and poor social interaction abilities. Hence, interaction with delinquents' peers is a major risk factor for gang membership (Thornberry *et al.*, 2003). On this note, we treat gang membership as an infection that spreads due to effective contact with peers whereas delinquent youths convert vulnerable youths through verbal and non-verbal communications. The choice of an infections disease model is motivated by research of (Lee and Sug, 2011; Sooknanan, Bhatt and Comissiong, 2012; Abdulrahman et al, 2013; Heesterbeek and Dietz, 1996; Adeboye, 2006).

A mathematical model on the perspective of juvenile crimes was developed by (Lee and Sug, 2011) with four (4) compartments of Susceptibles (S), Non-delinquent gang members (G), Delinquent gang memembers (D), and Gang members that are Law enforced (L). They obtained the basic reproduction number, R_C and established the conditions for local stability of gang-free equilibrium. In a similar development, (Sooknanan, Bhatt and Comissiong, 2013) developed a deterministic model with four (4) compartments of Non-susceptibles (N), Susceptibles (S), Gang members (G) and Recovered (R). They obtained the basic reproduction number, R_C and established the conditions for local stability of gang-free equilibrium. In this work, we therefore complement and extend the works of the aforementioned authors by having five (5) compartments of Susceptibles (S), Non-delinquent gang members (G), Delinquent gang memembers (D), Gang members that are law-enforced (L) and Gang members that are recovered (R). We also incorporated standard incidence rate, and the effect of counter-gang strategies which are enhanced by sensitization coverage, Reality Therapy, and Aggression Replacement Training (ART).

Materials and Methods

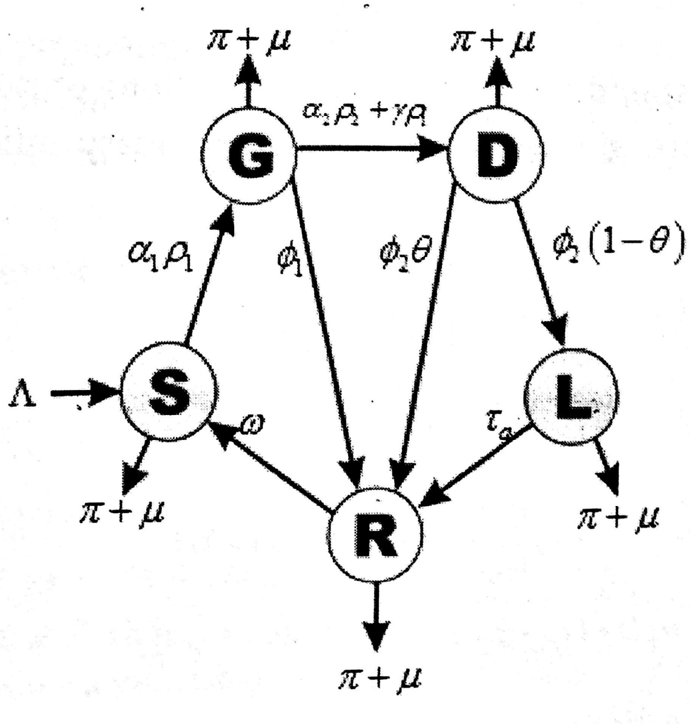

Fig. 1: Schematic representation of the model

In this model, individuals are adolescents between the ages of 13 and 18 years in the low socioeconomic class (i.e. poor neighborhood, school and family environments). The at-risk susceptible population, S are generated from daily recruitment of individuals who aged-in and those who recover from infection at the rate Λ and ω respectively. They acquired infection and move to the G compartment via infection from G and D, given by the incidence rate $\alpha_1\rho_1$ where $\alpha_1 = \frac{\beta_1 G + \beta_2 D}{N}$ and $\rho_1 = 1-\tau_s$. Gang members in all the compartments age-out or die naturally at the rate $\pi+\mu$. Some members of G move to D by committing crimes. These crimes may be motivated by delinquent behavior by delinquent peers at the peer pressure rate $\alpha_2\rho_2$ where $\alpha_2 = \frac{\beta_3 GD}{N}$ and $\rho_2 = 1-\tau_r$ or by personal issues such as quest for money, girl-friend factor or family problem modeled by the rate $\gamma\rho_1$.

The corresponding mathematical equations of the above schematic diagram are given by a system of ordinary differential equations below:

$$\frac{dS}{dt} = \Lambda - \frac{(\beta_1 G + \beta_2 D)(1-\tau_s)S}{N} + \omega R - (\pi+\mu)S \qquad (1)$$

$$\frac{dG}{dt} = \frac{(\beta_1 G + \beta_2 D)(1-\tau_s)S}{N} - \frac{\beta_3 G(1-\tau_r)D}{N} - \left[\gamma(1-\tau_s)+\phi_1+\pi+\mu\right]G \qquad (2)$$

$$\frac{dD}{dt} = \frac{\beta_3 G(1-\tau_r)D}{N} + \gamma(1-\tau_s)G - (\phi_2+\pi+\mu)D \qquad (3)$$

$$\frac{dL}{dt} = \phi_2(1-\theta)D - (\tau_a+\pi+\mu)L \qquad (4)$$

$$\frac{dR}{dt} = \phi_1 G + \phi_2\theta D + \tau_a L - (\omega+\pi+\mu)R \qquad (5)$$

where

$$N = S + G + D + L + R \tag{6}$$

and

$$\frac{dN}{dt} = \Lambda - (\pi + \mu) N \tag{7}$$

in the biological-feasible region:

$$\Omega = \left\{ \begin{matrix} (S,G,D,L,R) \in \Re_+^5 : S \geq 0, G \geq 0, D \geq 0, L \geq 0, R \geq 0; \\ N = S + G + D + L + R \end{matrix} \right\} \tag{8}$$

which can be shown to be positively invariant with respect to the equations (1) - (5).

The symbols used in the model are as follows:

S	SUSCEPTIBLE POPULATION
G	Non-delinquent gang members
D	Delinquent gang members
L	Delinquent gang members that are arrested and law-enforced
R	Recovered population
N	Total population
Λ	Human recruitment rate
β_1	Effective contact rate between S and G
β_2	Effective contact rate between S and D
β_3	Effective peer pressure rate between G and D
γ	Additional rate of progression from G to D
ϕ_1	Rate of movement from G to R
ϕ_2	Rate of movement from D to R or L where θ is proportion of D that recovers while is $(1-\theta)$ is proportion of D that moves to L.
τ_s	Rate of applying Sensitization to S
τ_r	Rate of applying Reality Therapy to G

τ_a	Rate of applying Aggression Replacement Training to D and L
ω	Rate of loss of immunity

Set

$$\left.\begin{aligned} \vartheta_1 &= 1-\tau_s \\ \vartheta_2 &= 1-\tau_r \\ \vartheta_3 &= 1-\theta \\ k_1 &= \pi+\mu \\ k_2 &= \gamma(1-\tau_s)+\phi_1+\pi+\mu \\ k_3 &= \phi_2+\pi+\mu \\ k_4 &= \tau_a+\pi+\mu \\ k_5 &= \omega+\pi+\mu \end{aligned}\right\} \quad (9)$$

Equation (1) – (5) becomes

$$\frac{dS}{dt} = \Lambda - \frac{(\beta_1 G+\beta_2 D)\vartheta_1 S}{N} + \omega R - k_1 S \quad (10)$$

$$\frac{dG}{dt} = \frac{(\beta_1 G+\beta_2 D)\vartheta_1 S}{N} - \frac{\beta_3 G\vartheta_2 D}{N} - k_2 G \quad (11)$$

$$\frac{dD}{dt} = \frac{\beta_3 G\vartheta_2 D}{N} + \gamma\vartheta_1 G - k_3 D \quad (12)$$

$$\frac{dL}{dt} = \phi_2\vartheta_3 D - k_4 L \quad (13)$$

$$\frac{dR}{dt} = \phi_1 G + \phi_2\theta D + \tau_a L - k_5 R \quad (14)$$

Model Analysis

Existence of Equilibrium States (E^*)

At the gang-free equilibrium state, we have absence of gang. Thus, all the infected classes will be zero and the entire population will comprise of only susceptible individuals.

$$\frac{dS}{dt}=\frac{dG}{dt}=\frac{dD}{dt}=\frac{dL}{dt}=\frac{dR}{dt}=0 \qquad (15)$$

At any arbitrary equilibrium state, let

$$(S,G,D,L,R)=(S^*,G^*,D^*,L^*,R^*) \qquad (16)$$

Thus equations (10) – (14) becomes

$$\Lambda-\frac{(\beta_1G^*+\beta_2D^*)\vartheta_1S^*}{N^*}+\omega R^*-k_1S^*=0 \qquad (17)$$

$$\frac{(\beta_1G^*+\beta_2D^*)\vartheta_1S^*}{N^*}-\frac{\beta_3G^*\vartheta_2D^*}{N^*}-k_2G^*=0 \qquad (18)$$

$$\frac{\beta_3G^*\vartheta_2D^*}{N^*}+\gamma\vartheta_1G^*-k_3D^*=0 \qquad (19)$$

$$\phi_2\vartheta_3D^*-k_4L^*=0 \qquad (20)$$

$$\phi_1G^*+\phi_2\theta D^*+\tau_aL^*-k_5R^*=0 \qquad (21)$$

From (20), we have

$$L^*=\frac{\phi_2\vartheta_3D^*}{k_4} \qquad (22)$$

From (19), we have

$$G^*=\frac{k_3D^*N^*}{\beta_3\vartheta_2D^*+\gamma\vartheta_1N^*} \qquad (23)$$

Substituting (22) and (23) into (21) gives

$$R^*=\frac{\left[k_3k_4\phi_1N^*+\eta_1\eta_2\right]D^*}{k_4k_5\eta_2} \qquad (24)$$

Where

$$\left.\begin{aligned}\eta_1&=k_4\phi_2\theta+\tau_a\phi_2\vartheta_3\\ \eta_2&=\beta_3\vartheta_2D^*+\gamma\vartheta_1N^*\end{aligned}\right\} \qquad (25)$$

Adding (17), (18) and (19) gives

$$\Lambda+\omega R^*-k_1S^*+(\gamma\vartheta_1-k_2)G^*-k_3D^*=0 \qquad (26)$$

Substituting (23) and (24) into (26) gives

$$S^* = \frac{\Lambda}{k_1} + \frac{\left\{\left[k_3 k_4 \phi_1 \omega N^* + \omega \eta_1 \eta_2\right] - k_4 k_5 \left[k_3 \left(k_2 - \gamma \vartheta_1\right) D^* N^* - k_3 \eta_2\right]\right\} D^*}{k_1 k_4 k_5 \eta_2} \quad (27)$$

Substituting (23) into (18) and simplifying gives

$$\left(k_3 \beta_1 \vartheta_1 S^* N^* - k_3 \beta_3 \vartheta_2 D^* N^* - k_2 k_3 N^{*2} + \beta_2 \beta_3 \vartheta_1 \vartheta_2 S^* D^* + \beta_2 \vartheta_1 \gamma \vartheta_1 S^* N^*\right) D^* = 0 \quad (28)$$

i.e

$$D^* = 0 \quad (29)$$

or

$$D^* = \frac{\left(k_3 \beta_1 \vartheta_1 + \beta_2 \vartheta_1^2 \gamma\right) S^* - k_2 k_3 N^*}{\beta_3 \vartheta_2 \left(k_3 N^* - \beta_2 \vartheta_1 S^*\right)} \quad (30)$$

Now, substituting (29) into (22), (23), (24) and (27) gives

$$G^* = L^* = R^* = 0 \quad (31)$$

and

$$S^* = \frac{\Lambda}{k_1} \quad (32)$$

From (30), we observe that D^* cannot be less than zero. Then $D^* = 0$ if

$$\left(k_3 \beta_1 \vartheta_1 + \beta_2 \vartheta_1^2 \gamma\right) S^* = k_2 k_3 N^* \quad (33)$$

This gives us (31); and $D^* > 0$ if

$$\frac{\left(k_3 \beta_1 \vartheta_1 + \beta_2 \vartheta_1^2 \gamma\right) S^*}{k_2 k_3 N^*} > 1 \quad (34)$$

which resulted into an equilibrium state where each of the sub-population is greater than zero.

Therefore, the system has two different equilibrium states, namely: the gang-free equilibrium in which all the infected compartments are zero and the endemic equilibrium in which all the compartments are greater than zero.

Existence of Gang-Free Equilibrium State, (E^0)

Lemma 1: A gang-free equilibrium state of the model exist at the point

$$E^0 = \left(S^0, G^0, D^0, L^0, R^0\right) = \left(\frac{\Lambda}{k_1}, 0, 0, 0, 0\right) \quad (35)$$

Proof: At the gang-free equilibrium state, let

$$\left(S, G, D, L, R\right) = \left(S^0, G^0, D^0, L^0, R^0\right) \quad (36)$$

Considering an arbitrary equilibrium at which equation (31) holds, substituting (36) into (17) – (21) gives

$$\Lambda - \frac{\left(\beta_1 G^0 + \beta_2 D^0\right)\vartheta_1 S^0}{N^0} + \omega R^0 - k_1 S^0 = 0 \quad (37)$$

$$\frac{\left(\beta_1 G^0 + \beta_2 D^0\right)\vartheta_1 S^0}{N^0} - \frac{\beta_3 G^0 \vartheta_2 D^0}{N^0} - k_2 G^0 = 0 \quad (38)$$

$$\frac{\beta_3 G^0 \vartheta_2 D^0}{N^0} + \gamma \vartheta_1 G^0 - k_3 D^0 = 0 \quad (39)$$

$$\phi_2 \vartheta_3 D^0 - k_4 L^0 = 0 \quad (40)$$

$$\phi_1 G^0 + \phi_2 \theta D^0 + \tau_a L^0 - k_5 R^0 = 0 \quad (41)$$

Now, from (31) and (32), we have

$$G^0 = L^0 = R^0 = 0 \quad (42)$$

and

$$S^0 = \frac{\Lambda}{k_1} \quad (43)$$

Hence, from (42) and (43) the lemma is proved.

Effective Reproduction Number (R_C)

Using the next generation operator technique described by (Diekmann & Heesterbeek, 2000) and subsequently analyzed by (Van de & Watmough, 2002), we obtained the Effective Reproductive Number (R_C) of the model (1) – (5), which is the spectral radius (ρ) of the next generation matrix, G.

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i.e (44)

$$R_c = \rho\left(FV^{-1}\right)$$

Now,

$$F = \begin{pmatrix} \beta_1\vartheta_1 & \beta_2\vartheta_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{45}$$

and

$$V = \begin{pmatrix} k_2 & 0 & 0 \\ -\gamma\vartheta_1 & k_3 & 0 \\ 0 & \phi_2\vartheta_2 & k_4 \end{pmatrix} \tag{46}$$

Thus

$$V^{-1} = \begin{pmatrix} \frac{1}{k_2} & 0 & 0 \\ \frac{\gamma\vartheta_1}{k_2k_3} & \frac{1}{k_3} & 0 \\ \frac{\gamma\phi_2\vartheta_1\vartheta_2}{k_2k_3k_4} & \frac{\phi_2\vartheta_3}{k_3k_4} & \frac{1}{k_4} \end{pmatrix} \tag{47}$$

and

$$FV^{-1} = \begin{pmatrix} \frac{k_3\vartheta_1\beta_1 + \gamma\vartheta_1^2\beta_2}{k_2k_3} & \frac{\vartheta_1\beta_2}{k_3} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{48}$$

Hence, the Effective Reproductive Number is given by

$$R_c = \frac{\vartheta_1\left(k_3\beta_1 + \gamma\vartheta_1\beta_2\right)}{k_2k_3} \tag{49}$$

Substituting the values of k_1, k_2, and ϑ_1 from (9), we have

$$R_C = \frac{\left[\gamma\left(1-\tau_s\right)+\phi_1+\pi+\mu\right]\beta_1 + \beta_2\gamma\left(1-\tau_s\right)}{\mu\left(\pi+\mu\right)\left[\gamma\left(1-\tau_s\right)+\phi_1+\pi+\mu\right]} \tag{50}$$

Local Stability of Gang-Free Equilibrium (E^0)

Theorem 1: The Gang-free equilibrium (E^0) of the model is Locally Asymptotically Stable (LAS) if $R_C < 1$.

Proof: We used the Jacobian stability approach to prove the stability of the gang-free equilibrium state.

Linearization of (10) – (14) at E^0 gives the Jacobian matrix

$$J(E^0)=\begin{pmatrix} -k_1 & -\beta_1\vartheta_1 & -\beta_2\vartheta_1 & 0 & \omega \\ 0 & \beta_1\vartheta_1-k_2 & \beta_2\vartheta_1 & 0 & 0 \\ 0 & \gamma\vartheta_1 & -k_3 & 0 & 0 \\ 0 & 0 & \phi_2\vartheta_3 & -k_4 & 0 \\ 0 & \phi_1 & \phi_2\theta & \tau_a & -k_5 \end{pmatrix} \quad (51)$$

Using elementary row-transformation on (51), we have

$$J(E^0)=\begin{pmatrix} -k_1 & -\beta_1\vartheta_1 & -\beta_2\vartheta_1 & 0 & \omega \\ 0 & -(k_2-\beta_1\vartheta_1) & \beta_2\vartheta_1 & 0 & 0 \\ 0 & 0 & \dfrac{k_3\vartheta_1\beta_1+\gamma\vartheta_1^2\beta_2-k_2k_3}{k_2-\beta_1\vartheta_1} & 0 & 0 \\ 0 & 0 & 0 & -k_4 & 0 \\ 0 & 0 & 0 & 0 & -k_5 \end{pmatrix} \quad (52)$$

and clearly, the eigenvalues are

$$\left.\begin{aligned} \lambda_1 &= -k_1 < 0 \\ \lambda_2 &= -(k_2-\beta_1\vartheta_1) < 0 \\ \lambda_3 &= \frac{k_3\vartheta_1\beta_1+\gamma\vartheta_1^2\beta_2-k_2k_3}{k_2-\beta_1\vartheta_1} \\ \lambda_4 &= -k_4 < 0 \\ \lambda_5 &= -k_5 < 0 \end{aligned}\right\} \quad (53)$$

Now, for λ_3 to be negative, we must have (54)

$$k_3\vartheta_1\beta_1+\gamma\vartheta_1^2\beta_2 < k_2k_3$$

i.e

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$$\frac{k_3 \vartheta_1 \beta_1 + \gamma \vartheta_1^2 \beta_2}{k_2 k_3} < 1 \tag{55}$$

Thus

$$R_C < 1 \tag{56}$$

As we can see from (53), all the eigenvalues are negative except for λ_3 which will be negative when $R_C < 1$. The epidemiological implication of this theorem is that juvenile gang crimes can be eliminated (control) from the population when $R_C < 1$, if the initial size of the sub-populations of the model are in the basin of attraction of the Gang-free Equilibrium.

Global Stability of Gang-Free Equilibrium (E^0)

In order to ensure that the gang-free equilibrium (GFE) is independent of the initial size of the sub-populations of the model, it is necessary to show that the GFE is globally asymptotically stable (GAS). There are many ways of proving the global stability of gang-free equilibrium which include among others the Lyapunov theorem and the Castillo-Chavez et al (2002) global stability theorem. We used the latter in this paper.

Theorem 2: The gang-free equilibrium, E_f of (1) – (5) is globally asymptotically stable (GAS) if $R_C < 1$.

Proof: To establish the global stability of the gang-free equilibrium, the two conditions (H1) and (H2) as in Castillo-Chavez *et al.* (2002) must be satisfied for $R_C < 1$.

The model equations (1) – (5) can be written in the form

$$X_1'(t) = F(X_1, X_2)$$

$$X_2'(t) = G(X_1, X_2);\ G(X_1, 0) = 0 \tag{57}$$

(58)

where $X_1 = (S^0, R^0)$ and $X_2 = (G^0, D^0, L^0)$ with the components of $X_1 \in \mathbb{R}^2$ denoting the uninfected individuals and the components of $X_2 \in \mathbb{R}^3$ denoting the infected individuals.

The gang-free equilibrium is now denoted as

$$E^0 = (X_1^*, 0) \tag{59}$$

where

$$X_1^* = (S^0, 0) \tag{60}$$

Now, to proof that the first condition, $(H1)$ for $X_1'(t) = F(X_1^*, 0)$ is true, i.e X_1^* is globally asymptotically stable.

We have linear differential equations as thus

$$X_1'(t) = F(X_1, 0) = \begin{pmatrix} \Lambda + \omega R^0 - k_1 S^0 \\ -k_5 R^0 \end{pmatrix} \tag{61}$$

Solving (61) gives

$$S^0(t) = \frac{\Lambda + \omega R^0}{k_1} + \left(\frac{\Lambda + \omega R^0}{k_1} \right) e^{-k_1 t} + S^0(0) e^{-k_1 t} \tag{62}$$

$$R^0(t) = R^0(0) e^{-k_5 t} \tag{63}$$

Now, clearly from (35), we have that $S^0(t) + G^0(t) + D^0(t) + L^0(t) \to S^0(t)$ as $t \to \infty$ regardless of the value of $S^0(0)$. Thus, $X_1^* = (S^0, 0) = \left(\frac{\Lambda}{k_1}, 0 \right)$ is globally asymptotically stable.

Next, to prove that the second condition $(H2)$ is true, that is

$$\hat{G}(X_1, X_2) = AX_2 - G(X_1, X_2) \tag{64}$$

We have

$$AX_2 = \begin{pmatrix} -(k_2 - \beta_1 \vartheta_1) & \beta_2 \vartheta_1 & 0 \\ \gamma \vartheta_1 & -k_3 & 0 \\ 0 & \phi_2 \vartheta_3 & -k_4 \end{pmatrix} \begin{pmatrix} G^0 \\ D^0 \\ L^0 \end{pmatrix} \tag{65}$$

and

$$G(X_1,X_2)=\begin{pmatrix} \frac{(\beta_1 G^0+\beta_2 D^0)\vartheta_1 S^0}{N^0}-\frac{\beta_3 G^0\vartheta_2 D^0}{N^0}-k_2 G^0 \\ \frac{\beta_3 G^0\vartheta_2 D^0}{N^0}+\gamma\vartheta_1 G^0-k_3 D^0 \\ \phi_2\vartheta_3 D^0-k_4 L^0 \end{pmatrix} \quad (66)$$

Substituting (65) and (66) into (64), we have

$$\hat{G}(X_1,X_2)=AX_2-G(X_1,X_2)=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (67)$$

It is thus obvious that $\hat{G}(X_1,X_2)=0$. Hence, the proof is complete.

Conclusion

In this paper, we developed a new deterministic model which incorporated some important factors that plays significant role in the recruitment dynamics and control of Juvenile Crimes. These factors are: standard incidence and the effect of counter-gang strategies which are enhanced by sensitization coverage, Reality Therapy, and Aggression Replacement Training (ART). We obtained the effective reproduction number (R_C). The analysis reveals that gang can be controlled if the effective reproduction number is less than unity regardless of the initial population profile. Thus, every effort must be put in place by all agencies concerned to prevent gang by reducing R_C strictly less than unity.

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