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A Mathematical Model For The Spread Of Typhoid Fever Incorporating The Carrier Compartment.

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ABSTRACT

In this work, a mathematical model of Typhoid Fever was developed incorporating the effect of carriers on the spread. The population was divided into four compartments namely Susceptible, Infected, Carrier, and Recovered compartments. The stability analysis of the disease-free equilibrium and the Endemic Equilibrium states were carried out. The stability of the Endemic Equilibrium state implies that for as long as carriers exist in a population, Typhoid fever cannot be completely eradicated.

Keywords: *Typhoid fever, Carries, Equilibrium state, Stability analysis.*

Introduction

Typhoid fever is a critical disorder related with fever caused by the salmonella typhi bacteria. It can also be caused by salmonella paratyphi, a related bacterium that usually causes a less severe disorder. The bacteria are deposited in water or food by a human carrier and are then spread to other people in the area. Typhoid fever infects 21 million people and kills 200,000 worldwide every year. Asymptomatic carriers are believed to play an essential role in the evolution and global transmission of typhi, and their presence greatly hinders the eradication of typhoid fever using treatment and vaccination (Roumagnac et al., 2006).

Typhoid fever is contracted by drinking or eating the bacteria in contaminated food or water. People with acute illness can contaminate the surrounding water supply through stool, which contains a high concentration of the bacteria. Contamination of the water supply can, in turn, taint the food supply. The bacteria can survive for weeks in water or dried sewage (WebMd.Com, 2017).

The bacterium that causes typhoid fever is spread through poor hygiene habits and public sanitation conditions, and sometimes through flying insects feeding on fees. Public education campaigns encouraging people to wash their hands after using the toilet and before handling food are important component in controlling spread of the disease. (medicinnet.com, 2017)

Even after treatment with antibiotics, a small number of people who recover from typhoid fever continue to harbor the bacteria in their intestinal tracts or gallbladders, often for years. These people, called chronic carriers, shed the bacteria in their feces and are capable of infecting others, although they no longer have signs or symptoms of the disease themselves (mayoclinic.org, 2017)

Due to the cost of typhoid fever treatment, prevention stands the most effective strategy in controlling the spread of the disease. There are two major preventive measures namely; behaviour change and vaccination. Behaviour change includes washing of hands after using the toilet, improvement in sanitation. (Anna, 2014).

Typhoid fever vaccine is safe, effective and available. However, despite their public-health benefit, individuals often refuse or avoid vaccinations which they perceive to be risky. Public enlightenment campaign on positive behaviour change and vaccination may have a positive effect on the disease transmission through decreasing effective contaminated faces, food, and drinking water and increasing effective immunization coverage rates.

During the last years Adetunde (2008), S. Mushayabasa et al., (2013), Mushayabasa (2012), Lauria et al., (2009). Gonzalez-Guzman (1989), Zaman, et ai., (2012), developed mathematical models to evaluate the effect of public health programs and provided long-term predictions regarding Typhoid Fever.

MATERIALS AND METHODS

The Model

The population was partitioned into four compartments; Susceptible compartment S(t), Infected compartment I(t), Carrier compartment C(t), and Recovered compartment R(t). This can be shown as a flow diagram in which the circles represent the different compartments and the arrows represent the transition between the compartments.

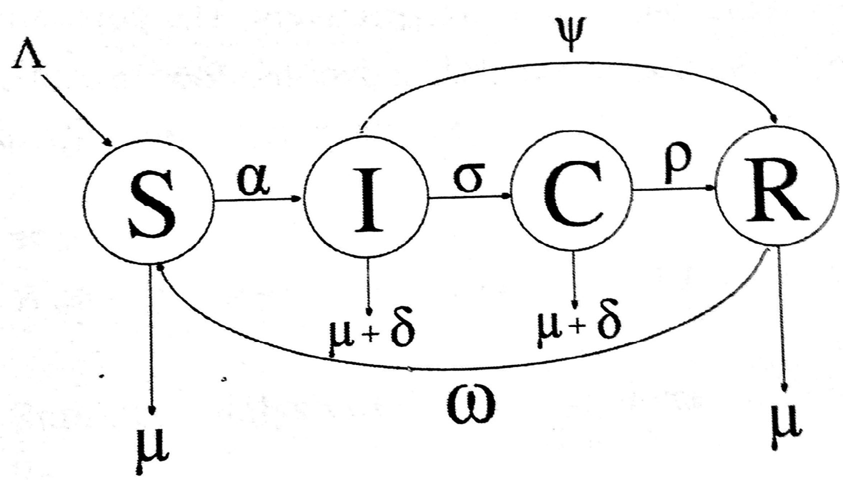

Figure 2.1 Schematic presentation of the model.

The population of the susceptible class increases with constant recruitment Λ, movement of individuals from the Recovered class into the Susceptible class at the rate ω as a result of winning off of partial immunity after recovery, the population of this Class decreases when individuals move from the susceptible class into the infected class I via interaction with Infected and Carriers at the rate α and further decreases with natural death at the rate μ.

The population of the infected Class increases with individuals coming in from the Susceptible class via interaction of the Susceptible individuals with the Infected and the Carrier individuals at the rate α and decreases with individuals recovering due to treatment at the rate ψ, progression into the Carrier class at the rate σ since a small number of people who recover from typhoid fever continue to harbor the bacteria in their intestinal tracts or gallbladders often for years, This Class further decreases with death due to infection at the rate δ and natural death at the rate μ.

The population of the Carrier class increases when the Infected individuals progress from the Infected class into the Carrier class at the rate σ and decreases as members of the Carrier class recovered the rate ρ, This Class further decreases with death due to infection at the rate δ and natural death at the rate μ.

The population of the recovered class increases with recovery of Infected and Carrier individuals at the rates ψ and ρ respectively. The population decreases as members of the Recovered class move into the susceptible class at the rate ω as a result of winning off of the partial immunity and due to natural death at the rate μ.

Model Equation

$$\frac{dS}{dt} = \wedge + \omega R - (\alpha I + \alpha C + \mu)S \qquad (2.1)$$

$$\frac{dI}{dt} = \alpha IS + \alpha CS - (\psi + \sigma + \mu + \delta)I \qquad (2.2)$$

$$\frac{dC}{dt} = \sigma I - (\rho + \mu + \delta)C \qquad (2.3)$$

$$\frac{dR}{dt} = \rho C + \psi I - (\omega + \mu)R \qquad (2.4)$$

Model Analysis

Existence of Equilibrium E*

At disease free equilibrium state;

Let $S = x_1$, $I = x_2$, $C = x_3$, $R = x_4$

$$\frac{dx_1}{dt} = \frac{dx_2}{dt} = \frac{dx_3}{dt} = \frac{dx_4}{dt} = 0$$

From (2.1) to (2.4)

$$\wedge + \omega x_4 - (\alpha x_2 + \alpha x_3 + \mu)x_1 = 0 \quad (2.5)$$

$$\alpha x_2 x_1 + \alpha x_3 x_1 - (\psi + \sigma + \mu + \delta)x_2 = 0 \quad (2.6)$$

$$\sigma x_2 - (\rho + \mu + \delta)x_3 = 0 \quad (2.7)$$

$$\rho x_3 + \psi x_2 - (\omega + \mu)x_4 = 0 \quad (2.8)$$

Solving (2.5) to (2.8) simultaneously, we obtained

$$(x_1, x_2, x_3, x_4) = \left(\frac{\wedge}{\mu,}, 0, 0, 0\right) \quad (2.9)$$

as the Disease Free Equilibrium State and.

$$x_1 = \frac{(\psi + \sigma + \mu + \delta)(\rho + \mu + \delta)}{\alpha(\rho + \mu + \delta) + \sigma\alpha} \quad (2.10)$$

$$x_2 = \frac{\sigma(\rho + \mu + \delta)(\omega + \mu)(\mu A - B)}{\omega B(\sigma\rho + \psi(\rho + \mu + \delta)) - (\omega + \mu)(\alpha(\rho + \mu + \delta) + \sigma\alpha)A} \quad (2.11)$$

$$x_3 = \frac{(\omega + \mu)(\mu A - B)\sigma}{(\omega B(\sigma\rho + \psi(\rho + \mu + \delta))) - (\omega + \mu)(\alpha(\rho + \mu + \delta) + \sigma\alpha)A} \quad (2.12)$$

$$x_4 = \frac{(\sigma\rho + \psi(\rho + \mu + \delta))(\mu A - B)}{\omega B(\sigma\rho + \psi(\rho + \mu + \delta)) - (\omega + \mu)(\alpha(\rho + \mu + \delta) + \sigma\alpha)} \quad (2.13)$$

as the Endemic Equilibrium State.

Where $A = (\psi + \sigma + \mu + \delta)(\rho + \mu + \delta)$ and $B = \alpha(\rho + \mu + \delta) + \sigma\alpha$

Stability Analysis of the Disease –Free Equilibrium (DFE)

Recall that the system of equation of the model at equilibrium state is;

$$\Lambda + \omega R - (\alpha I + \alpha C + \mu)S = 0 \quad (3.1)$$

$$\alpha IS + \alpha CS - (\psi + \sigma + \mu + \delta)I = 0 \quad (3.2)$$

$$\sigma I - (\rho + \mu + \delta)C = 0 \quad (3.3)$$

$$\rho C + \psi I - (\omega + \mu)R = 0 \quad (3.4)$$

The Jacobian matrix of the system of equation is given by;

$$J = \begin{bmatrix} -(\alpha I + \alpha C + \mu) & -\alpha S & -\alpha S & \omega \\ \alpha I + \alpha C & -(\psi + \sigma + \mu + \delta) & \alpha S & 0 \\ 0 & \sigma & -(\rho + \mu + \delta) & 0 \\ 0 & \psi & \rho & -(\omega + \mu) \end{bmatrix} \quad (3.5)$$

The characteristic equation is given by $|J-\lambda I|$

$$\begin{vmatrix} -(\alpha I+\alpha C+\mu)-\lambda & -\alpha S & \alpha S & \omega \\ \alpha(I+C) & -(\psi+\sigma+\mu+\delta)-\lambda & \alpha S & 0 \\ 0 & \sigma & -(\rho+\mu+\delta)-\lambda & 0 \\ 0 & \psi & \rho & -(\omega+\mu)-\lambda \end{vmatrix} = 0$$

$$[-(\alpha I+\alpha C+\mu)-\lambda]\begin{vmatrix} -(\psi+\sigma+\mu+\delta)-\lambda & \alpha S & 0 \\ \sigma & -(\rho+\mu+\delta)-\lambda & 0 \\ \psi & \rho & -(\omega+\mu)-\lambda \end{vmatrix} + \alpha S\begin{vmatrix} \alpha(I+C) & \alpha S & 0 \\ 0 & -(\rho+\mu+\delta)-\lambda & 0 \\ 0 & \rho & -(\omega+\mu)-\lambda \end{vmatrix}$$

$$+\alpha S\begin{vmatrix} \alpha(I+C) & -(\psi+\sigma+\mu+\delta)-\lambda & 0 \\ 0 & \sigma & 0 \\ 0 & \psi & -(\omega+\mu)-\lambda \end{vmatrix} - \omega\begin{vmatrix} \alpha(I+C) & -(\psi+\sigma+\mu+\delta)-\lambda & \alpha S \\ 0 & \sigma & -(\rho+\mu+\delta)-\lambda \\ 0 & \psi & \rho \end{vmatrix} = 0$$

$$\begin{aligned} &[-(\alpha I+\alpha C+\mu)-\lambda]\{[-(\psi+\sigma+\mu+\delta)-\lambda][-(\rho+\mu+\delta)-\lambda][-(\omega+\mu)-\lambda]-\alpha S\sigma[-(\omega+\mu)-\lambda]\} \\ &+\alpha S\{[\alpha(I+C)][-(\rho+\mu+\delta)-\lambda][-(\omega+\mu)-\lambda]\}+\alpha S\{[\alpha(I+C)]\sigma[-(\omega+\mu)-\lambda]\} \\ &-\omega\{[\alpha(I+C)][\sigma\rho-\psi(-(\rho+\mu+\delta)-\lambda)]\}=0 \end{aligned} \quad (3.6)$$

$$\begin{aligned} &[-(\alpha(I+C)+\mu)-\lambda][-(\omega+\mu)-\lambda]\{[-(\psi+\sigma+\mu+\delta)-\lambda][-(\rho+\mu+\delta)-\lambda]-\alpha S\sigma\} \\ &+\alpha S[\alpha(I+C)][-(\omega+\mu)-\lambda]\{[-(\rho+\mu+\delta)-\lambda]+\sigma\} \\ &-\omega\{[\alpha(I+C)][\sigma\rho-\psi(-(\rho+\mu+\delta)-\lambda)]\}=0 \end{aligned} \quad (3.7)$$

Recall that at the disease free equilibrium;

$$(S,I,C,R)=(x_1,x_2,x_3,x_4)=\left(\frac{\Lambda}{\mu},0,0,0\right) \quad (3.8)$$

Substituting (3.8) into (3.7)
We have

$$[-\mu-\lambda][-(\omega+\mu)-\lambda]\left\{[-(\psi+\sigma+\mu+\delta)-\lambda][-(\rho+\mu+\delta)-\lambda]-\alpha\frac{\Lambda}{\mu}\sigma\right\}=0 \quad (3.9)$$

Either

$$[-\mu-\lambda]=0 \quad (3.10)$$

or

$$[-(\omega+\mu)-\lambda]=0 \quad (3.11)$$

or

$$[-(\psi+\sigma+\mu+\delta)-\lambda][-(\rho+\mu+\delta)-\lambda]-\alpha\frac{\Lambda}{\mu}\sigma=0 \quad (3.12)$$

$\Rightarrow$

$$\lambda_1=-\mu \quad (3.13)$$

$$\lambda_2=-(\omega+\mu) \quad (3.14)$$

From (3.12)

$$(\psi+\sigma+\mu+\delta)(\rho+\mu+\delta)+(\psi+\sigma+\mu+\delta)\lambda+(\rho+\mu+\delta)\lambda+\lambda^2-\frac{\alpha\Lambda\sigma}{\mu}=0$$

$$\lambda^2+[(\psi+\sigma+\mu+\delta)+(\rho+\mu+\delta)]\lambda+\left[(\psi+\sigma+\mu+\delta)(\rho+\mu+\delta)-\frac{\alpha\Lambda\sigma}{\mu}\right]=0$$

$$\lambda^2+[(\psi+\sigma+\mu+\delta)+(\rho+\mu+\delta)]\lambda+\frac{(\psi+\sigma+\mu+\delta)(\rho+\mu+\delta)-\alpha\Lambda\sigma}{\mu}=0$$

$$\mu\lambda^2+\mu[(\psi+\sigma+\mu+\delta)+(\rho+\mu+\delta)]\lambda+(\psi+\sigma+\mu+\delta)(\rho+\mu+\delta)-\alpha\Lambda\sigma=0$$

$$\lambda_3=\frac{-\mu[(\psi+\sigma+\mu+\delta)+(\rho+\mu+\delta)]-\sqrt{(\mu[(\psi+\sigma+\mu+\delta)+(\rho+\mu+\delta)])^2-4\mu[(\psi+\sigma+\mu+\delta)(\rho+\mu+\delta)-\alpha\Lambda\sigma]}}{2\mu}$$

(3.15)

$$\lambda_4=\frac{-\mu[(\psi+\sigma+\mu+\delta)+(\rho+\mu+\delta)]+\sqrt{(\mu[(\psi+\sigma+\mu+\delta)+(\rho+\mu+\delta)])^2-4\mu[(\psi+\sigma+\mu+\delta)(\rho+\mu+\delta)-\alpha\Lambda\sigma]}}{2\mu}$$

(3.16)

From (3.13) to (3.14) λ_1, λ_2 *and* $\lambda_3<0$

λ_4 will be negative if and only if

$$\sqrt{(\mu[(\psi+\sigma+\mu+\delta)+(\rho+\mu+\delta)])^2-4\mu[(\psi+\sigma+\mu+\delta)(\rho+\mu+\delta)-\alpha\Lambda\sigma]}<\mu[(\psi+\sigma+\mu+\delta)+(\rho+\mu+\delta)]$$

Hence, the disease free equilibrium state is stable if

$$\sqrt{(\mu[(\psi+\sigma+\mu+\delta)+(\rho+\mu+\delta)])^2-4\mu[(\psi+\sigma+\mu+\delta)(\rho+\mu+\delta)-\alpha\Lambda\sigma]}<\mu[(\psi+\sigma+\mu+\delta)+(\rho+\mu+\delta)]$$

(3.17)

otherwise it will be unstable.

Stability Analysis of the Endemic Equilibrium State (EE)

We apply Bellman and Cooke's theorem of stability (Bellman and Cooke, 1963).

Theorem 4.1

Let

$$H(Z) = P(Z, e^{z}) \tag{4.1}$$

Where $p(z,w)$ is a polynomial with principal term

Suppose $H(iy), y \in R$ is separated in to real and imaginary parts

$$H(iw) = F(y) + iG(y) \tag{4.2}$$

If all Zeros of $H(z)$ have negative real parts, then zeros of F(y) and G(y) are real, simple and alternate and

$$F(0)G'(0) - F'(0)G(0) > 0 \text{ For all } y \in R \tag{4.3}$$

We consider the characteristic equation (3.7) in the form $H(\lambda)$

$$\begin{aligned} H(\lambda) = & \left[-(\alpha(I+C)+\mu)-\lambda\right]\left[-(\omega+\mu)-\lambda\right]\left\{\left[-(\psi+\sigma+\mu+\delta)-\lambda\right]\left[-(\rho+\mu+\delta)-\lambda\right]-\alpha S\sigma\right\} \\ & +\alpha S\left[\alpha(I+C)\right]\left[-(\omega+\mu)-\lambda\right]\left\{\left[-(\rho+\mu+\delta)-\lambda\right]+\sigma\right\} \\ & -\omega\left\{\left[\alpha(I+C)\right]\left[\sigma\rho-\psi\left(-(\rho+\mu+\delta)-\lambda\right)\right]\right\} = 0 \end{aligned}$$

Expanding and rearranging in ascending powers of λ

$$\begin{aligned} H(\lambda) = & \lambda^4(\rho+\mu+\delta)(\omega+\mu) + \lambda^3\begin{Bmatrix} (\alpha(I+C)+\mu)(\omega+\mu)(\alpha(I+C)+\mu) \\ +(\psi+\sigma+\mu+\delta)(\rho+\mu+\delta)(\omega+\mu) \\ +(\psi+\sigma+\mu+\delta)(\omega+\mu)+(\omega+\mu) \end{Bmatrix} \\ & +\lambda^2\begin{Bmatrix} (\alpha(I+C)+\mu)(\omega+\mu)(\psi+\sigma+\mu+\delta)(\alpha(I+C)+\mu) \\ +(\alpha(I+C)+\mu)(\omega+\mu)(\rho+\mu+\delta)(\alpha(I+C)+\mu)-\alpha S\sigma+\lambda^2\alpha^2 S(I+C) \end{Bmatrix} \\ & +\lambda\begin{Bmatrix} (\alpha(I+C)+\mu)(\omega+\mu)(\psi+\sigma+\mu+\delta)(\rho+\mu+\delta)(\alpha(I+C)+\mu) \\ -\alpha S\sigma(\alpha(I+C)+\mu)-(\omega+\mu)\alpha^2 S(I+C)-\alpha S\sigma(\omega+\mu) \\ +(\rho+\mu+\delta)\alpha^2 S(I+C)-\sigma\alpha^2 S(I+C) \end{Bmatrix} \\ & +(\omega+\mu)(\rho+\mu+\delta)\alpha^2 S(I+C)-\sigma(\omega+\mu)\alpha^2 S(I+C)-\alpha S\sigma(\alpha(I+C)+\mu)(\omega+\mu) \end{aligned}$$

Substituting $\lambda = ip$ we have,

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$$H(ip) = (ip)^4(\rho+\mu+\delta)(\omega+\mu) + (ip)^3\begin{Bmatrix}(\alpha(I+C)+\mu)(\omega+\mu)(\alpha(I+C)+\mu) \\ +(\psi+\sigma+\mu+\delta)(\rho+\mu+\delta)(\omega+\mu) \\ +(\psi+\sigma+\mu+\delta)(\omega+\mu)+(\omega+\mu)\end{Bmatrix}$$
$$+(ip)^2\begin{Bmatrix}(\alpha(I+C)+\mu)(\omega+\mu)(\psi+\sigma+\mu+\delta)(\alpha(I+C)+\mu) \\ +(\alpha(I+C)+\mu)(\omega+\mu)(\rho+\mu+\delta)(\alpha(I+C)+\mu)-\alpha S\sigma+\lambda^2\alpha^2 S(I+C)\end{Bmatrix}$$
$$+(ip)\begin{Bmatrix}(\alpha(I+C)+\mu)(\omega+\mu)(\psi+\sigma+\mu+\delta)(\rho+\mu+\delta)(\alpha(I+C)+\mu) \\ -\alpha S\sigma(\alpha(I+C)+\mu)-(\omega+\mu)\alpha^2 S(I+C)-\alpha S\sigma(\omega+\mu) \\ +(\rho+\mu+\delta)\alpha^2 S(I+C)-\sigma\alpha^2 S(I+C)\end{Bmatrix}$$
$$+(\omega+\mu)(\rho+\mu+\delta)\alpha^2 S(I+C)-\sigma(\omega+\mu)\alpha^2 S(I+C)-\alpha S\sigma(\alpha(I+C)+\mu)(\omega+\mu)$$

$$H(ip) = p^4(\rho+\mu+\delta)(\omega+\mu) - ip^3\begin{Bmatrix}(\alpha(I+C)+\mu)(\omega+\mu)(\alpha(I+C)+\mu) \\ +(\psi+\sigma+\mu+\delta)(\rho+\mu+\delta)(\omega+\mu) \\ +(\psi+\sigma+\mu+\delta)(\omega+\mu)+(\omega+\mu)\end{Bmatrix}$$
$$-p^2\begin{Bmatrix}(\alpha(I+C)+\mu)(\omega+\mu)(\psi+\sigma+\mu+\delta)(\alpha(I+C)+\mu) \\ +(\alpha(I+C)+\mu)(\omega+\mu)(\rho+\mu+\delta)(\alpha(I+C)+\mu)-\alpha S\sigma+\lambda^2\alpha^2 S(I+C)\end{Bmatrix} \qquad (4.5)$$
$$+ip\begin{Bmatrix}(\alpha(I+C)+\mu)(\omega+\mu)(\psi+\sigma+\mu+\delta)(\rho+\mu+\delta)(\alpha(I+C)+\mu) \\ -\alpha S\sigma(\alpha(I+C)+\mu)-(\omega+\mu)\alpha^2 S(I+C)-\alpha S\sigma(\omega+\mu) \\ +(\rho+\mu+\delta)\alpha^2 S(I+C)-\sigma\alpha^2 S(I+C)\end{Bmatrix}$$
$$+(\omega+\mu)(\rho+\mu+\delta)\alpha^2 S(I+C)-\sigma(\omega+\mu)\alpha^2 S(I+C)-\alpha S\sigma(\alpha(I+C)+\mu)(\omega+\mu)$$

Resolving into real and imaginary parts

$H(ip) = F(p) + iG(p)$

Where $F(p)$ and $G(p)$ are given respectively as

$$F(p) = p^4(\rho+\mu+\delta)(\omega+\mu)$$
$$-p^2\begin{Bmatrix}(\alpha(I+C)+\mu)(\omega+\mu)(\psi+\sigma+\mu+\delta)(\alpha(I+C)+\mu) \\ +(\alpha(I+C)+\mu)(\omega+\mu)(\rho+\mu+\delta)(\alpha(I+C)+\mu)-\alpha S\sigma+\lambda^2\alpha^2 S(I+C)\end{Bmatrix}$$
$$+(\omega+\mu)(\rho+\mu+\delta)\alpha^2 S(I+C)-\sigma(\omega+\mu)\alpha^2 S(I+C)-\alpha S\sigma(\alpha(I+C)+\mu)(\omega+\mu)$$

(4.6)

and

$$G(p) = -p^3\begin{Bmatrix}(\alpha(I+C)+\mu)(\omega+\mu)(\alpha(I+C)+\mu) \\ +(\psi+\sigma+\mu+\delta)(\rho+\mu+\delta)(\omega+\mu) \\ +(\psi+\sigma+\mu+\delta)(\omega+\mu)+(\omega+\mu)\end{Bmatrix} \qquad (4.7)$$
$$+p\begin{Bmatrix}(\alpha(I+C)+\mu)(\omega+\mu)(\psi+\sigma+\mu+\delta)(\rho+\mu+\delta)(\alpha(I+C)+\mu) \\ -\alpha S\sigma(\alpha(I+C)+\mu)-(\omega+\mu)\alpha^2 S(I+C)-\alpha S\sigma(\omega+\mu) \\ +(\rho+\mu+\delta)\alpha^2 S(I+C)-\sigma\alpha^2 S(I+C)\end{Bmatrix}$$

Differentiating with respect to p we obtained

$$F'(p) = 4p^3(\rho+\mu+\delta)(\omega+\mu) \\ -2p\left\{\begin{array}{l}(\alpha(I+C)+\mu)(\omega+\mu)(\psi+\sigma+\mu+\delta)(\alpha(I+C)+\mu) \\ +(\alpha(I+C)+\mu)(\omega+\mu)(\rho+\mu+\delta)(\alpha(I+C)+\mu)-\alpha S\sigma+\lambda^2\alpha^2 S(I+C)\end{array}\right\}$$

(4.8)

and

$$G'(p) = -3p^2\left\{\begin{array}{l}(\alpha(I+C)+\mu)(\omega+\mu)(\alpha(I+C)+\mu) \\ +(\psi+\sigma+\mu+\delta)(\rho+\mu+\delta)(\omega+\mu) \\ +(\psi+\sigma+\mu+\delta)(\omega+\mu)+(\omega+\mu)\end{array}\right\} \\ +\left\{\begin{array}{l}(\alpha(I+C)+\mu)(\omega+\mu)(\psi+\sigma+\mu+\delta)(\rho+\mu+\delta)(\alpha(I+C)+\mu) \\ -\alpha S\sigma(\alpha(I+C)+\mu)-(\omega+\mu)\alpha^2 S(I+C)-\alpha S\sigma(\omega+\mu) \\ +(\rho+\mu+\delta)\alpha^2 S(I+C)-\sigma\alpha^2 S(I+C)\end{array}\right\} \quad (4.9)$$

Setting $p = 0$ we have

$$F(0) = (\omega+\mu)(\rho+\mu+\delta)\alpha^2 S(I+C)-\sigma(\omega+\mu)\alpha^2 S(I+C) \\ -\alpha S\sigma(\alpha(I+C)+\mu)(\omega+\mu) \quad (4.10)$$

$$G(0) = 0 \quad (4.11)$$

$$F'(0) = 0 \quad (4.12)$$

$$G'(0) = (\alpha(I+C)+\mu)(\omega+\mu)(\psi+\sigma+\mu+\delta)(\rho+\mu+\delta)(\alpha(I+C)+\mu) \\ -\alpha S\sigma(\alpha(I+C)+\mu)-(\omega+\mu)\alpha^2 S(I+C)-\alpha S\sigma(\omega+\mu) \\ +(\rho+\mu+\delta)\alpha^2 S(I+C)-\sigma\alpha^2 S(I+C) \quad (4.13)$$

The condition for stability according to Bellman and Cooke's theorem of stability is

$$F(0)G'(0)-F'(0)G(0)>0 \quad (4.14)$$

Since $G(0)=0$, (4.14) becomes

$$F(0)G'(0)>0 \quad (4.15)$$

The condition for (4.15) to hold is

$$SignF(0) = SignG'(0)$$

Recall that at the Endemic Equilibrium State

$$S = x_1 = \frac{(\psi+\sigma+\mu+\delta)(\rho+\mu+\delta)}{\alpha(\rho+\mu+\delta)+\sigma\alpha} \quad (4.16)$$

$$I = x_2 = \frac{\sigma(\rho+\mu+\delta)(\omega+\mu)(\mu A-B)}{\omega B(\sigma\rho+\psi(\rho+\mu+\delta))-(\omega+\mu)(\alpha(\rho+\mu+\delta)+\sigma\alpha)A} \quad (4.17)$$

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$$C = x_3 = \frac{(\omega+\mu)(\mu A - B)\sigma}{(\omega B(\sigma\rho+\psi(\rho+\mu+\delta))) - (\omega+\mu)(\alpha(\rho+\mu+\delta)+\sigma\alpha)A} \quad (4.18)$$

$$R = x_4 = \frac{(\sigma\rho+\psi(\rho+\mu+\delta))(\mu A - B)}{\omega B(\sigma\rho+\psi(\rho+\mu+\delta)) - (\omega+\mu)(\alpha(\rho+\mu+\delta)+\sigma\alpha)} \quad (4.19)$$

Where $A = (\psi+\sigma+\mu+\delta)(\rho+\mu+\delta)$ and $B = \alpha(\rho+\mu+\delta)+\sigma\alpha$

From (4.17) and (4.18),

$$I + C = \frac{(\omega+\mu)(\mu A - B)[\sigma(\rho+\mu+\delta)+\sigma]}{\omega B(\sigma\rho+\psi(\rho+\mu+\delta)) - (\omega+\mu)(\alpha(\rho+\mu+\delta)+\sigma\alpha)A} \quad (4.20)$$

Substituting (4.16) and (4.20) into $F(0)$ and $G'(0)$, we obtained,

$$F(0) = \left\{ \begin{array}{l} (\omega+\mu)(\rho+\mu+\delta)\alpha^2\left(\frac{(\psi+\sigma+\mu+\delta)(\rho+\mu+\delta)}{\alpha(\rho+\mu+\delta)+\sigma\alpha}\right)\left(\frac{(\omega+\mu)(\mu A-B)[\sigma(\rho+\mu+\delta)+\sigma]}{\omega B(\sigma\rho+\psi(\rho+\mu+\delta))-(\omega+\mu)(\alpha(\rho+\mu+\delta)+\sigma\alpha)A}\right) \\ -\sigma(\omega+\mu)\alpha^2\left(\frac{(\psi+\sigma+\mu+\delta)(\rho+\mu+\delta)}{\alpha(\rho+\mu+\delta)+\sigma\alpha}\right)\left(\frac{(\omega+\mu)(\mu A-B)[\sigma(\rho+\mu+\delta)+\sigma]}{\omega B(\sigma\rho+\psi(\rho+\mu+\delta))-(\omega+\mu)(\alpha(\rho+\mu+\delta)+\sigma\alpha)A}\right) \\ -\alpha\left(\frac{(\psi+\sigma+\mu+\delta)(\rho+\mu+\delta)}{\alpha(\rho+\mu+\delta)+\sigma\alpha}\right)\sigma\left(\alpha\left(\frac{(\omega+\mu)(\mu A-B)[\sigma(\rho+\mu+\delta)+\sigma]}{\omega B(\sigma\rho+\psi(\rho+\mu+\delta))-(\omega+\mu)(\alpha(\rho+\mu+\delta)+\sigma\alpha)A}\right)+\mu\right)(\omega+\mu) \end{array} \right\}$$

(4.21)

and

$$G'(0) = \left\{ \begin{array}{l} \left(\alpha\left(\frac{(\omega+\mu)(\mu A-B)[\sigma(\rho+\mu+\delta)+\sigma]}{\omega B(\sigma\rho+\psi(\rho+\mu+\delta))-(\omega+\mu)(\alpha(\rho+\mu+\delta)+\sigma\alpha)A}\right)+\mu\right)(\omega+\mu)(\psi+\sigma+\mu+\delta) \\ (\rho+\mu+\delta)\left(\alpha\left(\frac{(\omega+\mu)(\mu A-B)[\sigma(\rho+\mu+\delta)+\sigma]}{\omega B(\sigma\rho+\psi(\rho+\mu+\delta))-(\omega+\mu)(\alpha(\rho+\mu+\delta)+\sigma\alpha)A}\right)+\mu\right) \\ -\alpha\left(\frac{(\psi+\sigma+\mu+\delta)(\rho+\mu+\delta)}{\alpha(\rho+\mu+\delta)+\sigma\alpha}\right)\sigma\left(\alpha\left(\frac{(\omega+\mu)(\mu A-B)[\sigma(\rho+\mu+\delta)+\sigma]}{\omega B(\sigma\rho+\psi(\rho+\mu+\delta))-(\omega+\mu)(\alpha(\rho+\mu+\delta)+\sigma\alpha)A}\right)+\mu\right) \\ -(\omega+\mu)\alpha^2\left(\frac{(\psi+\sigma+\mu+\delta)(\rho+\mu+\delta)}{\alpha(\rho+\mu+\delta)+\sigma\alpha}\right)\left(\frac{(\omega+\mu)(\mu A-B)[\sigma(\rho+\mu+\delta)+\sigma]}{\omega B(\sigma\rho+\psi(\rho+\mu+\delta))-(\omega+\mu)(\alpha(\rho+\mu+\delta)+\sigma\alpha)A}\right) \\ -\alpha\left(\frac{(\psi+\sigma+\mu+\delta)(\rho+\mu+\delta)}{\alpha(\rho+\mu+\delta)+\sigma\alpha}\right)\sigma(\omega+\mu) \\ +(\rho+\mu+\delta)\alpha^2\left(\frac{(\psi+\sigma+\mu+\delta)(\rho+\mu+\delta)}{\alpha(\rho+\mu+\delta)+\sigma\alpha}\right)\left(\frac{(\omega+\mu)(\mu A-B)[\sigma(\rho+\mu+\delta)+\sigma]}{\omega B(\sigma\rho+\psi(\rho+\mu+\delta))-(\omega+\mu)(\alpha(\rho+\mu+\delta)+\sigma\alpha)A}\right) \\ -\sigma\alpha^2\left(\frac{(\psi+\sigma+\mu+\delta)(\rho+\mu+\delta)}{\alpha(\rho+\mu+\delta)+\sigma\alpha}\right)\left(\frac{(\omega+\mu)(\mu A-B)[\sigma(\rho+\mu+\delta)+\sigma]}{\omega B(\sigma\rho+\psi(\rho+\mu+\delta))-(\omega+\mu)(\alpha(\rho+\mu+\delta)+\sigma\alpha)A}\right) \end{array} \right.$$

(4.22)

In order to have insight into the signs of $F(0)$ and $G'(0)$, we used hypothetical values for the parameters using a mathematical software (Maple 16).

Maple Codes for Computation of $F(0)$ and $G'(0)$

> restart

> Computation of $F(0)$ and $G(0)$

$\Lambda := 6 : \alpha := 0.02 : \psi := 0.15 : \rho := 0.1 : \mu := 0.014 : \delta := 0.001 : \sigma := -0.1 : \omega := 0.5 :$

for i **from** 0 **to** 9 **do** $\sigma := 0.1 + \sigma$; $K1 := \psi + \sigma + \mu + \delta : K2 := \rho + \mu + \delta : A := K1 \cdot K2 : B := \alpha \cdot K2 + \sigma \cdot \alpha : S := \dfrac{K1 \cdot K2}{\alpha \cdot K2 + \sigma \cdot \alpha} : Ic := \dfrac{(\omega + \mu) \cdot (\mu \cdot A - B) \cdot (\sigma \cdot K2 + \sigma)}{\omega \cdot B \cdot (\sigma \cdot \rho + \psi \cdot K2) - (\omega + \mu) \cdot (\alpha \cdot K1 + \sigma \cdot \alpha) \cdot A} :$

$F := (\omega + \mu) \cdot \alpha^2 \cdot S \cdot Ic \cdot (K2 - \sigma) - \alpha \cdot S \cdot \sigma \cdot (\alpha \cdot Ic + \mu) \cdot (\omega + \mu)$; $G := (\alpha \cdot Ic + \mu) \cdot (\omega + \mu) \cdot K1 \cdot K2 \cdot (\alpha \cdot Ic + \mu)$

$- \alpha \cdot S \cdot \sigma \cdot (\alpha \cdot Ic + \mu) - (\omega + \mu) \cdot \alpha^2 \cdot S \cdot Ic - \alpha \cdot S \cdot \sigma \cdot (\omega + \mu) + K2 \cdot \alpha^2 \cdot S \cdot Ic - \sigma \cdot \alpha^2 \cdot S \cdot Ic$; **end do**

Table 4.1: Computed Values of $F(0)$ and $G'(0)$

$\sigma := 0.$	$F := 0.$	$G := 0.000001911617400$
$\sigma := 0.1$	$F := -0.0005950742099$	$G := -0.01410697297$
$\sigma := 0.2$	$F := -0.002213130972$	$G := -0.02479854296$
$\sigma := 0.3$	$F := -0.003994386356$	$G := -0.03484068314$
$\sigma := 0.4$	$F := -0.005831710251$	$G := -0.04464076489$
$\sigma := 0.5$	$F := -0.007694885431$	$G := -0.05432366512$
$\sigma := 0.6$	$F := -0.009572094474$	$G := -0.06394082790$
$\sigma := 0.7$	$F := -0.01145776778$	$G := -0.07351739038$
$\sigma := 0.8$	$F := -0.01334893754$	$G := -0.08306710927$
$\sigma := 0.9$	$F := -0.01524387775$	$G := -0.09259815188$

CONCLUSION AND RECOMMENDATION.

Conclusion

In this study, we developed a model for Typhoid Fever with effects of Carriers on the spread. The stability analysis of the Disease Free

Equilibrium State (DFE) of the model shows that it will be conditionally stable if and only if

$$\sqrt{(\mu[(\psi+\sigma+\mu+\delta)+(\rho+\mu+\delta)])^2-4\mu[(\psi+\sigma+\mu+\delta)(\rho+\mu+\delta)-\alpha\Lambda\sigma]}<\mu[(\psi+\sigma+\mu+\delta)+(\rho+\mu+\delta)]$$

The analysis of the Endemic Equilibrium State (EE) is stable which implies that for as long as the carrier compartment exists, Typhoid Fever cannot be completely eradicated from the population.

Recommendation

Government should create awareness and sensitize the public on the transmission and spread of Typhoid Fever. Prevention is still the best control measure for Typhoid fever, hence emphasis should be placed on good sanitation and hygiene.

REFERENCES

Adetunde , I. A. (2008). *A Mathematical models for the dynamics of typhoid fever in Kassena Nankana district of upper cast of Ghana,* J. Mod. Math. Stat. 2(2), 45-49.

Bellman, R. & Cooke, K. (1963). *Differential equations*. Academic Press London.Pp.14-45

Gonzalez- Guzman(1989). "An epidemiological model for direct and indirect transmission of typhoid fever", Mathematical Bioscience, vol.96, no. 1,pp. 33-46.

Lauria, D. T., Maskery, B., Poulos, C., & Whittington, D.(2009). "An optimization model for reducing typhoid cases in developing countires without increasing public spending" Vaccine, vol.27,no. 10,pp.1609-1621.

Mayo Clinic (2017) Typhoid fever - Mayo Clinic http://www.mayoclinic.org/diseases-conditions/typhoid-fever/basics/causes/con-20028553. Retrieved 28-01-2017

Mushayabasa, S.(2012). *A simple epidemiological model for typhoid with saturated incidence rate and treatment effect,* International Journal of Biological, Veterinary, Agricultural and Food Engineering Vol:6 No:6.

Mushayabasa, S.C. P. Bhunu, and E.T. Ngarakana-Gwasira(2013). "Mathematical analysis of a typhoid model with carrier, direct and indirect disease transmiddion" *International Journal of Mathematical Science and Engineering Applications,* vol, 7, no.1,pp. 79-90.

Roumagnac, P. F. Weill, C. Dolecek, S.Baker, S. Brises, N. T. Chinh, T. A. Hong Le, C.J. Acosta, J. Farrar, G. Dougan & M. Achtman (2006), "Evolutionary History of Salmonella Typhi", Science, Vol. 314,N.5803, Pp. 1301-1304. "Typhoid". Merriam Webster Dictionary, Retrieved 2013-06-24.

WebMd.Com (2017). Typhoid Fever Causes, Symptoms, Treatment and Vaccine www.webmd.com/a-to-z-guides/typhoid-fever. Retrieved 28-01-2017.

www.medicinenet.com /typhoid fever/rticle.htm-43k: Typhoid fever symptoms, cause and treatment by medicinent.com. Retrieved 28-01-2017.

Zaman, G. et al.(2012), Modeling Dynamical Interactions between Leptospirosis infected Vector and Human Population, Appl. Math. Sci., 6, 2012, no. 26, 1286-1302.