ISSN – 2278-8881

Publications & Research International

African Scholars Journal of pure and Applied Science (JPAS-1)

Solution of a Deterministic Model for the Spread of Typhoid Fever Incorporating the Carrier Compartment Using Differential Transformation Method (Dtm).

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ABSTRACT

In this work, a deterministic model of Typhoid Fever was developed incorporating the effect of carriers on the spread. The population was divided into four compartments namely Susceptible, Infected, Carrier, and Recovered compartments. The system of four non linear differential equations was solved using Differential transformation method. It was observed that the rate of progression from Infected Compartment into Carrier Compartment plays crucial role on the spread of Typhoid fever.

Key words: *Salmonella Typhi, Typhoid fever, Carries, Analytical solution. Differential transformation.*

Introduction

Typhoid fever is a systemic infection caused by *Salmonella* Typhi, usually through ingestion of contaminated food or water. The acute illness is characterized by prolonged fever, headache, nausea, loss of appetite and constipation or sometimes diarrhoea. Symptoms are often non-specific and clinically non-distinguishable from other febrile illnesses. However, clinical severity varies and severe cases may lead to serious complications or even death. It occurs predominantly in association with poor sanitation and lack of clean drinking water.

According to the most recent estimates (published in 2014), approximately 21 million cases and 222 000 typhoid-related deaths occur annually worldwide. (World Health Organisation, 2017)

About 3%-5% of patients become carriers of the bacteria after the acute illness. Some patients suffer a very mild illness that goes unrecognized. These patients can become long-term carriers of the bacteria. The Bacteria multiplies in the gallbladder, bile ducts, or liver and passes into the bowel. The bacteria can survive for weeks in water or dried sewage. These chronic carriers may have no symptoms and can be the source of new outbreaks of typhoid fever for many years. (Medicinnet.com, 2017)

Approximately 3 to 5 percent of people may still carry the typhoid fever bacteria, even if symptoms go away with treatment. In carriers, it is possible for the illness to return or be passed on to other people. These people are known as typhoid fever carriers. The most famous typhoid fever carrier was Mary Mallon, the infamous Typhoid Mary (Diseases.emedtv.com, 2017).

The global burden of disease estimates for typhoid were based on a total of 22 community-based incidence studies with 19 from continents other than Africa and only three from Africa. On the basis of these data and a prediction rule based on climatic and socio-economic features, continental estimates of disease burden were derived (Crump *et al.* 2004). These estimates suggested a moderate incidence of typhoid of 10–100 cases/100,000 person years in most African countries, with the incidence highest in childhood

Differential Transformation Method (DTM) is one of the method use to solve linear and nonlinear differential equations. It was first proposed by Zhou (1986) for solving linear and nonlinear initial value problems in electrical circuit analysis. The DTM construct a semi analytical numerical technique that uses Taylor series for the solution of differential equations in the form of a polynomial.

DTM is a very effective and powerful tool for solving different kinds of differential equations. This technique has been used to solve different kind of problems such as: Fractional differential equations ((Arikoglu and

Ozkol, 2007) and (*Momani et al.* 2008)), Differential algebraic equations (Ayaz, 2004), Nonlinear oscillatory system, (Moustafa, 2008), Quadratic Riccati differential equation (Biazar and Eslami, 2004), Numerical solution of Susceptible Infected Recovered (SIR) model (Akinboro *et al.*(2014), Solution of prey and predator problem (Batiha, 2015), Fourth-order parabolic partial differential equations (Soltanalizadeh, 2012), Volterra integral equations (Odibat, 2008) and Difference equations (Arikoglu and Ozkol, 2006).

The main advantage of this method is that it can be applied directly to linear and nonlinear Ordinary Differential Equations (ODEs) without linearization, discretization or perturbation.

An arbitrary function *f(t)* can be expanded in Taylor series about a point *t=0* as

$$f(t)=\sum_{k=0}^{\infty}\frac{t^k}{k!}\left[\frac{d^k f}{dt^k}\right]_{t=0} \qquad (1.1)$$

The differential transformation of ***f(t)*** is defined as

$$F(t)=\frac{1}{k!}\left[\frac{d^k f}{dt^k}\right]_{t=0} \qquad (1.2)$$

Then the inverse differential transform is

$$f(t)=\sum_{k=0}^{\infty}t^k F(t) \qquad (1.3)$$

Hassan (2008), listed the fundamental mathematical operations performed by differential transformation in the table below where ***y(t)*** and ***g(t)*** are two uncorrelated functions of ***t*** and ***Y(k)*** and ***G(k)*** are the transformed functions corresponding to ***y(t)*** and ***g(t)***.

Table 1.1: The Fundamental Mathematical Operations by Differential Transformation Method (DTM).

Original Function	Transformed Function
$y(t) = f(t) \pm g(t)$	$Y(k) = F(k) \pm G(k)$
$y(t) = af(t)$	$Y(k) = aF(k)$
$y(t) = \frac{df(t)}{dt}$	$Y(k) = (k+1)F(k+1)$
$y(t) = \frac{d^2 f(t)}{dt^2}$	$Y(k) = (k+1)(k+2)F(k+2)$
$y(t) = \frac{d^m f(t)}{dt^m}$	$Y(k) = (k+1)(k+2)\ldots(k+m)F(k+m)$
$y(t) = 1$	$Y(k) = \delta(k)$
$y(t) = t$	$Y(k) = \delta(k-1)$
$y(t) = t^m$	$Y(k) = \delta(k-m) = \begin{cases} 1, & k = m \\ 0, & k \neq m \end{cases}$
$y(t) = f(t)g(t)$	$Y(k) = \sum_{m=0}^{k} G(m)f(k-m)$
$y(t) = e^{(\lambda t)}$	$Y(k) = \frac{\lambda^k}{k!}$
$y(t) = (1+t)^m$	$Y(k) = \frac{m(m-1)\ldots(m-k+1)}{k!}$

Source: Hassan, 2008

MATERIALS AND METHODS

The Model

The population was partitioned into four compartments; Susceptible compartment S(t), Infected compartment I(t), Carrier compartment C(t), and Recovered compartment R(t).

The population of the susceptible class increases with constant recruitment Λ, movement of individuals from the Recovered class into the Susceptible class at the rate ω as a result of winning off of partial immunity after recovery, the population of this Class decreases when individuals move from the susceptible class into the infected class I via interaction with Infected and Carriers at the rate α and further decreases with natural death at the rate μ.

The population of the infected Class increases with individuals coming in from the Susceptible class via interaction of the Susceptible individuals with the Infected and the Carrier individuals at the rate α and decreases with individuals recovering due to treatment at the rate ψ, progression into the Carrier class at the rate σ since a small number of people who recover from typhoid fever continue to harbor the bacteria in their intestinal tracts or gallbladders often for years, This Class further decreases with death due to infection at the rate δ and natural death at the rate μ.

The population of the Carrier class increases when the Infected individuals progress from the Infected class into the Carrier class at the rate σ and decreases as members of the Carrier class recovered the rate ρ, This Class further decreases with death due to infection at the rate δ and natural death at the rate μ.

The population of the recovered class increases with recovery of Infected and Carrier individuals at the rates ψ and ρ respectively. The population decreases as members of the Recovered class move into the susceptible class at the rate ω as a result of winning off of the partial immunity and due to natural death at the rate μ.

Model Equation

$$\frac{dS}{dt} = \Lambda + \omega R - (\alpha I + \alpha C + \mu)S \qquad (2.1)$$

$$\frac{dI}{dt} = \alpha IS + \alpha CS - (\psi + \sigma + \mu + \delta)I \qquad (2.2)$$

$$\frac{dC}{dt} = \sigma I - (\rho + \mu)C \qquad (2.3)$$

$$\frac{dR}{dt} = \rho C + \psi I - (\omega + \mu)R \qquad (2.4)$$

Analytical Solution of the Model Equations.

Let the model equation be a function ***q(t)***, ***q(t)*** can be expanded in Taylor series about a point ***t=0*** as

$$q(t)=\sum_{k=0}^{\infty}\frac{t^k}{k!}\left[\frac{d^kq}{dt^k}\right]_{t=0} \quad (2.5)$$

Where,

$q(t)=s(t)=i(t)=c(t)=r(t)$ (2.6)

The differential transformation of ***q(t)*** is defined as

$$Q(t)=\frac{1}{k!}\left[\frac{d^kq}{dt^k}\right]_{t=0} \quad (2.7)$$

Where,

$Q(t)=S(t)=I(t)=C(t)=R(t)$ (2.8)

Then the inverse differential transform is

$$q(t)=\sum_{k=0}^{\infty}t^kQ(t) \quad (2.9)$$

Using the fundamental operations of differential transformation method in table 1.1, we obtained the following recurrence relation of equations (2.1) to (2.4) as

$$S(k+1)=\frac{1}{k+1}\left[\Lambda+\omega R(k)-\alpha\sum_{m=0}^{k}S(m)I(k-m)-\alpha\sum_{m=0}^{k}S(m)C(k-m)-\mu S(k)\right] \quad (2.10)$$

$$I(k+1)=\frac{1}{k+1}\left[\alpha\sum_{m=0}^{k}S(m)I(k-m)+\alpha\sum_{m=0}^{k}S(m)C(k-m)-(\psi+\sigma+\mu+\delta)I(k)\right] \quad (2.11)$$

$$C(k+1)=\frac{1}{k+1}[\sigma I(k)-(\rho+\mu)C(k)] \quad (2.12)$$

$$R(k+1)=\frac{1}{k+1}[\rho C(k)+\psi I(k)-(\omega+\mu)R(k)] \quad (2.13)$$

With initial conditions

S(0)=2000, I(0)=15, C(0)=20, R(0)=200. (2.14)

The parameter values are

$\Lambda = 27, \omega = 0.5, \alpha = 0.003, \mu = 0.013, \rho = 0.001, \delta = 0.011$ (2.15)

For $k = 0,1,2,3,\ldots$ (2.16)

Substituting $k=0$ into (2.10) to (2.13) gives

$$S(1)=[\Lambda+\omega R(0)-\alpha S(0)I(0)-\alpha S(0)C(0)-\mu S(0)] \quad (2.17)$$

$$I(1)=[\alpha S(0)I(0)+\alpha S(0)C(0)-(\psi+\sigma+\mu+\delta)I(0)] \quad (2.18)$$

$$C(1) = [\sigma I(0) - (\rho + \mu)C(0)] \qquad (2.19)$$

$$R(1) = [\rho C(0) + \psi I(0) - (\omega + \mu)R(0)] \qquad (2.20)$$

Substituting $k=1$ into (2.10) to (2.13) gives

$$S(2) = \frac{1}{2}[\Lambda + \omega R(1) - \alpha[S(0)I(1) + S(1)I(0)] - \alpha[S(0)C(1) + S(1)C(0)] - \mu S(1)] \qquad (2.21)$$

$$I(2) = \frac{1}{2}[\alpha[S(0)I(1) + S(1)I(0)] + \alpha[S(0)C(1) + S(1)C(0)] - (\psi + \sigma + \mu + \delta)I(1)] \qquad (2.22)$$

$$C(2) = \frac{1}{2}[\sigma I(1) - (\rho + \mu)C(1)] \qquad (2.23)$$

$$R(2) = \frac{1}{2}[\rho C(1) + \psi I(1) - (\omega + \mu)R(1)] \qquad (2.24)$$

Substituting $k=2$ into (2.10) to (2.13) gives

$$S(3) = \frac{1}{3}\begin{bmatrix} \Lambda + \omega R(2) - \alpha[S(0)I(2) + S(1)I(1) + S(2)I(0)] \\ -\alpha[S(0)C(2) + S(1)C(1) + S(2)C(0)] - \mu S(2) \end{bmatrix} \qquad (2.25)$$

$$I(3) = \frac{1}{3}\begin{bmatrix} \alpha[S(0)I(2) + S(1)I(1) + S(2)I(0)] + \alpha[S(0)C(2) \\ +S(1)C(1) + S(2)C(0)] - (\psi + \sigma + \mu + \delta)I(2) \end{bmatrix} \qquad (2.26)$$

$$C(3) = \frac{1}{3}[\sigma I(2) - (\rho + \mu)C(2)] \qquad (2.27)$$

$$R(3) = \frac{1}{3}[\rho C(2) + \psi I(2) - (\omega + \mu)R(2)] \qquad (2.28)$$

Substituting $k=3$ into (2.10) to (2.13) gives

$$S(4) = \frac{1}{4}\begin{bmatrix} \Lambda + \omega R(3) - \alpha[S(0)I(3) + S(1)I(2) + S(2)I(1) + S(3)I(0)] \\ -\alpha[S(0)C(3) + S(1)C(2) + S(2)C(1) + S(3)C[0] - \mu S(3) \end{bmatrix} \qquad (2.29)$$

$$I(4)=\frac{1}{4}\begin{bmatrix}\alpha[S(0)I(3)+S(1)I(2)+S(2)I(1)+S(3)I(0)]\\+\alpha[S(0)C(3)+S(1)C(2)+S(2)C(1)+S(3)C(0)]-(\psi+\sigma+\mu+\delta)I(3)\end{bmatrix}$$

(2.30)

$$C(4)=\frac{1}{4}[\sigma I(3)-(\rho+\mu)C(3)]$$

(2.31)

$$R(4)=\frac{1}{4}[\rho C(3)+\psi I(3)-(\omega+\mu)R(3)]$$

(2.32)

Case 1

$\sigma = 0.016, \psi = 0.060$ (Low rate of progression from infectious compartment to carrier compartment).

(2.33)

Substituting (2.14), (2.15) and (2.33) into (2.17) to (2.32) using a mathematical software (Maple), we obtained,

$S1$=-109.000, $I1$= 208.500, $C1$= - 0.040, $R1$ = 81.400

(2.34)

$S2$ = -585.0990000, $I2$ = 609.2325000, $C2$ = 1.668280000, $R2$ = 488.2762720, (2.35)

$S3$ = -1087.686147, $I3$ = 1158.293205, $C3$ = 3.241454693, $R3$ = 5866.251763 (2.36)

$S4$ =-828.8243364, $I4$ = 1543.426957, $C4$ = 4.621827729, $R4$ = 101175.0004 (2.37)

Substituting (2.14) and (2.34) to (2.37) into (2.9) using maple software gives,

$$s(t)=\sum_{k=0}^{\infty}S(k)t^k = 2000 - 109.000t - 585.0990000t^2 - 1085.686147t^3 - 828.8243364t^4 + \dots$$

(2.38)

$$s(t) = \sum_{k=0}^{\infty} S(k)t^k = 2000 - 109.000t - 616.8990000t^2 - 1189.446077t^3 - 1616.906226t^4 +$$
...(2.56)

$$i(t) = \sum_{k=0}^{\infty} I(k)t^k = 15 + 208.500t + 611.0325000t^2 + 1168.986405t^3 + 1562.944538t^4 +$$
...(2.57)

$$c(t) = \sum_{k=0}^{\infty} C(k)t^k = 20 + 0.560t + 5.834080000t^2 + 11.37871429t^3 + 16.32598417t^4 +$$
...(2.58)

$$r(t) = \sum_{k=0}^{\infty} R(k)t^k = 200 - 38.600t - 70.59090800t^2 - 275.6218257t^3 - 1576.426392t^4 + \ldots$$
(2.59)

Results and Discussion.

Equations (2.38) to (2.41), (2.47) to (2.50) and (2.56) to (2.59) were coded using maple mathematical software (**Appendix** A) and the results presented as the following graphs.

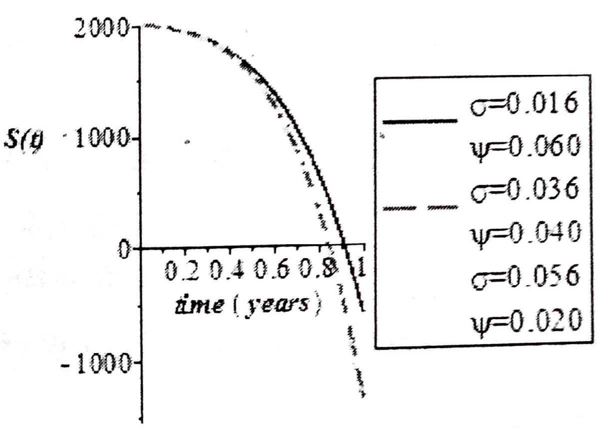

Figure 1: Showing graphs of Susceptible Compartments for Case 1, Case 2 and Case 3

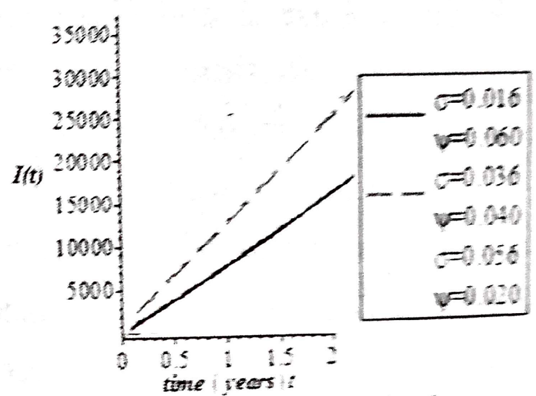

Figure 2: Showing graphs of Infected Compartments for Csae 1, Case 2 and Case 3

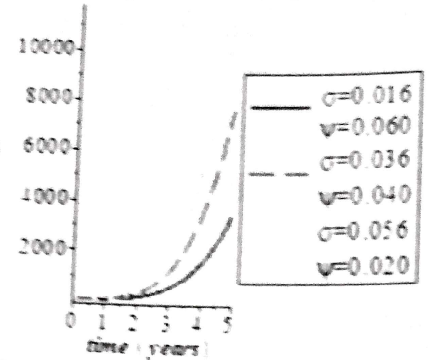

Figure 3: Showing graphs of Carrier Compartments for Case 1, Case 2 and Case 3

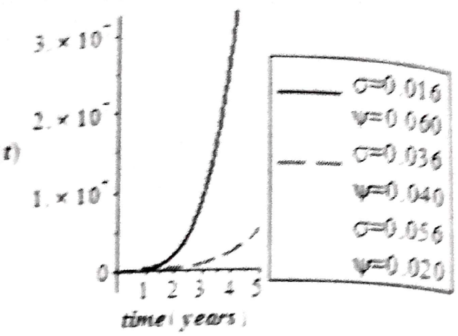

Figure 4: Showing graphs of Recovered Compartments for Case 1, Case 2 and Case 3

Figure 1 shows the graphical profile of Susceptible Compartment for Case 1, Case 2 and Case 3. It was observed that Susceptible individuals decreases with time in each case, but the rate of increment is higher in **Case 3** (i.e when $\sigma = 0.056, \psi = 0.020$ (High rate of progression from Infectious compartment to Carrier compartment)). This is because with High σ and low ψ there will be more Carriers consequently leading to increase in the chances of infecting the Susceptible Compartment.

Figure 2 shows the graphical profile for the Infected Compartment for Case 1, Case 2 and Case 3. It was observed that Infected individuals increases with time in each case, but the rate of decrease is higher in **Case 3** (i.e when $\sigma = 0.056, \psi = 0.020$ (High rate of progression from Infectious compartment to Carrier compartment)). This is because with High σ and low ψ there will be more Carriers consequently leading to increase in the chances of infecting the Susceptible Compartment thereby moving them into the Infected Compartment.

Figure 3 shows the graphical profile of the Carrier Compartment for Case 1, Case 2 and Case 3. It was observed that Carrier individuals increases with time in each case, but the rate of increment is higher in **Case 3** (i.e when $\sigma = 0.056, \psi = 0.020$ (High rate of progression from Infectious compartment to Carrier compartment)). This is because with High σ and low ψ more Infected Individuals will move into Carrier Compartment.

. Figure 3 shows the graphical profile of the Recovered Compartment for Case 1, Case 2 and Case 3. We observed that recovery is higher in **case 1** when $\sigma = 0.016, \psi = 0.060$ (Low rate of progression from Infectious Compartment to Carrier compartment).

Conclusion

In this research, we presented a deterministic model of the dynamics of typhoid fever. The analytical solution shows that the presence of carriers in the population will make it difficult for Typhoid fever to be completely eradicated once it is introduced,

Recommendation

In many developing nations, the public health goals that can help prevent and control typhoid fever such as safe drinking water, improved sanitation and adequate medical care may be difficult to achieve, hence vaccinating high-risk populations is the best way to control typhoid fever.

Appendix A (Maple codes for analytical solution of the model equations)

> *Restart*

Restart

> $\Lambda := 27 : \omega := 0.5 : \alpha := 0.003 : \psi := 0.060 : \rho := 0.001 : \mu := 0.013 : \delta := 0.011 : \sigma := 0.016 :$

> $S0 := 2000 : I0 := 15 : C0 := 20 : R0 := 200 :$

> $S1 := (\Lambda + \omega \cdot R0 - \alpha \cdot S0 \cdot I0 - \alpha \cdot S0 \cdot C0 - \mu \cdot S0);$

$S1 := -109.000$

> $I1 := (\alpha \cdot S0 \cdot I0 + \alpha \cdot S0 \cdot C0 - (\psi + \sigma + \mu + \delta) \cdot I0);$

$I1 := 208.500$

> $C1 := (\sigma \cdot I0 - (\rho + \mu) \cdot C0);$

$C1 := -0.040$

> $R1 := (\rho \cdot C0 + \psi \cdot I0 - (\omega + \mu)) \cdot R0;$

$R1 := 81.400$

> $S2 := \frac{1}{2} \cdot (\Lambda + \omega \cdot R1 - \alpha \cdot (S0 \cdot I1 + S1 \cdot I0) - \alpha \cdot (S0 \cdot C1 + S1 \cdot C0) - \mu \cdot S1);$

$S2 := -585.0990000$

> $I2 := \frac{1}{2}\cdot(\alpha\cdot(S0\cdot I1 + S1\cdot I0) + \alpha\cdot(S0\cdot C1 + S1\cdot C0) - (\psi + \sigma + \mu + \delta)\cdot I1);$

$I2 := 609.2325000$

> $C2 := \frac{1}{2}\cdot(\sigma\cdot I1 - (\rho + \mu)\cdot C1);$

$C2 := 1.668280000$

> $R2 := \frac{1}{2}\cdot(\rho\cdot C1 + \psi\cdot I1 - (\omega + \mu))\cdot R1;$

$R2 := 488.2762720$

> $S3 := \frac{1}{3}\cdot(\Lambda + \omega\cdot R2 - \alpha\cdot(S0\cdot I2 + S1\cdot I1 + S2\cdot I0) - \alpha\cdot(S0\cdot C2 + S1\cdot C1 + S2\cdot C0) - \mu\cdot S2);$

$S3 := -1085.686147$

> $I3 := \frac{1}{3}\cdot(\alpha\cdot(S0\cdot I2 + S1\cdot I1 + S2\cdot I0) + \alpha\cdot(S0\cdot C2 + S1\cdot C1 + S2\cdot C0) - (\psi + \sigma + \mu + \delta)\cdot I2);$

$I3 := 1158.293205$

> $C3 := \frac{1}{3}\cdot(\sigma\cdot I2 - (\rho + \mu)\cdot C2);$

$C3 := 3.241454693$

> $R3 := \frac{1}{3}\cdot(\rho\cdot C2 + \psi\cdot I2 - (\omega + \mu))\cdot R2;$

$R3 := 5866.251763$

> $S4 := \frac{1}{4}\cdot(\Lambda + \omega\cdot R3 - \alpha\cdot(S0\cdot I3 + S1\cdot I2 + S2\cdot I1 + S3\cdot I0) - \alpha\cdot(S0\cdot C3 + S1\cdot C2 + S2\cdot C1 + S3\cdot C0) - \mu\cdot S3);$

$S4 := -828.8243364$

> $I4 := \frac{1}{4}\cdot(\alpha\cdot(S0\cdot I3 + S1\cdot I2 + S2\cdot I1 + S3\cdot I0) + \alpha\cdot(S0\cdot C3 + S1\cdot C2 + S2\cdot C1 + S3\cdot C0) - (\psi + \sigma + \mu + \delta)\cdot I3);$

$I4 := 1543.426957$

> $C4 := \frac{1}{4}\cdot(\sigma\cdot I3 - (\rho + \mu)\cdot C3);$

$C4 := 4.621827729$

> $R4 := \frac{1}{4}\cdot(\rho\cdot C3 + \psi\cdot I3 - (\omega + \mu))\cdot R3;$

$R4 := 1.011750004\ 10^5$

> $A1 := S0 + S1 \cdot t + S2 \cdot t^2 + S3 \cdot t^3 + S4 \cdot t^4;$

$$A1 := 2000 - 109.000\,t - 585.0990000\,t^2 - 1085.686147\,t^3 - 828.8243364\,t^4$$

> $B1 := I0 + I1 \cdot t + I2 \cdot t^2 + I3 \cdot t^3 + I4 \cdot t^4;$

$$B1 := 15 + 208.500\,t + 609.2325000\,t^2 + 1158.293205\,t^3 + 1543.426957\,t^4$$

> $Q1 := C0 + C1 \cdot t + C2 \cdot t^2 + C3 \cdot t^3 + C4 \cdot t^4;$

$$Q1 := 20 - 0.040\,t + 1.668280000\,t^2 + 3.241454693\,t^3 + 4.621827729\,t^4$$

> $D1 := R0 + R1 \cdot t + R2 \cdot t^2 + R3 \cdot t^3 + R4 \cdot t^4;$

$$D1 := 200 + 81.400\,t + 488.2762720\,t^2 + 5866.251763\,t^3 + 1.011750004\,10^5\,t^4$$

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