ON THE OPERATING CHARACTERISTICS OF QUEUING SYSTEM FOR AN NNPC MEGA STATION IN NIGERIA

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Abstract

In this work, an understanding is sought of Queuing characteristics at an NNPC Mega station in Nigeria when petrol is easily available so as to compare with the queuing characteristics when there is petrol scarcity which is a recurring decimal in the country. The Mega station with queuing discipline of First-In First-Out, a service mechanism of single-queue multiple-channels and a system capacity of an infinite source has distinct operating characteristic with traffic intensity being 0.77. Further analysis of the queuing characteristics revealed that, the average number of vehicles in queue is from 2 to 3 while the average time a vehicle spends in queue is 1.58 minutes. The probability of a vehicle queuing on arrival is 0.5993 while there is a 0.4007 probability that a vehicle may not queue on arrival. It was concluded that with 1.8 minutes, a vehicle spends more time in service than on the queue and since the number of vehicles in the system, 4 to 5, is $\geq$ the number of active servers, 3; there is negligible queue at NNPC Mega station Minna when there is no fuel scarcity.

Introduction

Queuing is a general phenomenon of life and it occurs in our day to day activities. Queues occur as a result of limited service facilities or failure of the system. Delays and queuing problems are most normal attributes not only in our daily life condition but also in more technical circumstances, such as in manufacturing, computer networking and telecommunications. Queuing models provide the analyst with a powerful tool for designing and evaluating the performance of queuing systems (Kandemir and Cavas, 2007).

Introduction Continue

Queuing theory determines the measures of performance of waiting lines such as the average waiting time in queue and the productivity of the service facility. Queuing models vary from single to multiple channels systems. According to Obamiro (2014), for many customers, waiting in lines or queuing is annoying or has negative experience. Queuing theory deals with the mathematical description of behavior of queues. Queuing theory can be applied to a variety of operational situations where it is not possible to predict accurately the arrival rate of customers and service rate of service facilities. There are many applications of queuing theory most of which have been well documented in the literature of Probability, Operations Research, and Management Science. Some of the application includes machine repairs, inventory control, scheduling patients in hospitals, banking sector, attending to customers in a restaurant, a petrol station, call centers etc. It is not only useful for performance assessment but also useful for security assessment (Nosek and Wilson, 2001).

The Queuing model is commonly labeled as M/M/c/K, where the first M represents Markovian exponential distribution of inter-arrival times, the second M represents Markovian exponential distribution of service times, c (a positive integer) represents the number of servers, and K is the specified number of customers in a queuing system. This general model contains only limited number of K customers in the system. However, if unlimited number of customers exist, which means $K = \infty$ then our model will be labeled as M/M/c (Hillier and Lieberman, 2001.)

This study focuses on Single-Queue Multiple-Server in a fuelling station using Nigeria National Petroleum Corporation (NNPC) Mega Filling Station Minna, Niger State, Nigeria as a case study. A queuing model is constructed so that queue lengths and waiting time can be predicted. The aim of the study is to determine the Queuing (operating) Characteristics at the NNPC Mega Station in the absence of petrol scarcity with the intention of comparing the behavior with that when there is scarcity of petrol in the country.

The choice of NNPC Mega station was predicated on the fact that, it is a reliable filling station in terms of stable fuel price and availability. It is well known and can be found in every state capital of the federation. It is the one filling station that you can be sure to get fuel during fuel scarcity and at the government regulated price since it is owned by the NNPC. The Mega Station in Minna was established on the 10^{th} March, 2008 with the aim of reducing scarcity of petrol in the state. It was established during the period of fuel scarcity in the country and it is expected to run 24 hours a day, 7 days a week. NNPC Mega Station Minna is located at off Nnamdi Azikiwe way (western bye pass) Minna with a total of 35 working staff.

It consists of five main platforms. Each of these platforms comprises two fuel dispensers. Every fuel dispenser comprises two nozzles, making a total of 20 nozzles. NNPC Mega Station Minna supplies petrol, diesel and kerosene. At the time of this study the prices were as follows:

i. Petrol - 143 Naira per liter
ii. Diesel - 215 Naira per liter
iii. Kerosene - 150 Naira per liter

The formulation of our model follows the M/M/s model which is based on the following assumptions:

i. Arrival of vehicles follow a Poisson process (random arrivals).

ii. The queue discipline is based on First-Come First-Served (FCFS) basis by any of the servers. There is no priority classification for any arrival.

iii. Service time is distributed exponentially with an average number of μ customers per unit time.

iv. There is no limit to the number of vehicles on queue (infinite).

vi. Server here represents an attendant in the system (petrol station).

vii. Service rate is independent of the length of the queue (service providers do not rush a customer because the line is long).

Queuing Terminologies

From the standpoint of analyzing queues, the arrival of customers is represented by the **interarrival** time between successive customers, and the service is described by the service time per customer. Generally, the **interarrival** and service times can be probabilistic, as in the operation of a post office, or deterministic, as in the arrival of applicants for job interviews. **Queue size** plays a role in the analysis of queues, and it may have a finite size, as in the buffer area between two successive machines, or it may be infinite, as in mail order facilities. Queue discipline, which represents the order in which customers are selected from a queue, is an important factor in the analysis of queuing models. The most common discipline is **First Come, First Served (FCFS)**.

Other disciplines include **Last Come, First Served (LCFS)** and **Service in Random Order (SIRO)**. Customers may also be selected from the queue based on some order of priority. For example, rush jobs at a shop are processed ahead of regular jobs. The **queuing behavior** of customers plays a role in waiting-line analysis. "Human" customers may **jockey** from one queue to another in the hope of reducing waiting time. They may also **balk** from joining a queue altogether because of anticipated long delay, or they may **renege** from a queue because they have been waiting too long.

In queuing theory, three models are generally of great concern: The Single-Queue Single-Server (M/M/1), the Single-Queue Multiple-Server (M/M/s) and the Multiple-Queue Multiple-Server ($M/M/\infty$) models (see figures 1, 2 and 3).

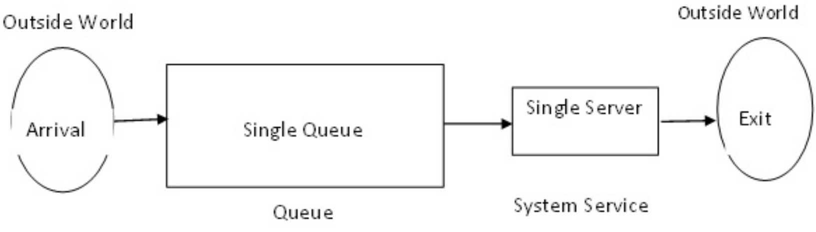


Figure 1: A Queue System Depicting Single-Queue Single-Channel Model

Materials and Method Continue
M/M/s

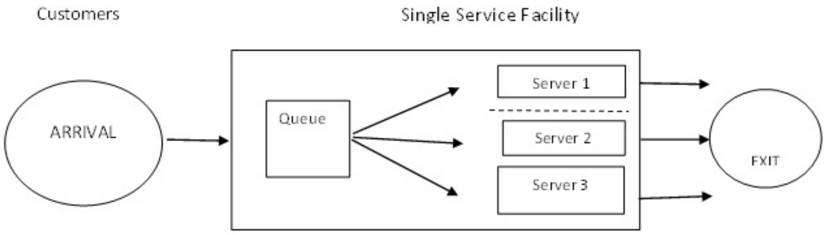


Figure 2: Single Stage Queuing Model with Single- Queue and Multiple Parallel Servers.

Materials and Method Continue
$M/M/\infty$

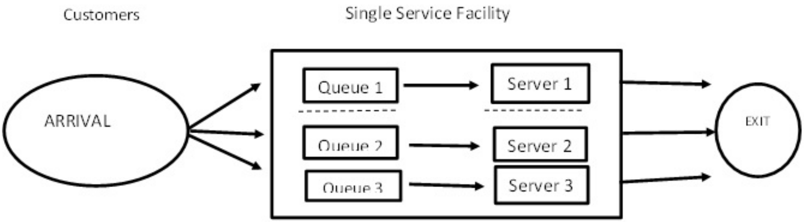


Figure 3: Single Stage Queuing Model with Multiple Queue and Multiple Parallel Servers.

The over all concern in all the models is the investigation into the operating characteristics of their **traffic intensity** which is a measure of the average occupancy of a facility during a specified time. If the traffic intensity is greater than 1, it means the arrival rate is beyond the service rate and queuing delay will grow without bound if the traffic intennsity stays the same.

The Single-Queue Single-Server (M/M/1) queuing model has the following variables and relationships:
Average Arrival Rate, λ. Average Service Rate, μ.
Utilization factor or Traffic Intensity,

$$\rho = \frac{\lambda}{\mu} \tag{1}$$

Probability of zero customers in the system:

$$\rho_0 = 1 - \rho \tag{2}$$

The probability of having n customers in the system:

$$\rho_n = \rho_0 \rho^n \tag{3}$$

The probability of having n or more customers in the system:

$$(\rho_n \geq n) = \rho^n \tag{4}$$

The average number of customers in the system:

$$L_s = \frac{\rho}{1-\rho} or L_s = \frac{\lambda}{\mu - \lambda} \tag{5}$$

The average number of customers in the queue:

$$L_q = \frac{\rho^2}{(1-\rho)} or L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} \tag{6}$$

The average waiting time of a vehicle in the queue:

$$W_q = \frac{\rho}{(1-\rho)} * \frac{1}{\mu} or W_q = \frac{\lambda}{\mu(\mu-\lambda)} \tag{7}$$

The average waiting time of a vehicle in the system,

$$W_s = \frac{1}{1-\rho} \times \frac{1}{\mu} \tag{8}$$

The M/M/s Model Depicted in Figure 2

The Single-Queue Multiple-Server (M/M/s) queuing model analyses the following variables:
Average Arrival Rate, λ. Average Service Rate, μ.
Utilization factor or Traffic Intensity,

$$\rho = \frac{\lambda}{s\mu} \tag{9}$$

where s = number of servers or channels.
Probability of zero customers in the system:

$$\rho_0 = \frac{s!(1-\rho)}{(\rho s)^s + s!(1-\lambda)[\sum_{n=0}^{n=s-1} \frac{1}{n!(\rho s)}]} \tag{10}$$

where n = integers from zero to one less than the number of servers

The probability of queuing on arrival:

$$= \frac{(\rho s)^s}{s!(1-\rho)}\rho_0 \tag{11}$$

The probability of not queuing on arrival:

$$= 1 - \frac{(\rho s)^s}{s!(1-\rho)}\rho_0 \tag{12}$$

The average number of vehicles in the system:

$$L_s = \frac{\rho(\rho s)^s}{s!(1-\rho)}\rho_0 + \rho s \tag{13}$$

The average number of vehicles in the queue:

$$L_q = \frac{\rho(\rho s)^s}{s!(1-\rho)}\rho_0 \tag{14}$$

The average waiting time of a vehicle in the queue:

$$W_q = \frac{(\rho s)^s}{s!(1-\rho)^2 s\mu}\rho_0 \tag{15}$$

The average waiting time a vehicle in the system:

$$W_s = \frac{(\rho s)^s}{s!(1-\rho)^2 s\mu}\rho_0 + \frac{1}{\mu} \tag{16}$$

(Soji, 2001)

The M/M/∞ queue is a multi-server queueing model where every arrival experiences immediate service and does not wait. In Kendall's notation it describes a system where arrivals are governed by a Poisson process, there are infinitely many servers, so jobs do not need to wait for a server. Each job has an exponentially distributed service time. It is a limit of the M/M/s queue model where the number of servers s becomes very large. The complexity of this model usually require a simulation (Harrison and Patel, 1992).

Using a Stop Watch, the primary data in Tables 1 and 2 were collected in October, 2018 by personal interviews and direct observation of vehicle arrivals grouped in one hour intervals from 8am to 6pm. Table 2 shows the duration of service for 10 vehicles based on the number of liters different vehicle owners purchased. Three Nozzles were working, representing three servers or chanels which are used for the formulation in this study.

Table 1: Table of Arrivals of Vehicles into the Queuing System

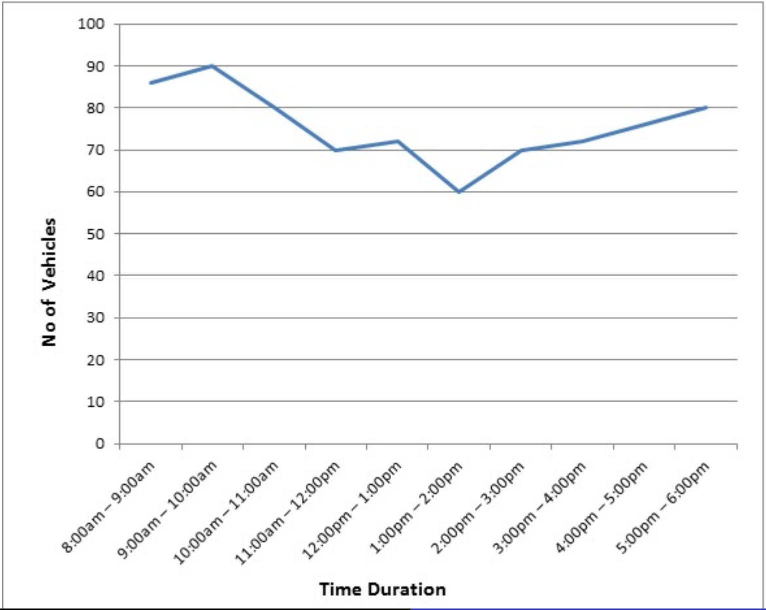
Time Duration	Frequency of Arrival (No of Vehicles)
8:00am - 9:00am	86
9:00am - 10:00am	90
10:00am - 11:00am	80
11:00am - 12:00pm	70
12:00pm - 1:00pm	72
1:00pm - 2:00pm	60
2:00pm - 3:00pm	70
3:00pm - 4:00pm	72
4:00pm - 5:00pm	76
5:00pm - 6:00pm	80

Table 2: Service of Vehicles on the Queue

Vehicles	No of Litres Served	Amount (N)	Duration of Service (N)
1^{st}	07.00	1,000	26
2^{nd}	63.07	9,020	189
3^{rd}	20.90	3,000	81
4^{th}	17.50	2,500	60
5^{th}	65.07	9,500	220
6^{th}	13.99	2,000	47
7^{th}	18.19	2,600	71
8^{th}	80.06	10,000	260
9^{th}	21.34	3,050	90
10^{th}	21.00	3,008	62
Total	328.12	45,679	1106

Results and Discussions

Figure 1: Line Graph of Vehicles Arrival Process



It can be obsrved from figure 1 that, more vehicles arrive NNPC Mega station Minna in the morning hours between 9:00 and 10:00 with average arrival rate of 90 vehicles per hour. The least arrival of 60 vehicles per hour can be seen to be between 1pm and 2pm

- **The average Arrival rate of vehicles per hour** (from table 1) is obtained to be:

$$\lambda = \frac{89+90+80+70+72+60+70+72+76+80}{10}$$

$$= \frac{756}{10} = 75.6 = 76 vehicles/hr. \quad (17)$$

- **The average Service rate of vehicles per hour** (from table 2) is obtained as follows:

If 7 liters of petrol are served at NNPC mega station Minna in 26 seconds, then, 1 liter will be served for, $1^{st} = \frac{26}{7} = 3.714$ Sec/liter. Similarly, the values for $2^{nd} - 10^{th}$ observations are obtained and summarized in table 3.

Table 3: Summary of the Duration of Service of Vehicles Per Liter

Vehicles	No of Litres Served	Duration of Service (secs)	Duration of Servi
1^{st}	07.00	26	3.714
2^{nd}	63.07	189	2.997
3^{rd}	20.90	81	3.876
4^{th}	17.50	60	3.429
5^{th}	65.07	220	3.810
6^{th}	13.99	47	3.360
7^{th}	18.19	71	3.903
8^{th}	80.06	260	3.248
9^{th}	21.34	90	4.217
10^{th}	21.00	62	2.952
Total	**328.12**	**1106**	**35.506**

It can be observed from table 3 that, on the average, it takes $\frac{35.506}{10} = 3.6$ seconds to serve a liter of petrol at NNPC mega station minna.

The average Service rate is:

$$\mu = \frac{1106}{10} = 110.6 \ Secs/vehicle = 33 \ vehicles/hr \qquad (18)$$

(If it takes 110.6 Seconds to serve 1 Vehicle, in 1 hour (3600 secs), approximately 33 vehicles/hour/server will be served)

The operating characteristics of the NNPC mega station Minna follows a M/M/s model which is a system of Single-Queue Multiple-Server system with (see section 2.2) 3 servers that were active at their full capacity at the time of this research.

(i) Traffic Intensity $(\rho) = \frac{\lambda}{s\mu}$. But $\lambda = 76, \mu = 33$ *and* $s = 3$. Thus,

$$(\rho) = \frac{76}{(3)(33)} = 0.77 \tag{19}$$

(ii) Probability of no vehicle in the system (ρ_0),

$$\rho_0 = 0.0671 \tag{20}$$

(iii) The average number of vehicles in the queue (L_q),

$$L_q = 2.0065 = 2 \ to \ 3 \ vehicles \tag{21}$$

(iv) The average number of vehicles in the system (L_s),

$$L_s = 4.3165 = 4 \ to \ 5 \ vehicles \tag{22}$$

(v) The Average Waiting Time of a vehicle in the Queue W_q,

$$W_q = 0.0263 \ hrs = 1.5793 \ minutes \tag{23}$$

(vi) The Averagge Waiting Time of a vehicle in the System, W_s,

$$W_s = 0.0566 \ hrs = 3.3962 \ minutes \tag{24}$$

(vii) The Probability of a vehicle Queuing on Arrival,

$$Prob = 0.5993 \tag{25}$$

(viii) The Probability of a vehicle not Queuing on Arrival in the System,

$$Prob = 1 - 0.5993 = 0.4007 \tag{26}$$

(ix) The Queuing Capacity of NNPC mega station Minna stands at:

$$\textbf{Capacity} = 3.396\ mins * 33\ cars\ per\ hour * 20\ provisionary\ channels$$

$$= 2,241.36 \tag{27}$$

Discussion of Results

With the probability of having no vehicle in the system being 0.1, it means that, there is 90% chance of always having a vehicle in the system. In other words, it is impossible not to see a vehicle in the station

The Average number of vehicles waiting in the queue is from 2 to 3, while the average number waiting in the system is from 4 to 5 vehicles. This means that, there is an average of 2 to 3 vehicles in service ($\rho s = 0.77 * 33 = 2.31$) since the number of vehicles in the system is made up of the number on the queue plus that in service.

The average waiting time of a vehicle on the queue before being served is 1.5793 minutes while the average waiting time in the system is 3.3962 minutes. This means that, on the average, a vehicle spends $W_s - L_s = 1.8169$ minutes in service at the Mega station

The probability of a vehicle queuing on arrival is 0.6 while the probability of not queuing on arrival is 0.4. This means that, there is a 60% chance of queuing on arrival and 40% chance of not queuing on arrival. In other words, a vehicle is more likely to queue on arrival than not to queue even though the waiting time is small.

Conclusion

The analysis of data collected in October 2018 shows that, 4 to 5 vehicles would averagely be seen spending 3 to 4 minutes each before leaving the system. With 1.8 minutes, a vehicle spends more time in service than on the queue. Since the number of vehicles in the system, 4 to 5, is $\geq$ the number of active servers, 3; it can be concluded that, there is negligible queue at the Mega station.
The study recommends a further study to investigate the inter-arrival times, consideration of more number of days for data collection on arrival rate, preferably, one week instead of just one day and more number of vehicles for data collection on service rate instead of just 10 vehicles.

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