

## Application of Differential Equation to Economics

Yusuf, A., Bolarin, G., and David, M., O.

Department of Mathematics, Federal University of Technology, PMB 65, Minna, 00176-0000  
Nigeria, Niger State, Nigeria

\*Corresponding Author: [yusuf.abdulhakeem@futminna.edu.ng](mailto:yusuf.abdulhakeem@futminna.edu.ng)

### Abstract

In recent years, many Business analyst and Economist discovered the abnormality in price of products or commodities, rather than a fixed price or a little increase, the price increases drastically affecting the demand or supply of goods with respect to time, which in turn inflates the price of such commodities, and affects the various policy alternatives. These problems were modeled into differential equations and the solutions were obtained. With a closer view at the Evans Price Adjustment Model, which is a determinant factor of Price as regards quantity of Demand  $D(t)$  and supply  $S(t)$  with respect to time  $t$ . Variation of time clearly shows that as time increases, price increases, supply increases and demand decreases.

**Keywords:** Revenue function, Marginal revenue, profile function, Evans Price model.

### 1.0 INTRODUCTION

The relationship between the financial sector, business and economic growth is an important issue that has been examined in a wide range of research papers, both theoretical and empirical. Many of them have focused on the impact of the financial sector through derivative on economic growth. Pioneering studies that highlight the role of the financial sector in the dynamism of the economy include Wicksell (1934), Goldsmith (1969), which found that the financial system serves as an engine driving the economic activity.

On the other hand, Levine (1991) points out that stock markets facilitate long-term investments, helping to reduce risk and simultaneously offering liquidity to savers and funding to companies. The author concludes that stock markets do contribute to economic growth. Moreover, Levine and Zervos (1998) highlight that a significant number of empirical studies support the existence of a relationship between capital markets and economic growth in the long term.

Derivatives markets have experienced robust growth in recent decades. In December 2008, the volume of derivatives worldwide was approximately USD 592 trillion, much higher than the gross domestic product (GDP) of the United States (the world's largest economy), which was just over 13.8 trillion in 2007. In 2003, 92% of the 500 largest firms in the world used derivatives to manage risk in several ways, especially interest rate risk, according to information provided by the BIS (Bank for International Settlements). The derivatives market is not only an enormous market, but also one that is growing dramatically. Derivative contracts increased more than sevenfold in the period 1998-2014. The role played by derivatives markets in boosting economic growth has been analyzed by authors such Sundaram (2013), Sipko (2011), Prabhaet *al.* (2014), among many others. Most have found a positive relationship between the development of the derivatives market and economic growth; however, a

worldwide analysis of such a relationship has yet to be carried out. This research paper examines the impact of derivatives markets on economic growth in four major world economies. Specifically, we assess the impact of variables such as the volume of the derivatives market in US dollars and the volume of the derivatives market as a proportion of GDP on economic growth over the period 2002-2014 and it is a new development in the literature

## 2.0 MATHEMATICAL FORMULATIONS

Consider the differential equation of the form

$$\begin{aligned} D(t) &= \alpha_0 + \alpha_1 p(t) + \alpha_2 p'(t), \alpha_0 > 0, \alpha_1 > 0, \alpha_2 > 0. \\ S(t) &= \beta_0 + \beta_1 p(t) + \beta_2 p'(t), \beta_0 > 0, \beta_1 > 0, \beta_2 < 0. \end{aligned} \tag{2.1}$$

If  $\alpha_2 = \beta_2 = 0$  we have the Evans price Adjustment Model in which  $\alpha_1 < 0$  since when price increases, demand decreases and  $\beta_1 > 0$  since when price increases, supply increases. In Allen's Model, co-efficients  $\alpha_2, \beta_2$  accounts for effect of speculation.

The Evans price adjustment model, assumes that the rate of change of price  $p$  with respect to time  $t$  is proportional to the shortage  $D - S$  so that

$$\frac{dp}{dt} = K(D - S) \tag{2.2}$$

$$D(t) = \alpha_0 + \alpha_1 p(t) \tag{2.3}$$

$$S(t) = \beta_0 + \beta_1 p(t) \tag{2.4}$$

From the above

$$D(t) = \alpha_0 + \alpha_1 p(t) + \alpha_2 p'(t) \tag{2.5}$$

$$S(t) = \beta_0 + \beta_1 p(t) + \beta_2 p'(t)$$

$$D(t) - S(t) = (\alpha_0 - \beta_0) + (\alpha_1 - \beta_1)p(t) + (\alpha_2 - \beta_2)p'(t) \tag{2.6}$$

At dynamic equilibrium  $\alpha_2 = 0 = \beta_2$

$$D(t) - S(t) = 0$$

$$\Rightarrow D(t) = S(t)$$

$$\alpha_0 + \alpha_1 p(t) = \beta_0 + \beta_1 p(t) \tag{2.7}$$

Let

$$D(t) = S(t) = p$$

$$\frac{dp}{dt} = 0$$

$$(a_0 - \beta_0) + (a_1 - \beta_1)p + (a_2 - \beta_2)\frac{dp}{dt} = 0$$

$$(a_2 - \beta_2)\frac{dp}{dt} + (a_1 - \beta_1)p = -(a_0 - \beta_0)$$

$$(a_2 - \beta_2)\frac{dp}{dt} + (a_1 - \beta_1)p = \beta_0 - a_0$$

Divide through by  $a_2 - \beta_2$  we have

$$\frac{dp}{dt} + \frac{(a_1 - \beta_1)}{(a_2 - \beta_2)}p = \frac{\beta_0 - a_0}{a_2 - \beta_2}$$

Let

$$\frac{a_1 - \beta_1}{a_2 - \beta_2} = \gamma$$

and

$$\frac{\beta_0 - a_0}{a_2 - \beta_2} = \xi$$

By substitution we have

$$\frac{dp}{dt} + \gamma p = \xi$$

By using application of linear differential equation of the form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

Using integrating factor

$$u(t) = e^{\int p dt} = e^{\gamma t}$$

The solution becomes

$$p(t) = e^{-\frac{\gamma}{2}t} \left( p_0 - \frac{\gamma}{2} \right) e^{-\frac{\gamma}{2}t}$$

1

2

3

4

5

6

7

8

9

10

11

12

13

14

15

16

17

18

19

20

21

22

23

24

25

$$t = \ln 6$$

Taking logarithms of both sides

$$0.01t = \ln 6$$

Taking  $\ln$  of both sides

$$e^{-0.01t} = 1.6$$

$$e^{-0.01t} = 1.6$$

$$\frac{e^{-0.01t}}{e^{-0.01t}} = \frac{0.2}{0.32}$$

Multiply both sides by  $\frac{1}{e^{-0.01t}}$

$$\frac{0.32e^{-0.01t}}{0.2e^{-0.01t}} = \frac{0.2}{0.2}$$

Divide both sides by 0.2

$$0.32e^{-0.01t} = 0.2e^{-0.01t}$$

$$P'(t) = -0.2e^{-0.01t} + 0.32e^{-0.01t} = 0$$

And then note that  $P'(t) = 0$  when

$$P'(t) = 20(-0.01e^{-0.01t}) - 16(-0.02e^{-0.02t}) = -0.2e^{-0.01t} + 0.32e^{-0.02t}$$

c) To maximize the price, we first compute the derivatives

That is, approximately 5 of runs per unit

$$P'(t) = 1 + 20e^{-0.01t} - 16e^{-0.02t} = 5.446$$

b) After 6 months ( $t = 6$ ) the price is

$$P(6) = 1 + 20e^{-0.06} - 16e^{-0.12}$$

And

$$\text{So that } t = 5 - 21 = -16$$

$$P'(t) = 1 + 20e^{-0.01t} - 16e^{-0.02t}$$

$t = 47$

(2.30)

The critical number corresponds to a maximum, the largest price occur after 47 months

Since

$$P(47) = 1 + 20e^{-0.01(47)} - 16e^{-0.02(47)} = 7.25$$

(2.31)

The largest unit price is approximately 7.25

Since

$$D(47) = 3 + 10e^{-0.01(47)} = 9.25$$

and

$$S(47) = 2 + P(47) = 2 + 7.25 = 9.25$$

(2.32)

It follows that approximately 9.250 units will be both demanded and supplied at the maximum price of 7.25 naira per unit

d) Since

$$\lim_{t \rightarrow \infty} P(t) = \lim_{t \rightarrow \infty} (1 + 20e^{-0.01t} - 16e^{-0.02t}) = 1 + 20(0) - 16(0) = 1$$

(2.33)

The Price tends towards 1 naira per unit in the "Long run".

Case 2:- Supply  $S(t)$  and demand  $D(t)$  functions for a commodity are given in terms of the unit price  $P(t)$  at time  $t$ . Assume that price changes at a rate proportional to the shortage  $D(t) - S(t)$  with

indicated constant of proportionality  $K$  and initial price  $P_0$  in each problem:

a. Set up and solve a differential equation for  $P(t)$

b. Find the unit price of the commodity when  $t = 4$

c. Determine what happens to the price at  $t \rightarrow \infty$ .

$$S(t) = 2 + 3P; D(t) = 10 - P(t); K = 0.02; P_0 = 1$$

From equation (3.1)

$$\frac{dP}{dt} = K(D - S)$$

(2.34)

$$\frac{dP}{dt} = K(D(t) - S(t)) = K(10 - P - (2 + 3P)) = K(10 - 2 - P - 3P)$$

(2.35)

$$\frac{dP}{dt} = K(8 - 4P)$$

Or equivalently,

$$S(t) = 1 + 4P(t), D(t) = 15 - 3P(t), k = 0.015, p = 5$$

Assume

Case 3:

The Price tends towards 2 rupee per unit in the "Long run"

$$\lim_{t \rightarrow \infty} P(t) = \lim_{t \rightarrow \infty} (2 - e^{-0.015t}) = 2 - 0 = 2 \tag{2.42}$$

c) Price  $P(t)$  as  $t \rightarrow \infty$  will become

That is, approximately 1.27 rupee per unit

$$P(4) = 2 - e^{-0.06} = 2 - e^{-0.06} \approx 1.2739 \tag{2.41}$$

b) After 4 months ( $t = 4$ ) the price is

$$P(t) = 2 - e^{-0.06} \tag{2.40}$$

And

$$\text{So that } c = 1 - 2 = -1$$

$$\text{Since the initial price is } P(0) = 1; \Rightarrow P(t) = 5 = 2 + e^{-0.06} = 3 - e^{-0.06} = 2 + e^{-0.06} \tag{2.39}$$

$$P(t) = \frac{1}{1} \int e^{0.015t} (0.16)e^{-0.015t} + c = e^{-0.015t} \left( \frac{0.16}{0.015} + c \right) = e^{-0.015t} (2e^{0.015t} + c) = 2 + e^{-0.015t} \tag{2.38}$$

So the general solution is

$$\text{b) The integrating factor is } f(t) = e^{\int 0.015 dt} = e^{0.015t} \tag{2.37}$$

a) Implying it is a first order linear differential equation with  $P'(t) = 0.015P(t) - 0.16$

$$\frac{dP}{dt} + 0.015P = 0.16 \tag{2.36}$$

$$\frac{dP}{dt} - 0.015P = -0.16 \tag{2.35}$$

(2.43)

$$\frac{dp}{dt} = K(D - S) = K(D(t) - S(t)) = K((15 - 3p) - (1 - 4p)) = K(15 - 3p - 1 + 4p) = K(14 - 7p)$$

Or equivalently,

$$\Rightarrow \frac{dp}{dt} = 0.015(14 - 7p) = 0.21 - 0.105p$$

$$\frac{dp}{dt} + 0.105p = 0.21$$

(2.44)

Implying it's a first order linear differential equation with  $P(t) = 0.105$  and  $Q(t) = 0.21$ . The integrating factor is  $I(t) = e^{\int 0.105 dt} = e^{0.105t}$

So the general solution is

$$P(t) = \frac{1}{e^{0.105t}} \left( \int e^{0.105t} (0.21) dt + c \right) = e^{-0.105t} \left( \frac{0.21 e^{0.105t}}{0.105} + c \right) = e^{-0.105t} (2e^{0.105t} + c) \quad (2.45)$$

Since the initial price is  $P(0) = 1$ ;

$$\Rightarrow P(0) = 1 = 2 + ce^{-0.105(0)} = 2 + ce^0 = 2 + c$$

(2.46)

So that  $c = 3 - 2 = 1$

And

$$P(t) = 2 + e^{-0.105t}$$

(2.47)

b) After 4 months ( $t = 4$ ) the price is

$$P(4) = 2 + e^{-0.105(4)} = 2 + e^{-0.42} \approx 2.6570$$

That is, approximately 2.6570 rupees per unit

c) Price  $P(t)$  as  $t \rightarrow \infty$  will become

$$\lim_{t \rightarrow \infty} P(t) = \lim_{t \rightarrow \infty} (2 + e^{-0.105t}) = 2 + 0 = 2$$



100

$$y'' + y' + y = 0$$

$$y_1 = e^{-x/2} \cos(\frac{\sqrt{3}}{2}x), y_2 = e^{-x/2} \sin(\frac{\sqrt{3}}{2}x)$$

General solution is  $y = c_1 y_1 + c_2 y_2$

$$y(0) = 1 \implies c_1 = 1$$

$$y'(0) = 0 \implies c_2 = 0$$

$$y = e^{-x/2} \cos(\frac{\sqrt{3}}{2}x)$$

is the general solution

(30)

The integrating factor is  $e^{\int p(x) dx}$

Implying it is a first order linear differential equation with  $P(x) = -2x$  and  $Q(x) = 1 - 4x^2$

(31)

$$\frac{dy}{dx} + 2xy = 1 - 4x^2$$

$$\frac{dy}{dx} + 2xy = 1 - 4x^2$$

Or equivalently

$$\frac{dy}{dx} + 2xy = 1 - 4x^2$$

$$\frac{d}{dx}(y e^{x^2}) = (1 - 4x^2) e^{x^2}$$

(32)

$$\frac{dy}{dx} + 2xy = 1 - 4x^2$$

From equation (31)

$$y e^{x^2} = \int (1 - 4x^2) e^{x^2} dx + C$$

where

$C$  is a constant

The first term on the right is the integral of  $e^{x^2}$

... ..

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100																																																																																																																																																																																																																																																
12.4000	12.4178	12.4357	12.4537	12.4718	12.4900	12.5083	12.5267	12.5452	12.5638	12.5825	12.6013	12.6202	12.6392	12.6583	12.6775	12.6968	12.7162	12.7357	12.7553	12.7750	12.7948	12.8147	12.8347	12.8548	12.8750	12.8953	12.9157	12.9362	12.9568	12.9775	12.9983	13.0192	13.0402	13.0613	13.0825	13.1038	13.1252	13.1467	13.1683	13.1900	13.2118	13.2337	13.2557	13.2778	13.2999	13.3221	13.3444	13.3668	13.3893	13.4119	13.4346	13.4574	13.4803	13.5033	13.5264	13.5496	13.5729	13.5963	13.6198	13.6434	13.6671	13.6909	13.7148	13.7388	13.7629	13.7871	13.8114	13.8358	13.8603	13.8849	13.9096	13.9344	13.9593	13.9843	14.0094	14.0346	14.0599	14.0853	14.1108	14.1364	14.1621	14.1879	14.2138	14.2398	14.2659	14.2921	14.3184	14.3448	14.3713	14.3979	14.4246	14.4514	14.4783	14.5053	14.5324	14.5596	14.5869	14.6143	14.6418	14.6694	14.6971	14.7249	14.7528	14.7808	14.8089	14.8371	14.8654	14.8938	14.9223	14.9509	14.9796	15.0084	15.0373	15.0663	15.0954	15.1246	15.1539	15.1833	15.2128	15.2424	15.2721	15.3019	15.3318	15.3618	15.3919	15.4221	15.4524	15.4828	15.5133	15.5439	15.5746	15.6054	15.6363	15.6673	15.6984	15.7296	15.7609	15.7923	15.8238	15.8554	15.8871	15.9189	15.9508	15.9828	16.0149	16.0471	16.0794	16.1118	16.1443	16.1769	16.2096	16.2424	16.2753	16.3083	16.3414	16.3746	16.4079	16.4413	16.4748	16.5084	16.5421	16.5759	16.6098	16.6438	16.6779	16.7121	16.7464	16.7808	16.8153	16.8499	16.8846	16.9194	16.9543	16.9893	17.0244	17.0596	17.0949	17.1303	17.1658	17.2014	17.2371	17.2729	17.3088	17.3448	17.3809	17.4171	17.4534	17.4898	17.5263	17.5629	17.5996	17.6364	17.6733	17.7103	17.7474	17.7846	17.8219	17.8593	17.8968	17.9344	17.9721	18.0099	18.0478	18.0858	18.1239	18.1621	18.1994	18.2368	18.2743	18.3119	18.3496	18.3874	18.4253	18.4633	18.5014	18.5396	18.5779	18.6163	18.6548	18.6934	18.7321	18.7709	18.8098	18.8488	18.8879	18.9271	18.9664	19.0058	19.0453	19.0849	19.1246	19.1644	19.2043	19.2443	19.2844	19.3246	19.3649	19.4053	19.4458	19.4864	19.5271	19.5679	19.6088	19.6498	19.6909	19.7321	19.7734	19.8148	19.8563	19.8979	19.9396	19.9814	20.0233	20.0653	20.1074	20.1496	20.1919	20.2343	20.2768	20.3194	20.3621	20.4049	20.4478	20.4908	20.5339	20.5771	20.6204	20.6638	20.7073	20.7509	20.7946	20.8384	20.8823	20.9263	20.9704	21.0146	21.0589	21.1033	21.1478	21.1924	21.2371	21.2819	21.3268	21.3718	21.4169	21.4621	21.5074	21.5528	21.5983	21.6439	21.6896	21.7354	21.7813	21.8273	21.8734	21.9196	21.9659	22.0123	22.0588	22.1054	22.1521	22.1989	22.2458	22.2928	22.3399	22.3871	22.4344	22.4818	22.5293	22.5769	22.6246	22.6724	22.7203	22.7683	22.8164	22.8646	22.9129	22.9613	23.0098	23.0584	23.1071	23.1559	23.2048	23.2538	23.3029	23.3521	23.4014	23.4508	23.4993	23.5489	23.5986	23.6484	23.6983	23.7483	23.7984	23.8486	23.8989	23.9493	24.0000

Table 3.1: Values of Price and Quantity Functions (D(t) and S(t)) with respect to time t

CASE 1

With a given price in the Future Price Adjustment Model in chapter three, it's a downward slope of Price in quantity of Demand (D) and supply (S) with respect to time. variation of time (t) shows that as time increases, price increases, supply increases and demand decreases. This is the main reason we have to use MATLAB.

3.1. RESULTS AND DISCUSSION

The Price and Quantity functions are given by the following:

$$D(t) = 20.0 - 0.0001t$$

$$S(t) = 10.0 + 0.0001t$$

There is equilibrium when the quantity demanded is equal to the quantity supplied.

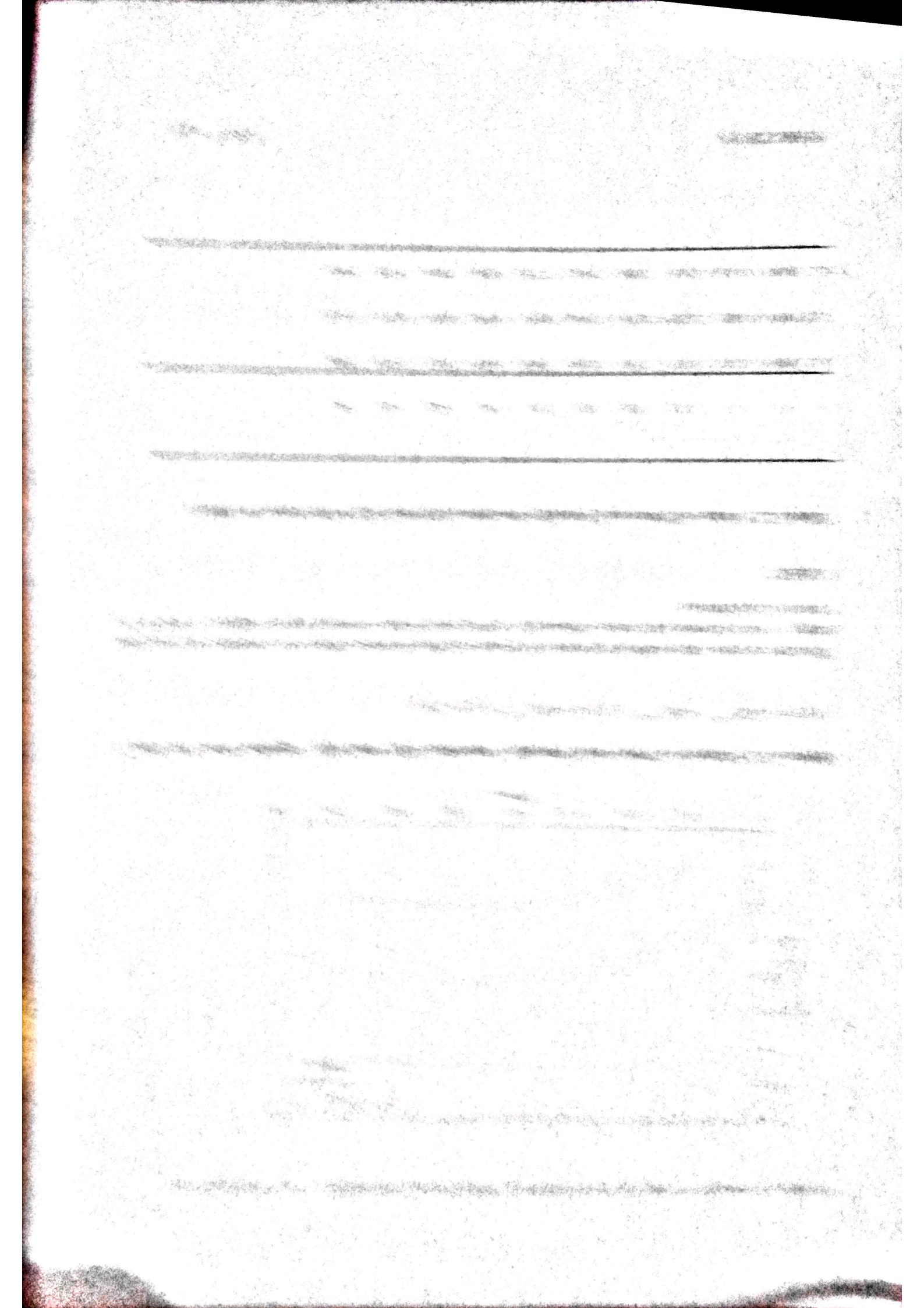
$$20.0 - 0.0001t = 10.0 + 0.0001t$$

$$10.0 = 0.0002t$$

$$t = \frac{10.0}{0.0002} = 50000$$

Q

$$P(t) = 20.0 - 0.0001t$$



\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

一、  
 二、  
 三、  
 四、  
 五、  
 六、  
 七、  
 八、  
 九、  
 十、

...

...

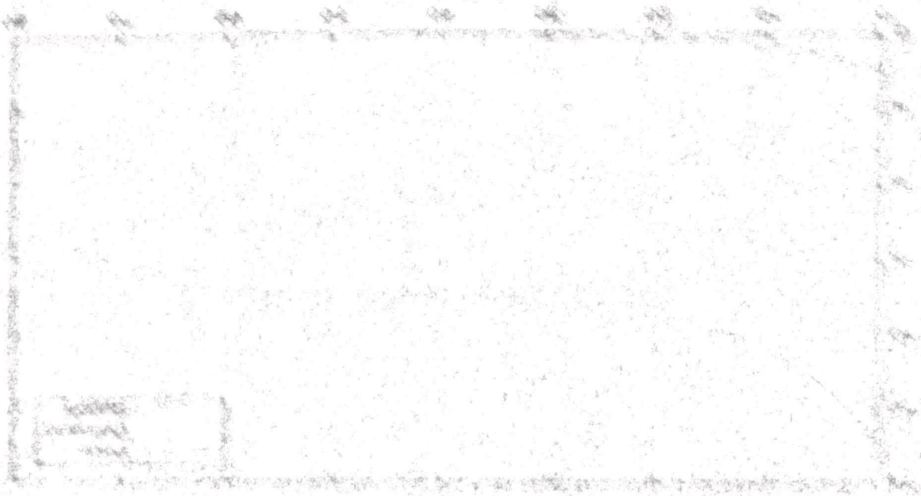
...

...

...

...

...



...

...

[The page contains several paragraphs of extremely faint, illegible text, likely bleed-through from the reverse side of the document. The text is organized into approximately five distinct sections, each separated by a blank line or a small gap. The characters are too light to be accurately transcribed.]