Application of Differential Equation to Economics

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Abstract

In recent years, many Business analyst and Economist discovered the abnormality in price of products or commodities, rather than a fixed price or a little increase, the price increases drastically affecting the demand or supply of goods with respect to time, which in turn inflates the price of such commodities, and affects the various policy alternatives. These problems were modeled into differential equations and the solutions were obtained. With a closer view at the Evans Price Adjustment Model, which is a determinant factor of Price as regards quantity of Demand $D(t)$ and supply $S(t)$ with respect to time t. Variation of time clearly shows that as time increases, price increases, supply increases and demand decreases.

Keywords: Revenue function, Marginal revenue, profile function, Evans Price model.

1.0 INTRODUCTION

The relationship between the financial sector, business and economic growth is an important issue that has been examined in a wide range of research papers, both theoretical and empirical. Many of them have focused on the impact of the financial sector through derivative on economic growth. Pioneering studies that highlight the role of the financial sector in the dynamism of the economy include Wicksell (1934), Goldsmith (1969), which found that the financial system serves as an engine driving the economic activity.

On the other hand, Levine (1991) points out that stock markets facilitate long-term investments, helping to reduce risk and simultaneously offering liquidity to savers and funding to companies. The author concludes that stock markets do contribute to economic growth. Moreover, Levine and Zervos (1998) highlight that a significant number of empirical studies support the existence of a relationship between capital markets and economic growth in the long term.

between capital markets and economic growth in recent decades. In December 2008, the volume Derivatives markets have experienced robust growth in recent decades. In December 2008, the volume of derivatives worldwide was approximately USD 592 trillion, much higher than the gross domestic product (GDP) of the United States (the world's largest economy), which was just over 13.8 trillion in 2007. In 2003, 92% of the 500 largest firms in the world used derivatives to manage risk in several ways, especially interest rate risk, according to information provided by the BIS (Bank for International Settlements). The derivatives market is not only an enormous market, but also one that is growing dramatically. Derivative contracts increased more than sevenfold in the period 1998-2014. The role played by derivatives markets in boosting economic growth has been analyzed by authors such Sundaram (2013), Sipko (2011), Prabha*et al.* (2014), among many others. Most have found a positive relationship between the development of the derivatives market and economic growth; however, a worldwide analysis of such a relationship has yet to be carried out. This research paper examines the impact of derivatives markets on economic growth in four major world economies. Specifically, we assess the impact of variables such as the volume of the derivatives market in US dollars and the volume of the derivatives market as a proportion of GDP on economic growth over the period 2002-2014 and it is a new development in the literature

2.0 MATHEMATICAL FORMULATIONS

Consider the differential equation of the form

$$\begin{aligned} D(t) &= \alpha_0 + \alpha_1 p(t) + \alpha_2 p'(t), \alpha_0 > 0, \alpha_1 > 0, \alpha_2 > 0. \\ S(t) &= \beta_0 + \beta_1 p(t) + \beta_2 p'(t), \beta_0 > 0, \beta_1 > 0, \beta_2 < 0. \end{aligned} \tag{2.1}$$

If $\alpha_2 = \beta_2 = 0$ we have the Evans price Adjustment Model in which $\alpha_1 < 0$ since when price increases, demand decreases and $\beta_1 > 0$ since when price increases, supply increases. In Allen's Model, co-efficients $\alpha_{2.\beta_2}$ accounts for effect of speculation.

The Evans price adjustment model, assumes that the rate of change of price p with respect to time t is proportional to the shortage $D-S$ so that

$$\frac{dp}{dt} = K(D-S) \tag{2.2}$$

$$D(t) = \alpha_0 + \alpha_1 p(t) \tag{2.3}$$

$$S(t) = \beta_0 + \beta_1 p(t) \tag{2.4}$$

From the above

$$\begin{aligned} D(t) &= \alpha_0 + \alpha_1 p(t) + \alpha_2 p'(t) \\ S(t) &= \beta_0 + \beta_1 p(t) + \beta_2 p'(t) \\ D(t) - S(t) &= (\alpha_0 - \beta_0) + (\alpha_1 - \beta_1) p(t) + (\alpha_2 - \beta_2) p'(t) \end{aligned} \tag{2.5}$$

$$\tag{2.6}$$

At dynamic equilibrium $\alpha_2 = 0 = \beta_2$

$D(t) - S(t) = 0$

$\Rightarrow D(t) = S(t)$

$$\alpha_0 + \alpha_1 p(t) = \beta_0 + \beta_1 p(t) \tag{2.7}$$

Let

$$D(t) = S(t) = p \tag{2.8}$$

$$\frac{dp}{dt} = 0$$

$$(\alpha_0 - \beta_0) + (\alpha_1 - \beta_1)p + (\alpha_2 - \beta_2)\frac{dp}{dt} = 0 \tag{2.9}$$

$$(\alpha_2 - \beta_2)\frac{dp}{dt} + (\alpha_1 - \beta_1)p = -(\alpha_0 - \beta_0) \tag{2.10}$$

$$(\alpha_2 - \beta_2)\frac{dp}{dt} + (\alpha_1 - \beta_1)p = \beta_0 - \alpha_0$$

Divide through by $\alpha_2 - \beta_2$ we have

$$\frac{dp}{dt} + \frac{(\alpha_1 - \beta_1)}{\alpha_2 - \beta_2}p = \frac{\beta_0 - \alpha_0}{\alpha_2 - \beta_2} \tag{2.11}$$

Let

$$\frac{\alpha_1 - \beta_1}{\alpha_2 - \beta_2} = \gamma \tag{2.12}$$

and

$$\frac{\beta_0 - \alpha_0}{\alpha_2 - \beta_2} = \xi$$

By substitution we have

$$\frac{dp}{dt} + \gamma p = \xi \tag{2.13}$$

By using application of linear differential equation of the form

$$\frac{dy}{dx} + p(x)y = Q(x) \tag{2.14}$$

Using integrating factor

$$\upsilon(t) = e^{\int \gamma dt} = e^{\gamma t} \tag{2.15}$$

The solution becomes

$$p(t) = \frac{\xi}{\gamma} + \left(p_0 - \frac{\xi}{\gamma}\right)e^{-\gamma t} \tag{2.16}$$

[illegible]

[illegible]

[illegible]

Where [illegible]

Case [illegible]

[illegible]

[illegible]

[illegible]

which we recognize as a [illegible] differential equation, with [illegible]

[illegible]

The integrating factor is

[illegible]

So the general solution is

[illegible]

[illegible]

Since the initial [illegible]

[illegible]

$P(0) = 5 = 1 + 20e^{-0.01(0)} + ce^{0} = 1 + 20 + c$ (2.23)

So that $c = 5 - 21 = -16$

And

$P(t) = 1 + 20e^{-0.01t} - 16e^{-0.02t}$ (2.24)

b) After 6 months $(t = 6)$ the price is

$$P(6) = 1 + 20e^{-0.01(6)} - 16e^{-0.02(6)} = 5.6446 \quad (2.25)$$

That is, approximately 5.64 naira per unit

c) To maximize the price, we first compute the derivatives

$$P'(t) = 20\left(-0.01e^{-0.01t}\right) - 16\left(-0.02e^{-0.02t}\right) = -0.2e^{-0.01t} + 0.32e^{-0.02t} \quad (2.26)$$

And then note that $P'(t) = 0$ when

$P'(t) = -0.2e^{-0.01t} + 0.32e^{-0.02t} = 0$ (2.27)

$0.32e^{-0.02t} = 0.2e^{-0.01t}$

Divide both sides by 0.2

$$\frac{0.32e^{-0.02t}}{0.2} = \frac{0.2e^{-0.01t}}{0.2} \quad (2.28)$$

Multiply both sides by $\frac{1}{e^{-0.02t}}$

$$\frac{e^{-0.01t}}{e^{-0.02t}} = \frac{0.32}{0.2}$$

$e^{-0.01t-(-0.02t)} = 1.6$ (2.29)

$e^{-0.01t+0.02t} = 1.6$

Taking $\ln$ of both sides

$0.01t = \ln 1.6$

Taking logarithms of both sides

$t = \ln 1.6$

$$t = 47 \qquad (2.30)$$

The critical number corresponds to a maximum, the largest price occur after 47 months

Since

$$P(47) = 1 + 20e^{-0.01(47)} - 16e^{-0.02(47)} = 7.25 \qquad (2.31)$$

The largest unit price is approximately 7.25

Since

$$\begin{aligned} &D(47) = 3 + 10e^{-0.01(47)} = 9.25 \\ &\text{and} \\ &S(47) = 2 + P(47) = 2 + 7.25 = 9.25 \end{aligned} \qquad (2.32)$$

It follows that approximately 9.250 units will be both demanded and supplied at the maximum price of 7.25 naira per unit

d) Since

$$\lim_{t\to\infty} P(t) = \lim\left(1 + 20e^{-0.01t} - 16e^{-0.02t}\right) = 1 + 20(0) - 16(0) = 1 \qquad (2.33)$$

The Price tends towards 1 naira per unit in the "Long run".

Case 2:- $Supply S(t)$ and $demand D(t)$ functions for a commodity are given in terms of the unit price $P(t)$ at time t. Assume that price changes at a rate proportional to the shortage $D(t) - S(t)$ with indicated constant of proportionality K and initial price P_0 in each problem:

a. Set up and solve a differential equation for $P(t)$
b. Find the unit price of the commodity when $t = 4$
c. Determine what happens to the price at $t \to \infty$.

$$S(t) = 2 + 3p; D(t) = 10 - p(t); k = 0.02; p_0 = 1$$

From equation (3.1)

$$\frac{dp}{dt} = K(D - S) \qquad (2.34)$$

$$\frac{dp}{dt} = k\left(D(t) - S(t)\right) = k\left((10 - p) - (2 + 3p)\right) = k\left(D(t) - S(t)\right) = k\left(10 - 2 - (p + 3p)\right) \qquad (2.35)$$

$$\frac{dp}{dt} = k(8 - 4p)$$

Or equivalently,

$$\frac{dp}{dt}=0.02(8-4p)=0.16-0.08p$$

$$\frac{dp}{dt}+0.08p=0.16 \tag{2.36}$$

a) Implying it's a first order linear differential equation with $P(t)=0.08$ and $Q(t)=0.16$

b) The integrating factor is $I(t)=e^{\int 0.08dt}=e^{0.08t}$ (2.37)

So the general solution is

$$P(t)=\frac{1}{e^{0.08t}}\left(\int e^{0.08t}(0.16)dt+c\right)=e^{-0.08t}\left(\frac{0.16e^{0.08t}}{0.08}+c\right)=e^{-0.08t}\left(2e^{0.08t}+c\right)=2+ce^{-0.08t} \tag{2.38}$$

Since the initial price is $P(0)=1$; $\Rightarrow P(0)=5=2+ce^{-0.08t}=2+ce^{-0.08(0)}=2+c$ (2.39)

So that $c=1-2=-1$

And

$$P(t)=2-e^{-0.08t} \tag{2.40}$$

b) After 4 months $(t=4)$ the price is

$$P(4)=2-e^{-0.08(4)}=2-e^{-0.32}\square 1.2739 \tag{2.41}$$

That is, approximately 1.27 naira per unit

c) Price $P(t)$ as $t\rightarrow\infty$ will become

$$\lim_{t\rightarrow\infty}P(t)=\lim\left(2-e^{-0.08t}\right)=2-0=2 \tag{2.42}$$

The Price tends towards 2 naira per unit in the "Long run".

Case 3:

Assume

$S(t)=1+4p(t);D(t)=15-3p(t);k=0.015;p_0=3$

$$\frac{dp}{dt}=K(D-S)$$

$$\frac{dp}{dt}=k(D(t)-S(t))=k((15-3p)-(1+4p))=k(15-3p-1-4p)=k(15-1-(3p+4p)) \qquad (2.43)$$

$$\frac{dp}{dt}=k(14-7p)$$

Or equivalently,

$$\Rightarrow \frac{dp}{dt}=0.015(14-7p)=0.21-0.105p$$

$$\frac{dp}{dt}+0.105p=0.21 \qquad (2.44)$$

Implying it's a first order linear differential equation with $P(t)=0.105 and Q(t)=0.21$ The integrating factor is $I(t)=e^{\int 0.105 dt}=e^{0.105t}$

So the general solution is

$$P(t)=\frac{1}{e^{0.105t}}\left(\int e^{0.105t}(0.21)dt+c\right)=e^{-0.105t}\left(\frac{0.21e^{0.105t}}{0.105}+c\right)=e^{-0.105t}\left(2e^{0.105t}+c\right)$$
$$=2+ce^{-0.105t} \qquad (2.45)$$

Since the initial price is $P(0)=1$;

$$\Rightarrow P(0)=3=2+ce^{-0.105t}=2+e^{-0.105(0)}=2+ce^{0}=2+c \qquad (2.46)$$

So that $c=3-2=1$

And

$$P(t)=2+e^{-0.105t} \qquad (2.47)$$

b) After 4 months $(t=4)$ the price is

$$P(4)=2+e^{-0.105(4)}=2+e^{-0.042} \square\ 2.6570$$

That is, approximately 2.6570 naira per unit

c) Price $P(t)$ as $t\to\infty$ will become

$$\lim_{t\to\infty} P(t)=\lim\left(2+e^{-0.105t}\right)=2+0=2$$

The Price tends towards 2 cents per unit in the "Long run"

Case 4

Assume

- [illegible]

From equation (2.3)

$$\frac{dp}{dt} = K(S - D)$$ (2.48)

$$\frac{dp}{dt} = [illegible]$$

$$\frac{dp}{dt} = [illegible]$$

Or equivalently,

$$\Rightarrow \frac{dp}{dt} = [illegible]$$

$$\frac{dp}{dt} + 0.02p = [illegible]$$ (2.49)

Implying it's a first order linear differential equation with [illegible]

The integrating factor is $I(t) = e^{\int 0.02dt} = e^{0.02t}$ (2.50)

So the general solution is

$$P(t) = [illegible]$$

$$= [illegible]$$

$$P(t) = [illegible]$$

- Since the initial price is $P(0) = 4$,

$$\Rightarrow P(0) = 4$$

$$= [illegible]$$

So that $c = 4 - 1 + 0.14 = 3.14$

And

$P(t) = 1 - 0.14e^{[illegible]} + 3.14e^{[illegible]}$,

Or

$$P(t) = 1 - \frac{1}{7}e^{[illegible]} + \frac{22}{7}e^{[illegible]} \qquad (2.51)$$

b) After 8 months $(t = 4)$ the price is

$$P(4) = 1 - 0.14e^{[illegible]} + 3.14e^{[illegible]} = 1 - 0.14e^{[illegible]} + 3.14e^{[illegible]} \approx 3.7695 \qquad (2.52)$$

This is, approximately 3.7695 dollars per unit

c) Price $P(t)$ as $t \to \infty$ will become

$$\lim_{t \to \infty} P(t) = \lim_{t \to \infty}\left(1 - 0.14e^{[illegible]} + 3.14e^{[illegible]}\right) = 1 - 0 - 0 = 1 \qquad (2.53)$$

∴ The Price tends towards 1 naira per unit in the "Long run"

3.0 RESULTS AND DISCUSSION

With a closer view at the Evans Price Adjustment Model in chapter three, it's a determinant factor of Price as regards quantity of Demand $D(t)$ and supply $S(t)$ with respect to time t variation of time clearly shows that as time increases, price increases, supply increases and demand decreases. This is the more reason we have to use MATLAB

CASE 1

Table 3.1: Values of Price and Quantity functions (D(t) and S(t)) with respect to time t

t	0	6	12	18	24	30	36	42	48	54
p(t)	5.0000	5.6446	6.1524	6.5426	6.8320	7.0354	7.1655	7.2336	7.2494	7.2214
D(t)	13.2000	12.4176	11.8692	11.5527	10.8663	10.4082	9.9768	9.5705	9.1878	8.8275
S(t)	7.2000	7.6446	8.1524	8.5426	8.8320	9.0354	9.1655	9.2336	9.2494	9.2214

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

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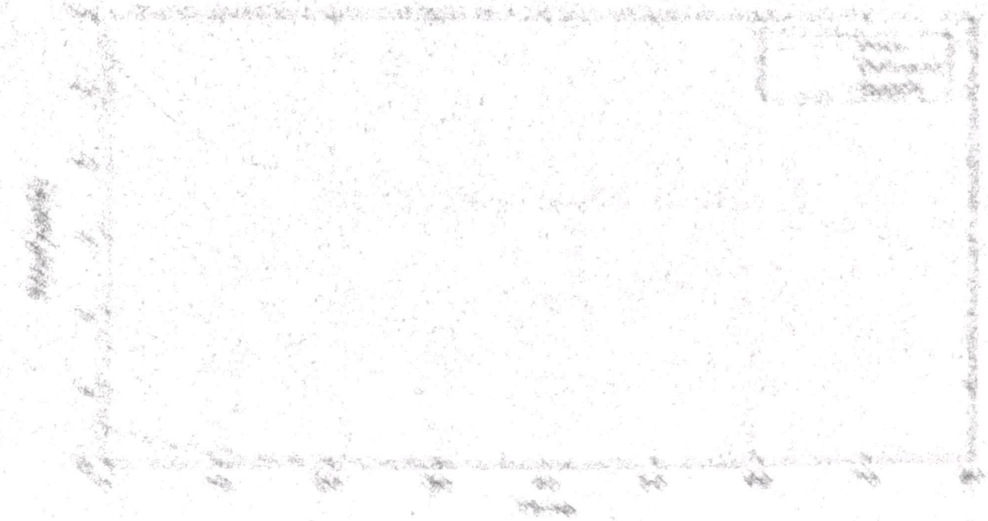

Figure 3.5 Graph of Price and Quantity functions (d(t) and s(t)) against time for case 3

[illegible]

The above shows that within the period of 8 years as time increases there is a decrease in price, and supply decreases drastically, while demand increases significantly, at the point where demand equals supply it tend to be point of Equilibrium.

CASE 4

Table 3.4: Values of Price and Quantity functions (d(t) and s(t)) with respect to time t

t	0	6	12	18	24	30	36	42	48	54
p(t)	4.0000	3.6478	3.2906	3.0834	2.8764	2.6964	2.4803	2.2932	2.1407	2.0058
d(t)	[illegible]	[illegible]	2.9966	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000
s(t)	[illegible]	5.4608	5.3304	5.0832	4.8764	4.6964	4.4803	4.2932	4.1407	4.0058