4.3 Game-theoretic solutions for resource allocation and tracking

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The application of game theory could provide revolutionary solutions to the military tasks of sensor and resource management. In abstract, games can be thought of as a series of strategic decisions, in which a player's action at any point is determined in the context of alternatives available to other players. They can be cooperative or adversarial. Crucially, players are often required to make decisions in the absence of communication with, or knowledge of, other player's strategies. This has obvious parallels with scenarios where own forces must maximise an outcome but are unable to communicate (e.g. cooperative identification of a target), or where the unpredictable actions of an adversary cannot be well-modelled (e.g. mitigation of RF spectrum denial techniques).

Game theory provides a formal mathematical framework for analysing conflict and cooperation between intelligent rational decision makers. An important concept in game theory is Nash equilibrium [30], [31], a balanced state in a game where no player has any incentive to deviate from their chosen strategy after considering all of their opponent's potential strategies. This means that at the Nash equilibrium, no player can benefit by unilaterally deviating from their strategy. Practically, decisions can be made which, while not optimal, will have a predictable impact.

Table 4.1: Simple cooperative sensor game payoffs. Rows give actions available to sensor 1 and columns actions for sensor 2. Each pair of numbers give the rewards for (sensor 1, sensor 2).

		Sensor 2 observes	
		Target 1	Target 2
Sensor 1 observes	Target 1	(10,10)	(1,4)
	Target 2	(4,1)	(5,5)

A pair of toy examples which demonstrate, in small part, this power and nuance are provided by the following simple games. They involve two illuminating sensors, who cannot communicate with one another, tasked with detecting two targets in a scene. One target is high value, the other less so. In the first instance it is presumed that if both sensors go after the same target then their cooperative illumination will increase the chances of detection, since bi-static or MIMO techniques can be used (see e.g. chapter 3). If only one sensor points at a target, the chances of detection for that target are reduced. This game is summarised in table 4.1 using arbitrary payoffs[1]. Target 1 is higher value than target 2. In this instance there are a pair of Nash equilibria, where both sensors point at the same target. Although observing target 2 is sub-optimal, no benefit would be derived by either sensor unilaterally altering its strategy. It's worth noting that the 'socially optimal' outcome isn't always an equilibrium state. This can be seen by way of a second example where sensors interfere with each other, reducing the benefit of observing the same target. This is enumerated in table 4.2 simply by reducing the joint reward for observing the same target[2]

It is evident from table 4.2 that the socially optimal out-

[1] This is a variant of the canonical coordination game, *stag hunt*.

[2] Based on the *prisoner's dilemma*

Table 4.2: Simple interfering sensor game payoffs. Rows give actions available to sensor 1 and columns actions for sensor 2. Each pair of numbers give the rewards for (sensor 1, sensor 2).

		Sensor 2 observes	
		Target 1	Target 2
Sensor 1 observes	Target 1	(3,3)	(1,4)
	Target 2	(4,1)	(2,2)

come is for both sensors to observe target 1. This is not an equilibrium state, however. Only where both sensors observe target 2 will no player derive benefit by switching to observe target 1. This is the only Nash equilibrium in this game. Therefore, regardless of the other player's action, a player in this game should observe target 2 even though mutual cooperation would provide a better utility for both players. This is the best outcome for each player given that they do not know what the other will do. The power of game theory is that it provides principled methods to arrive at such equilibria and so derive beneficial strategies in the absence of communication.

4.3.1 Exploiting game theory for defence

Game theoretic ideas have applications in radar, where waveforms can be chosen for a particular purpose (e.g. to maximise the detection of a target). These choices must often be made in the absence of communication with allies or in the presence of adversaries. This concept has its most potent example in radar jamming, an adversarial game where players seek to minimise their detectability or maximise their chances of detecting an adversary. Wider applicability is possible in multi-function radar, adaptive beamforming, passive bi-static or multi-static design under uncertainty, imperfect sensor measurements and radar clutter. These applications are all relevant to the DE&S Future Combat Air System (FCAS) programme where decisions on individual sensing options will need to be made autonomously and in the presence of adversaries.

Game theory can also be used for resource allocation and detection in sensor networks. Here, tactics to dynamically optimise detection performance in a network where nodes (adversarial or coalition) are unaware of each other's strategies, but react to each other's actions, must be derived.

UDRC researchers have used game theory for analysing interaction of sensors in a network and to develop distributed resource allocation techniques. As in the toy examples, the socially optimal outcome can be obtained if there is cooperation between sensors, and in these cases, a centralised resource allocation based on *convex optimisation*[3] will provide this solution. However, a centralised approach to resource allocation may not be desirable or feasible if there is no communication between sensors or if the communication links are intermittent or insecure. The UDRC work therefore focussed on autonomous decentralised resource allocation schemes and used game theory as the means to address these problems. As has been seen, the game-theoretic method may not necessarily provide the globally optimal solution. It is designed, however, to provide a robust solution. In addition to resource allocation techniques, the UDRC team also developed game-theoretic methods for sensor detection-to-track association for multi-target tracking. This problem is a combinatorial optimisation problem (c.f. §4.2). Game theory was shown, by UDRC researchers, to provide an efficient method to solve this problem and to outperform many other methods in terms of computational complexity [32], [33].

[3]Optimisation of a so-called convex function, where there is a single maximum or minimum

Game theoretic resource allocation techniques

UDRC researchers developed distributed resource allocation algorithms using methods based on so-called *potential games*[4]. These were tested on waveform allocation problems and showed improved performance measured in terms of signal to disturbance ratio compared to benchmark techniques [34], [35]. The uniqueness of an equilibrium was proved in [34] for a scenario where allied, non-communicating radars aim to select optimal waveforms by maximising signal to disturbance ratio. This demonstrated sensors interacting strategically without the need to exchange any information. To quantify the performance, a sensor network consisting of three groups of radars was simulated. The radars within the same group could coordinate their waveform allocation, but they could not communicate with radars in other groups.

The UDRC has also developed power allocation techniques for distributed sensors [36], [37]. The researchers performed extensive Nash equilibrium analysis to demonstrate existence and uniqueness of equilibrium power allocation. This rigorous mathematical analysis demonstrated that an active sensor could use signals transmitted by others in the same group as signals of opportunity [38]. Hence, without explicit coordination, certain sensors need not illuminate targets but could act purely passively, thus deriving military benefit through resource saving and maintaining covertness. Specifically, in the case when exactly n radars in a group of M achieve the desired signal-to-interference-plus-noise ratio (SINR), then at least $M-n$ radars in that cluster remain inactive. The sensors that are inactive are determined only by the target and clutter characteristics, and are independent of the actions of the other groups and the corresponding clutter. This observation leads to the conclusion that the identity of the illuminating source is not part of the game. This observation was key for the proof of

[4]This is a game in which the incentive of each player can be expressed by the same mathematical function.

Nash equilibrium [38]. UDRC researchers showed that at the Nash equilibrium one of the radars in each group opts to remain silent, i.e. zero transmission power, but uses signal from the other radar in that group as the signal of opportunity to obtain the desired SINR for target detection [39].

Multiple sensors and multiple targets

Beamforming techniques for two-dimensional phased-MIMO arrays have been developed in [40], [41]. The UDRC further extended the power allocation and beamforming methods for a sensor network with multiple targets, consisting of both surveillance and tracking sensors using non-cooperative, partially cooperative and Stackelberg game[5] methods [42]. The primary objective of each player is to minimise its transmission power while attaining an optimal beamforming strategy and satisfying a certain detection criterion for each of the targets. Initially, UDRC researchers considered a strategic non-cooperative game, where there is no communication between the various players. Here each sensor selfishly determines its optimal beam and power allocation. This was refined into a more coordinated game incorporating a pricing mechanism. Introducing a price in the utility determination for each player enforced a minimisation in the interference induced in other sensors and increased the social utility of the system. Subsequently, the UDRC team formulated a Stackelberg game by adding a surveillance sensor to the system model, which played the role of the leader, with the remaining sensors as followers. The leader applied a pricing policy for interference charged to the followers aiming at maximizing its profit while keeping the incoming interference under a certain threshold. The proof of the existence and uniqueness of the Nash equilibrium for each scenario was also presented in [42].

[5] A Stackelberg game is a type of *leader-follower* game; a game in which one player (the leader) makes a move which is observed by the other players (followers) who then react to this move.

Robust waveform design for cognitive radars

The UDRC team developed robust waveform techniques for multi-static cognitive radars in a signal-dependent clutter environment [43], [44]. In cognitive radar design second order statistics related to clutter are often assumed to be known. This is unrealistic, as exact knowledge of the clutter parameters is difficult to obtain in practical scenarios. Hence this work addressed waveform design in the presence of uncertainty in the clutter environment, and developed both worst-case and probabilistic robust waveform design techniques. As existing methods in the literature are over-conservative and generic, UDRC researchers proposed a new approach which directly incorporated uncertainty in the radar cross-section and Doppler parameters of the clutter. Using appropriate (Taylor series) approximations, a clutter-specific stochastic optimisation was made that, while maximising the SINR of a particular radar, was able to ensure the other radars in the network reliably achieve a desired SINR [45].

Game theoretic data association for multi-target tracking

UDRC researchers developed a game theoretic approach to solve the data association problem for a varying number of targets in multi-target tracking scenarios [32], [33]. This algorithm used a filtering method to generate initial track hypotheses. The game theoretic method was then used to perform target to track association. The use of a game theory allows for computationally tractable data association in very complicated scenarios.

The UDRC team developed two tracking methods based on sequential Monte Carlo methods to produce state estimates of multiple targets [47], [48]. A further innovative multi-target tracking algorithm was developed, allowing multiple extended targets to be tracked [49]. This is particularly useful for tar-

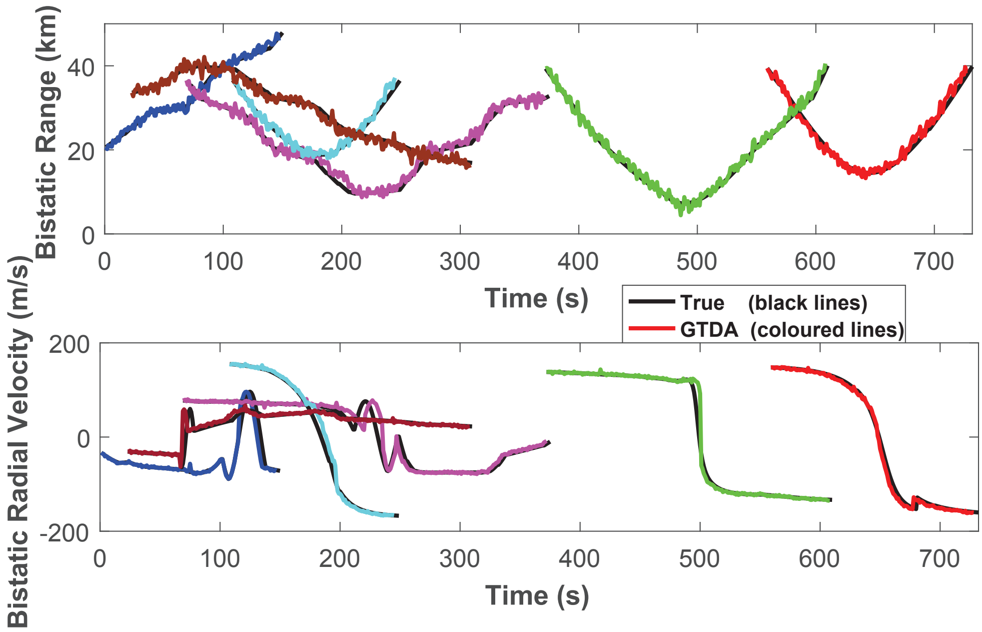

Figure 4.8: Results of game-theoretic data association. Solid black lines (visible beneath the coloured lines) represent the true flight paths on the range and radial velocity maps obtained from a live flight tracker [46]. The coloured lines denote the output of the tracker and the game-theoretic data association method.

gets with irregular shapes or extensions which produce multiple detections per scan.

To obtain target to track associations the problem of data association was formulated as a game between multiple and varying numbers of tracks (the players). To exercise the method, a passive radar experiment was devised. Aeroplanes were detected using signals of opportunity (TV transmitters) together with a low-cost antenna and an SDR to capture the signals. The UDRC technique achieved good target to track association in [33]. Figure 4.8 shows results obtained from the experiment. Notice that there are a total of six targets throughout the duration of the experiment. Three of the targets (cyan, green, red) have a U-like trajectory in range correspond to targets moving in a straight line past the closest point to the passive radar receiver. Zero (radial) velocity (bottom graph) corresponds to when targets are closest in range to the transmit-receive set-up. The other three targets have irregular trajectories indicating targets moving away after having taken off, or taking position to land at a local airport. The range and radial velocities of the true flight path (black) and the target-state-estimate after game-theoretic data association (GTDA: coloured) are shown. These results demonstrate that the proposed GTDA technique is able to properly associate the target state estimates of different targets with their corresponding tracks [32].

4.3.2 Enabling contract on temporal anomaly detection

Researchers at Loughborough participated in and won the temporal anomaly detection challenge set during the anomaly detection workshop in 2014 (see table 1.6). They were subsequently contracted to develop that submission further in collaboration with Dstl's Counter Terrorism and Security Division. Their solution used model-based spectral estimation and machine learning methods to automatically detect anomalies in temporal data. The new methods employed various statistical measures, including higher-order statistics based on support vector machines, to detect anomalies without any prior knowledge of their characteristics (i.e. in the absence of any signatures). The techniques proposed by the UDRC researchers were able to determine the start and end times of the anomalies (as required) and their frequencies with the desired accuracy. The algorithms demonstrated the ability to detect anomalies in low-to-moderate SNR environments, and when the underlying frequency of the signal drifted.

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