Yakubu Y., Aliyu Z. Q., Usman A., and Evans O. P. "A Study of the Effects of Baking Materials and Oven Temperature on Cake Height: Split-plot Central Composite Design Approach". Pg 809-826

SPSBIC 2017

4 - 5 May, Minna Nigeria

SCHOOL OF PHYSCIAL SCENCES
1[ST] BIENNIAL INTERNATIONAL CONFERENCE

PROCEEDINGS

Theme:
Science Technology and Innovation (STI):
The Vision for Poverty Reduction and Sustainable Development

FEDERAL UNIVERSITY OF TECHNOLOGY
MINNA, NIGER STATE, NIGERIA

PREFACE

This is the first international Conference organized by the school of Physical Sciences of the Federal University of Technology. Minna Nigeria the school is relatively new and consisting of the Departments of Physics. Chemistry. Mathematics. Statistics. Geology and Geography. It was exercised from the former school of Natural and Applied Sciences on the 6th of November 2014.

The school of Physical Sciences 1st Biennial International Conference is an interdisciplinary forum for the presentation of new ideas. recent developments and research findings in the field of Science and Technology. The Conference provides a platform to scholars. researchers in the academics and other establishments to meet. share and discuss how science and technology can help reduce poverty and bring about sustainable development. Submissions were received both nationally and internationally and severally reviewed by our international program committee. All contributions are neither published elsewhere nor submitted for publication as asserted by contributor.

We wish to express our gratitude to the school for challenging us to organize the first international conference. Special thanks to the Dean of the School Prof. A. S. Abubakar. Special thanks to all members of the organizing committee and sub-committee for their dedication. determination and sacrifice towards achieving a fruitful and successful conference.

The Local Organizing Committee Chairman
Kasim Uthman Isah (PhD).

A STUDY OF THE EFFECTS OF BAKING MATERIALS AND OVEN TEMPERATURE ON CAKE HEIGHT: SPLIT-PLOT CENTRAL COMPOSITE DESIGN APPROACH

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Abstract

Most people love a delicious piece of cake. But what is the secret to a good cake? While many may comment on flavor, frostings or other attributes, most people agree on one thing: texture. The texture of the cake needs to be "fluffy". While there are exceptions to the traditional textures, one of the easiest ways to get the right texture of a cake is by having the cake rise to a maximum height. By having the tallest cake with the same amount of starting material, more air is allowed into the cake, thereby creating the "fluffy" texture that people desire. In this work, a central composite design (CCD) experiment within a split-plot structure was conducted to assess the impact of some baking materials on cake height. The experimental factors (cake-baking materials) include oven temperature (factor A), amount of flour (factor B), baking powder (factor C), and amount of milk (factor D) with a fixed amount of other necessary ingredients present. The generated data were analyzed using Design Expert (version 10) statistical package. The restricted maximum likelihood (REML) estimates of the variance components were first obtained and the generalized least squares (GLS) estimator was used to obtain the factor effects estimates. The set of levels of these factors that yields optimum value of the cake height (the stationary point) was then sought using optimization facility of the statistical package. It was observed that each of the linear terms: A, B, C, D, the interaction term: BC and the quadratic terms: A^2, C^2, and D^2 contributes significantly to the height of the cake. The fitted generalized least squares model accounted for 95% of the total variation in the cake height. The estimated optimum cake height was found to be $\hat{y} = 11.047$ at the stationary point: $oven\ temp = 250^oC$, $amount\ of\ flour = 1.5\ cups, baking\ powder = 1.5\ teaspoonfull, and\ milk = 0.75cup$.

Keywords: Cake height, Split-plot CCD, Experiment, Design

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1. Introduction

Most people love a delicious piece of cake. However, the secret to a good cake as most people often agree is texture. The texture of the cake needs to be "fluffy". While there are exceptions to the traditional textures, one of the easiest ways to get the right texture of a cake is by having the cake rise to a maximum height. By having the tallest cake with the same amount of starting material, more air is allowed into the cake, thereby creating the "fluffy" texture that people desire. In this work, a study was carried out on the impact of some cake-baking materials on cake height using statistically-designed experimental technique.

Statistically designed experiment is an indispensable technique in the design stage of a product or a process for investigating the effects of several factors on a quality characteristic of interest. These experiments play a key role in the design of new products, improvement of existing ones as well as the design and development of manufacturing processes to produce them, which are the crucial activities in most industrial organizations today. In most industries today, for instance, biotechnology and pharmaceuticals, medical devices, electronics and chemical industries etc., experimental design methodology has resulted in shorter design and development time for new products as well as products that are easier to manufacture, products with higher reliability and enhanced field performance, products that meet or even exceed customer requirements. The tools required for adequate selection of a design and the subsequent fitting and evaluation of the hypothesized model using the data generated by the design, have been developed in an area of experimental design known as **response surface methodology (RSM)**.

Response Surface Methodology is an area of experimental design which consists of a group of mathematical and statistical techniques used in the development of an adequate functional relationship between a response of interest, y, and a number of associated control (or input) variables denoted by $x_1, x_2, \ldots, x_k$ (Myers *et all*, 2009). The most extensive applications of RSM are in the industrial world, particularly in situations where potential influence of several process variables on some quality characteristic of the process is being investigated. RSM is widely used to explore and to optimize response surfaces in these experiments. RSM is sequential in nature and so the experimenter begins with a screening experiment to identify important factors. Follow-up experiments then seek to improve the performance of the response. This process allows the experimenter to learn about the process or system under study as the investigation proceeds. Response Surface Methodology is useful for developing, improving, and optimizing the response variable especially when treatments are from a continuous set of values. One of the most commonly-used response surface design is the second-order design such as the central composite design (CCD) and the Box-Behnken design (BBD), which were introduced, respectively, by Box and Wilson (1951), and Box and Behnken (1960). The variables in these designs are always completely randomized.

The CCD consists of factors with five levels that involve three components. They are:

i. a complete (or a fraction of) 2^k factorial design with factor levels coded as -1, 1 (called the factorial portion),
ii. an axial portion consisting of $2k$ points arranged so that two points are chosen on the coordinate axis of each control variable at a distance of α from the design center,
iii. n_0 center points.

Thus the total number of points in a CCD is $n = 2^k + 2k + n_0$. These three components of the design play important and somewhat different roles.

i. The factorial portion consists of n_f equally-spaced points that contribute to the estimation of linear terms and are the only points that contribute to the estimation of the interaction terms in the model.
ii. The axial portion consists of points lying on the axis of each input variable, and in this portion of the design, the factors are not varying simultaneously but rather in a one-factor-at- a-time array. Thus no information regarding the interaction effect is provided by this portion of the design. However, the axial portion allows for efficient estimation of pure

quadratic terms in the model, and without these points, only the sum of the quadratic terms, i.e., $\sum_{i=1}^{k} \beta_{ii}$, can be estimated.

iii. The center runs provide an internal estimate of error (i.e., the pure error), and efficiently provide information about the existence of curvature in the system. If curvature is found in the system, the addition of axial points allows for efficient estimation of the pure quadratic terms.

This design technique was used in this work but conducted within a split-plot structure.

One difficulty in applying classical response surface designs is that they inherently assume that all factors are equally easy to manipulate, thereby allowing for complete randomization of the experimental run order. In practice most industrial experiments cannot be completely randomized due to the presence of factors with levels that are difficult to change (called hard-to-change (HTC) factors) and those with levels that are easy to change (called easy-to-change (ETC) factors). Thus factors with HTC levels cannot be completely randomized and once an experiment includes such factors, split-plot design approach is used, in which the experimental runs are performed in groups, where, in a group, the levels of the HTC factors are not reset from run to run. This creates dependence among the runs in one group, thereby leading to clusters of correlated errors and responses.

There are two separate randomizations for every split-plot design- whole plot factor levels are randomly assigned to the whole plots using a different randomization for each block; subplot factor levels are randomly assigned within each whole plot using a separate randomization for each whole plot. This leads to two error terms for effects comparison, one for the whole-plot treatments (σ_{Υ}^{2}), and one for the subplot treatments $(\sigma_{\varepsilon}^{2})$ as well as the interaction between whole-plot treatments and subplot treatments.

Split-plot central composite designs consist of four different categories of points. These include the factorial portion (f), which consists of n_f equally-spaced points that contribute to the estimation of linear and interaction terms in the model, axial point (whole-plot(α) and subplot(β)), which consists of points lying on the coordinate axis of each input variable, and which allow for efficient estimation of pure quadratic terms in the model and center (c) points, which provide an internal estimate of error (i.e., the pure error), and efficiently provide information about the existence of curvature in the system. If curvature is found in the system, the addition of axial points allows for efficient estimation of the pure quadratic terms.

This work carried out the split-plot central composite design experiment that investigates the effects of oven temperature (factor A), amount of flour (factor B), baking powder (factor C), and amount of milk (factor D) on cake height and then locates the set of levels of these factors that optimizes the predicted cake height (the stationary points). The oven temperature is a HTC factor while the other three are ETC factors. Therefore, the central composite design (CCD) experiment was conducted with a split-plot approach using these factors with a fixed amount of other necessary ingredients present. The generated data were analyzed with the aid of Design Expert (version 10) statistical package using restricted maximum likelihood (REML) approach.

1.1 Statistical model and notations

As stated earlier, the responses from the split-plot experiment are correlated and this violates the assumption of independence of the ordinary least squares or OLS estimator. Therefore generalized least squares or GLS estimates are better since they account for the correlation between these observations and are generally more precise. The generalized least squares (GLS) model for a split-plot response surface design is

$$y = X\beta + Z\gamma + \varepsilon \tag{1.1}$$

where **y** is the N x 1 vector of responses, **X** is the N x p overall model matrix, **β** is the p x 1 vector of regression coefficients, **Z** is an N x b incidence matrix assigning observations to each of the b whole plots; γ is the N x 1 vector of whole-plot error terms, ε is the N x 1 vector of subplot error terms. It is assumed that $\gamma_i \sim N(0, \sigma_\gamma^2)$, $\varepsilon_{ij} \sim N(0, \sigma_\varepsilon^2)$, $cov(\gamma_i, \varepsilon_{ij}) = 0$.

The variance - covariance matrix for the observation vector **y** is

$$Var(y) = V = \sigma_\varepsilon^2 I_n + \sigma_\gamma^2 ZZ'$$

$$= \sigma_\varepsilon^2 (I_n + dZZ')$$

where $d = \frac{\sigma_\gamma^2}{\sigma_\varepsilon^2}$ gives the relative magnitude of the two variance components. The matrix ZZ' is a block diagonal matrix with diagonal matrices of $J_{n1}, J_{n2}, \ldots, J_{nz}$, where J_{ni} is an n_i x n_i matrix of 1's and n_i is the number of observations in the ith whole-plot.

The diagonal elements of V are the variances of the responses and the off- diagonal elements are the covariances between pairs of responses. The nonzero off-diagonal elements correspond to pairs of responses from within a given whole plot while the zero off-diagonal elements correspond to pairs of runs from two different whole plots.

The generalized least squares (GLS) estimates are calculated from

$$\hat{\beta}_{GLS} = (X'V^{-1}X)^{-1}X'V^{-1}y \tag{1.2}$$

Thus,

$$Var(\hat{\beta}_{GLS}) = (X'V^{-1}X)^{-1}$$

$$\hat{y} = X(X'V^{-1}X)^{-1}X'V^{-1}y = Hy$$

where H is the 'hat' matrix.

2. LITERATURE REVIEW

Research works on impact of a split-plot structure on response surface designs began in the 1990s and most of these work focused on two-level fractional factorial designs run as split-plots. Box (1996) explains that completely randomized experiments are often impractical in industry and indicates that split-plot experiments are often very efficient and easier to run.

Bisgaard and Steinberg (1997) look at the design and analysis of prototype experiments. They present examples that use split-plot designs and show clearly how to carry out a two-stage analysis of the data. Bisgaard (2000) uses two-level fractional factorials to construct split-plot designs and gives general expressions for deriving alias structures based on the group structure of the arrays.

Letsinger, Myers and Lentner (1996) is the first paper to exclusively focus on restricted randomization in response surface methodology. The authors describe an experiment from the chemical industry which investigates the effects of five process variables (temperature 1, temperature 2, humidity 1, humidity 2, and pressure) on a certain quality characteristic. They used a modified central composite design but the experimental runs were not completely randomized due to the fact that the levels of temperature 1 and pressure were hard to change. Thus, the split-plot design approach was used in which the levels of the hard-to-change factors were changed as little as possible.

A sequential strategy for designing multi stratum designs, special cases of which are split-plot designs, was presented by Trinca and Gilmour (2001). The authors describe an experiment to investigate the effects of five factors on protein extraction. The factors were the feed position for the inflow of the mixture, the feed flow rate, the gas flow rate, the concentration of protein A and the concentration of protein B, each factor consisting of three levels. The feed position consists of hard-to-change levels and is therefore the whole plot factor. The merit of split-plot approach was fully utilized in this experiment since two experimental runs instead of one could be performed on one single day.

Kowalski (2002) considers split-plot experiments in robust parameter design. He constructs 24-run designs in two ways: using the properties of a balanced incomplete block design and by semifolding a 16 run design.

Vining, Kowalski and Montgomery (2005), hereafter referred to as VKM, show how to modify the standard central composite design (CCD) and Box-Behnken design (BBD) to accommodate a split-plot structure. The authors then establish the general conditions under which the ordinary least squares estimates of the model are equivalent to the generalized least squares estimates and therefore are best linear unbiased.

3. MATERIALS AND METHODS

The materials used for baking all the cakes include flour, baking powder, milk and oven temperature, with fixed amount of other necessary ingredients. Oven temperature was included because the temperature at which the batches are baked strongly impacts cake height. It is a HTC factor since its levels cannot be frequently changed as it takes some time to stabilize, while the other three are ETC factors as their levels can easily be changed during randomization. Each of the factors consists of five (5) levels and these levels were coded using

$$x_i = \frac{2X_i-(X_{il}+X_{ih})}{X_{ih}-X_{il}} \qquad (3.1)$$

Where x_i is the coded level of the ith factor, $i = 1,2,\ldots,k$; X_{il} and X_{ih} are, respectively, the actual low and high levels of the ith factor. The choice of factors, levels and factor ranges are listed in TABLE 3.1.

Table3.1: Choice of factors, levels and ranges

Factor	coded levels				
	-1	-1	0	1	1

Amount of Flour(X_1)	1/2 cup	1/2 cup	1 cup	1.5 cups	1.5 cups
Amt of Baking Powder(X_2)	1/2 teaspoon	1/2 teaspoon	1 teaspoon	1.5 teaspoon	1.5 teasp
Amount of Milk(X_3)	1/4 cup	1/4 cup	1/2 cup	3/4 cups	3/4 cups
Temperature of Oven(z_1)	150C	150C	200C	250C	250C

For every cake, the butter and sugar were first mixed together in the same Kitchen-aid mixer on the same speed setting. The flour, baking powder and salt were sifted together and added with the specified amount of milk and mixed at the same speed. This was then poured into the cake pan and a spatula was used to scrape all into the pan. All cakes were baked from the same batch of ingredients and the same cake pans were used for each run.

3.1 Performing the Experiment

A four-factor central composite experiment was designed with a split-plot structure in one whole-plot variable and three subplot variables, and with single replication of factorial and axial parts. Each factor consists of five (5) levels: -1, +1, $-\alpha$, $+\alpha$, and 0. At each of the factorial points involving the HTC factor, the subplot runs are the factorial points in the ETC factors. At each of the two axial points that make up whole plots (i.e., $z_1 = -\alpha, +\alpha$), eight replicates of the center of the subplot factors (i.e., $x_1 = 0, x_2 = 0, x_3 = 0$) were run. Two replicates of the whole-plot center points ($z_1 = 0$) were run, one consisting of the six (6) axial points in the ETC factors while the other containing four(4) replicates of the center of the subplot factors ($x_1 = 0, x_2 = 0, x_3 = 0$). Thus each of the whole-plot factorial and axial portions of the design is of size eight while the first and second replicates of the whole-plot center portion are, respectively, of size six and four. Thus the resulting split-plot central composite design (CCD) has $n_f = 16$ factorial points, $n_\alpha = 16$ whole-plot axial points, $n_\beta = 6$ subplot axial points and $n_c = 4$ center points. Thus there were $N = 42$ total design points in 6 whole plots as given in Table 3.2. DESIGN-EXPERT (version 10) statistical package was used to create the design layout. The fully-randomized order of runs of the experiment was given in the appendix.

Table 3.2: Design Matrix for **D(1,3)** split-plot CCD

Wp	z1	x1	x2	x3	n
1	−1	±1	±1	±1	8
2	+1	±1	±1	±1	8
3	$-\alpha$	0	0	0	8
4	$+\alpha$	0	0	0	8
5	0	$\pm\beta$	0	0	2
	0	0	$\pm\beta$	0	2

	0	0	0	$\pm\beta$	2
6	0	0	0	0	4

At each setting (level) of the HTC factor (temperature), all possible combinations of the levels of the ETC factors (flour, baking powder and milk) were baked at the same time. This was followed by another (single) run of the oven at the other temperature setting and the process was continued until all the temperature factor levels were completely run. All cakes were baked for 28 minutes to minimize baking time as a nuisance factor. After every run of the oven, its temperature was allowed to stabilize (i.e., cool down to 0°C) before another level was randomly selected for another run. Thus the experiment took us six days to complete. All cakes were measured using the same ruler and measurements were taken in centimeters (cm). No unusual behavior was observed during the experimental runs.

The design consists of two different randomization structure:- temperature factor-levels were randomly and independently assigned to the whole-plots; within each whole plot, the ETC factor level combinations were randomly and independently assigned to the subplots using a different randomization technique. Thus levels of the whole-plot factors were not reset for each run of the subplot factors and this leads to two error terms for effects comparison, one for the whole-plot treatments (σ_γ^2), and one for the subplot treatments (σ_ε^2), as well as the interaction between whole-plot treatments and subplot treatments.

3.2 Data Analysis

The generated data were analyzed using Design Expert (version 10) statistical package. **Restricted maximum likelihood** estimation (REML) technique was used to estimate the whole-plot and subplot variance components (σ_γ^2 and σ_ε^2, respectively). REML estimates for these variance components can be obtained by maximizing the restricted log likelihood function:

$$l_g = -\frac{1}{2}\log|V| - \frac{1}{2}\log|X'V^{-1}X| - \frac{1}{2}r'V^{-1}r - \frac{n-p}{2}\log(2\pi) \quad (3.2)$$

Where $\boldsymbol{X}$ and $\boldsymbol{V}$ are as defined in section 1.1 above; $r = y - X(X'V^{-1}X)^{-1}X'V^{-1}y$ and p is the number of parameters in $\boldsymbol{\beta}$ (Goos and Jones, 2011). These estimates were obtained here with the aid of the statistical package.

The Generalized Least Squares (GLS) estimator given in equation (1.2), which is a version of least squares that allows us to account for covariances among the responses, such as might be present in a mixed effects model, was used to estimate the factor effects. The standard errors for the factor effects were then computed as the square root of the diagonal elements of the covariance matrix $(X'V^{-1}X)^{-1}$.

Optimization facility of the statistical package was then used to obtain the set of levels of the factors that yields optimum value of the cake height (the stationary point).

4. RESULTS AND DISCUSSION

We first looked at the analysis of variance (ANOVA) for our fitted model. This ANOVA confirms the adequacy of our fitted model, as given in the Table below.

TABLE 4.1: ANOVA (REML)

Source	Term df	Error df	F	p-value Prob > F	
Whole-plot	2	27	11	0.0003	significant
a-Oven temperature	*1*	*27*	*8.14*	*0.0082*	
a^2	*1*	*27*	*13.87*	*0.0009*	
Subplot	12	27	42.19	< 0.0001	significant
B-Quantity of flour	*1*	*27*	*355.82*	*< 0.0001*	
C-Amount of baking powder	*1*	*27*	*10.92*	*0.0027*	
D-Amount of milk	*1*	*27*	*17.86*	*0.0002*	
aB	*1*	*27*	*2.27*	*0.1432*	
aC	*1*	*27*	*2.03*	*0.1653*	
aD	*1*	*27*	*0.015*	*0.9036*	
BC	*1*	*27*	*8.37*	*0.0075*	
BD	*1*	*27*	*0.081*	*0.7776*	
CD	*1*	*27*	*0.042*	*0.8401*	
B^2	*1*	*27*	*0.71*	*0.4081*	
C^2	*1*	*27*	*6.97*	*0.0136*	
D^2	*1*	*27*	*6.97*	*0.0136*	

The fourth column of the table gives the computed F-values while the fifth column gives the probability values. From this column we can see that each of the terms: a, B, C, D, BC, a^2, C^2, and D^2 contributed significantly to the goodness-of-fit of the model. That is, each of the oven temperature, amount of flour, amount of baking powder and amount of milk had significant effect on the cake height, with p-values far less than 0.05. Also, the interaction effect of the quantity of flour and amount of baking powder was highly significant; the quadratic effects of the oven temperature, amount of baking powder and quantity of milk were also highly significant. All other terms were not significant.

Next we considered the group (or whole-plot) and residual (or subplot) variance components. These are variations that were not explained by the model in terms of the factors, as given in the Table below.

TABLE4.2: Variance Components

Source	Variance	StdErr	95% CI Low	95% CI High
Group	*0.000*	*0*	*0*	*0*
Residual	*0.38*	*0.1*	*0.24*	*0.7*
Total	0.38			

The group (or whole-plot error) variance is due to resetting of a hard-to-change factor level (in this work, oven temperature). The computed group variance was zero, and this signifies that the whole-plot model explained all of the variation between the whole plots. The residual (or subplot error) variance is due to each of the subplot runs. In this work, the residual variance was 0.38.

This fitted model explains 95% of the total variability in the cake height ($R^2 = 0.95$). Thus only 5% was not accounted for by the model, also, the adjusted R^2 was 0.92 ($Adj.\ R^2 = 0.92$), as given at the bottom of Table 4.3. These statistics signify that the fitted model was good, that is, the model have captured most of the variation in the data.

Then we looked at the computed regression coefficients and variance inflation factors (VIF) as given in Table 4.3 below. The VIF measures how much the variance of the model is inflated by the lack of orthogonality in the design. It indicates the extent to which multicollinearity (correlation among predictors) is present in a regression analysis.

TABLE 4.3: Estimated regression coefficients

Source	Coefficient Estimate	Standard Error	VIF
Intercept	5.727822581	0.19865203	
Whole-plot Terms:			
a-Oven temperature	-0.309375	0.1084569	1
a^2	-0.861391129	0.23130547	1.082949309
Subplot Terms:			
B-Quantity of flour	2.727777778	0.1446092	1
C-Amount of baking powder	0.477777778	0.1446092	1
D-Amount of milk	0.611111111	0.1446092	1
aB	0.23125	0.15338122	1
aC	-0.21875	0.15338122	1
aD	0.01875	0.15338122	1

BC	0.44375	0.15338122	1
BD	-0.04375	0.15338122	1
CD	0.03125	0.15338122	1
B^2	0.303629032	0.36128969	3.566820276
C^2	0.953629032	0.36128969	3.566820276
D^2	0.953629032	0.36128969	3.566820276

$\boldsymbol{R^2 = 0.95}$, **Adjusted** $\boldsymbol{R^2 = 0.92}$

From the last column of this table, all the VIFs are equal to one except for the last three terms of the model. This indicates that each of the first eleven predictors was orthogonal to (i.e., not correlated with) all the other predictors in the model. Each of the last three predictors has VIF equal to 3.56, which indicates that it was moderately correlated with all the other predictors. Thus there was no case of multicollinearity in the data, and we have the fitted second-order model for the data as:

$$Height = 5.73 - 0.31z_1 + 2.73x_1 + 0.48x_2 + 0.61x_3 + 0.23z_1x_1 - 0.22z_1x_2 + 0.019z_1x_3 + 0.44x_1x_2 - 0.044x_1x_3 + 0.031x_2x_3 - 0.86z_1^2 + 0.30x_1^2 + 0.95x_2^2 + 0.95x_3^2 \quad (4.1)$$

Each of the terms in this model has the same p-value as in Table 4.1. Thus the effects of the terms: a, B, C, D, BC, a^2, C^2, and D^2 were each significant while that of the remaining terms were not significant.

4.1 *Diagnostic plots*

Here we diagnose the statistical properties of the above fitted model. Figure 4.1 below gives the normal probability plot of the residuals, which indicates whether the residuals follow a normal distribution.

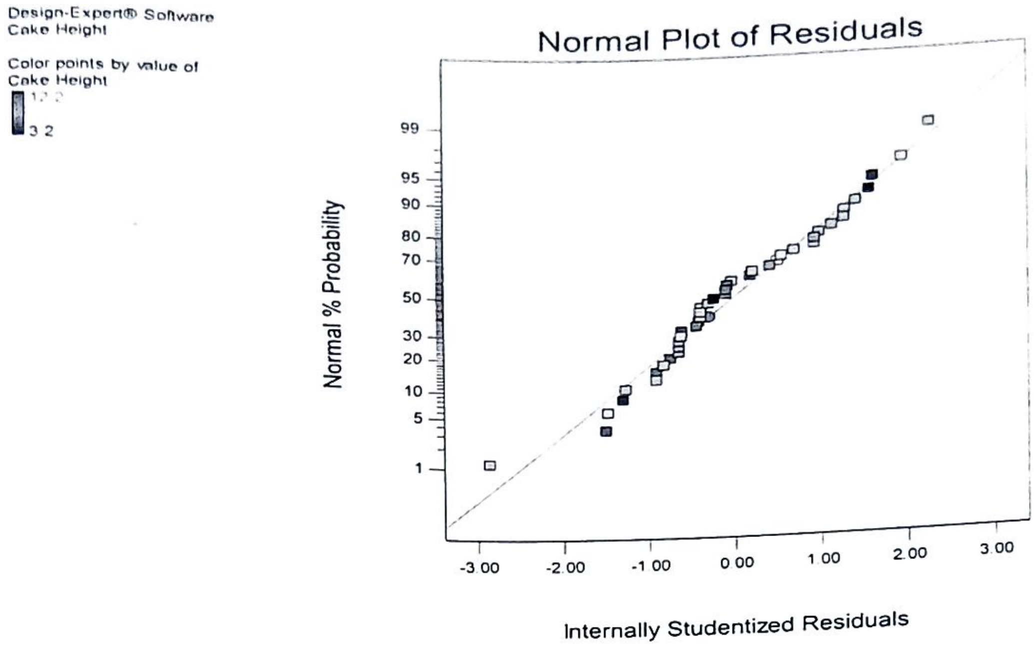

Fig.4.1: Normal probability plot of the residuals

From this plot, all the points fall on the straight line. This indicates that, to some extent, the residuals were normally distributed. Thus there are no problems with our data. next we looked at the plot of the residuals versus the predicted values, as given in figure4.2 below.

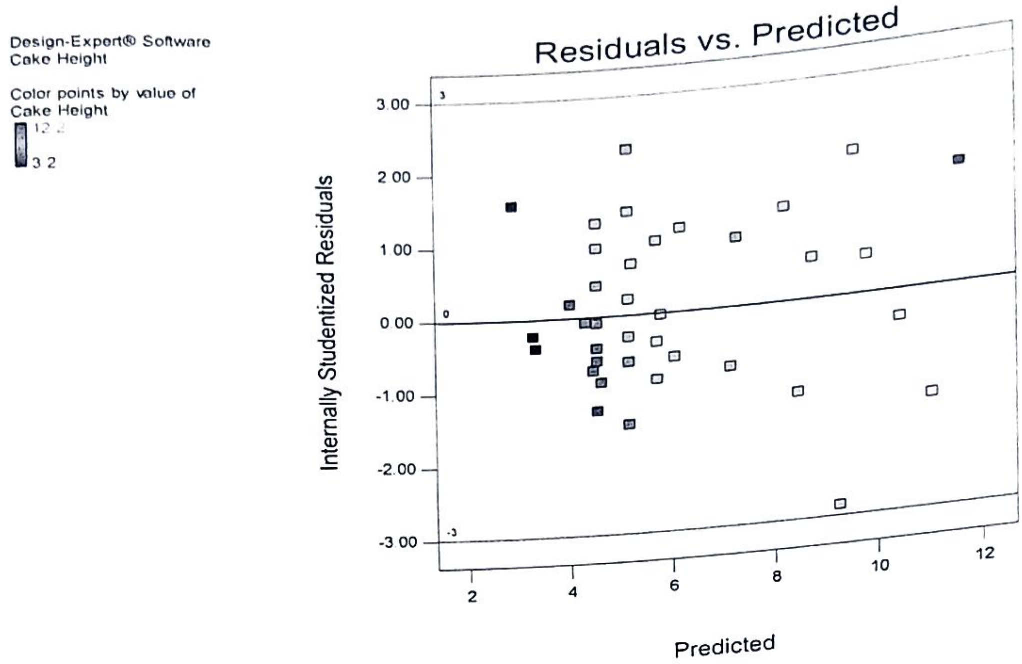

Fig.4.2: Residuals Vs Predicted plot

This plot is a visual check for the assumption of constant variance. As can be directly seen, this plot is a random scatter with a consistent top to bottom range of residuals across the predictions on the X axis. Thus we can conclude here that our model satisfied the constant variance assumption.

Next we looked at the plot of the residuals versus the experimental run order, as given in figure 4.3 below.

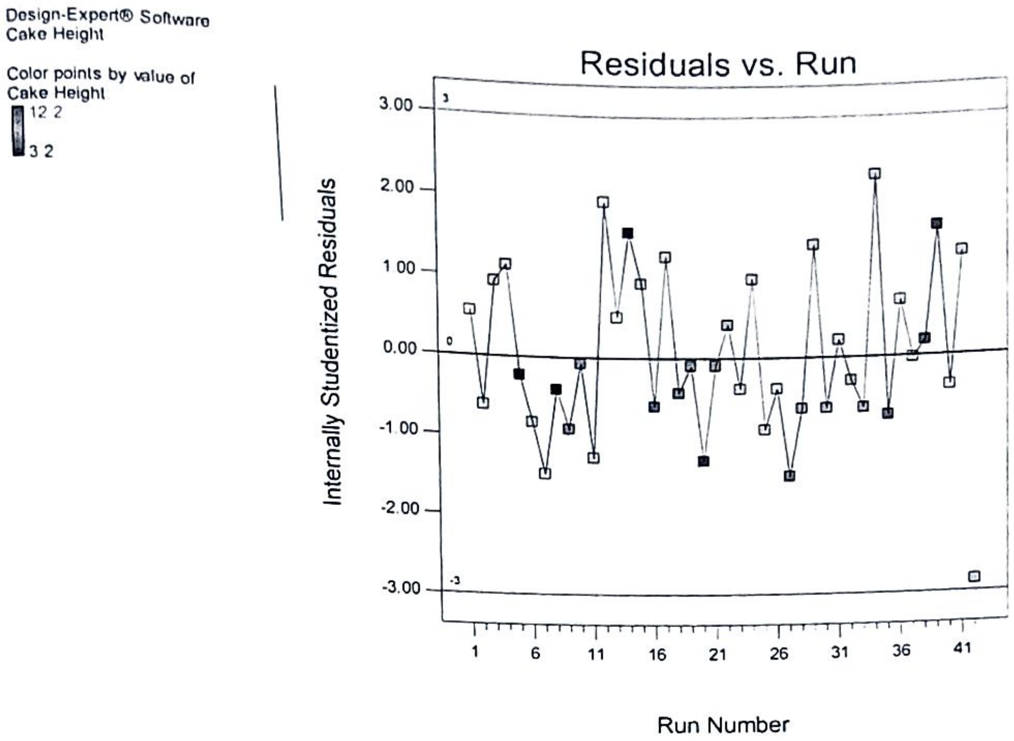

Fig.4.3: Residuals versus experimental run order

This plot provides a check for lurking variables that may have influenced the response during the experiment.

As can be directly observed, the plot showed a random scatter without any trend. Thus there were no any lurking variable in the background.

Lastly, we looked at the plot of the predicted values versus the actual values, as given in the figure 4.4 below.

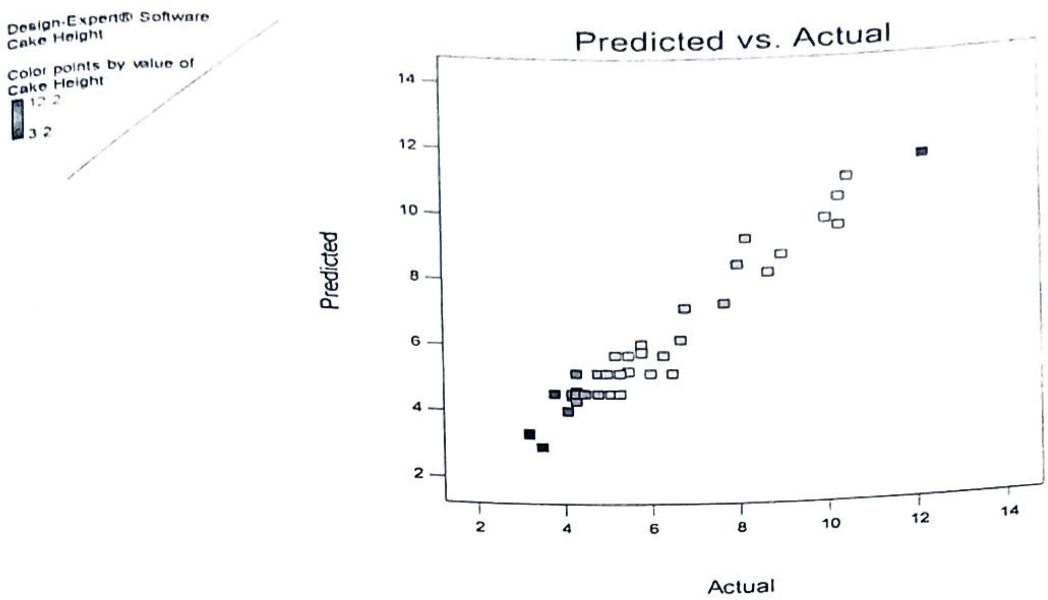

Fig.4.4: predicted versus actual values plot

This graph plots the observed (actual) response values versus the predicted response values. The plot help us to detect observations that that were not well predicted by the model. From this figure, the data points were evenly split by the 45 degree line. Thus our fitted model was okay.

5. Conclusion

This study has investigated the potential effects of baking materials and oven temperature on cake height through the conduct of the response surface experiment within a split-plot structure. By conducting the response surface experiment within the split-plot structure, the number of times the oven temperature has to be reset was reduced by sorting the design into whole-plot groups. Since randomization was restricted by sorting, a restricted maximum likelihood (REML) analysis was applied to obtain the estimates of the variance components and the generalized least squares (GLS) estimator was used to obtain estimates of the effects. The predictors were not linearly correlated and so there was no case of multicollinearity; thus the parameter values were independently estimated. The baking materials considered as well as the oven temperature play significant roles in determining the cake height, which is one of the easiest ways to get the right texture of a cake. The fitted generalized least squares (GLS) model accounted for 95% of the variability in the cake height.

Numerical optimization resulted in the set of the factor levels that will yield optimum value of the cake height. The statistical properties of the fitted model were further diagnosed and these revealed no problems with the data. The conducted split-plot CCD allowed the experimenters to save time and money and run a design which might well have been infeasible if fully randomized.

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Appendix

Design matrix and run- order of the data

				Factor 1	Factor 2	Factor 3	Factor 4
Std	Group	Run	Space Type	a:Oven temperature	B:Quantity of flour	C:Amount of baking powder	D:Amount of milk

38	1	1	Axial	0	1	0	0
41	1	2	Axial	0	0	0	-1
42	1	3	Axial	0	0	0	1
39	1	4	Axial	0	0	-1	0
37	1	5	Axial	0	-1	0	0
40	1	6	Axial	0	0	1	0
16	2	7	Factorial	1	1	1	1
9	2	8	Factorial	1	-1	-1	-1
13	2	9	Factorial	1	-1	-1	1
15	2	10	Factorial	1	-1	1	1
10	2	11	Factorial	1	1	-1	-1
14	2	12	Factorial	1	1	-1	1
12	2	13	Factorial	1	1	1	-1
11	2	14	Factorial	1	-1	1	-1
29	3	15	Axial	1	0	0	0
31	3	16	Axial	1	0	0	0
33	3	17	Axial	1	0	0	0
34	3	18	Axial	1	0	0	0
32	3	19	Axial	1	0	0	0
30	3	20	Axial	1	0	0	0
35	3	21	Axial	1	0	0	0
36	3	22	Axial	1	0	0	0
19	4	23	Center	0	0	0	0
17	4	24	Center	0	0	0	0
20	4	25	Center	0	0	0	0
18	4	26	Center	0	0	0	0
22	5	27	Axial	-1	0	0	0
25	5	28	Axial	-1	0	0	0
23	5	29	Axial	-1	0	0	0

24	5	30	Axial	-1	0	0	0
26	5	31	Axial	-1	0	0	0
27	5	32	Axial	-1	0	0	0
21	5	33	Axial	-1	0	0	0
28	5	34	Axial	-1	0	0	0
3	6	35	Factorial	-1	-1	1	-1
5	6	36	Factorial	-1	-1	-1	1
7	6	37	Factorial	-1	-1	1	1
1	6	38	Factorial	-1	-1	-1	-1
8	6	39	Factorial	-1	1	1	1
4	6	40	Factorial	-1	1	1	-1
2	6	41	Factorial	-1	1	-1	-1
6	6	42	Factorial	-1	1	-1	1