Odd Generalized Exponential Kumaraswamy distribution: its properties and application to real life data

By

***N. K. Kaile[1], **A. Isah and *H. G. Dikko**
*Department of statistics,
Ahmadu Bello University, Zaria-Nigeria
**Department of statistics,
Federal University of Technology, Minna-Nigeria
E-mail: naskaile20@gmail.com

ABSTRACT

In this study, a new distribution called the Odd Generalized Exponential Kumaraswamy (OGE-K) distribution is defined and its properties investigated. The properties of the new distribution verified includes its probability density function, moment, moment generating function, characteristic function, quantile function, reliability analysis and order statistics. The maximum likelihood estimation procedure is used to estimate the parameters of the new distribution. Application of real data set indicates that the proposed distribution would serve as a good alternative to Kumaraswamy distribution among others to model real- life data in many areas.

Keywords: Order Statistics, Quantile function, Maximum Likelihood Estimation and characteristic function.

INTRODUCTION

The Kumaraswamy distribution is the most widely applied statistical distribution in hydrological problems and many natural phenomena. The Kumaraswamy distribution is very similar to the Beta distribution but has the important advantage of an invertible closed form cumulative distribution function. Kumaraswamy (1976, 1978) has shown that the well-known probability distribution functions such as the normal, log-normal, beta and empirical distributions such as Johnson's and polynomial-transformed-normal, do not fit well hydrological data, (such as daily rainfall and daily stream flow) and developed a new probability density function known as the sine power probability density function. Kumaraswamy (1980) developed a more general probability density function for double bounded random processes, which is known as Kumaraswamy's distribution. This distribution is applicable to many natural phenomena whose outcomes have lower and upper bounds, such as the heights of individuals, scores obtained on a test, atmospheric temperature and hydrological data. Furthermore, this distribution could be appropriate in situations where scientists use probability distributions which have infinite lower and/or upper bounds to fit data, when in reality the bounds are finite. The probability density function and cumulative density function of kumarswamy distribution are respectively given below;

$$f(x) = abx^{a}(1-x^{a})^{b-1} \tag{1}$$

and

$$F(x) = 1-(1-x^{a})^{b} \tag{2}$$

for $0 \le X \le 1$, $a > 0, b > 0$, kumarswamy (a, b), where a and b are positive shapes parameters. It has many of the same properties as the beta distribution but has some advantages in terms of tractability. This distribution appears to have received considerable interest in hydrology and related areas.

Maiti and Pramanik (2015) came up with a new family of distributions called Odd Generalized Exponential Family of distributions. The cdf and pdf of this family are respectively given in equations (3) and (4):

$$F(x) = 1-\exp\left\{-\lambda\left(\frac{G(x)}{1-G(x)}\right)\right\} \tag{3}$$

$$f(x) = \lambda g(x)\left[1-G(x)\right]^{2}\exp\left\{-\lambda\left(\frac{G(x)}{1-G(x)}\right)\right\} \tag{4}$$

where $G(x)$ and $g(x)$ are the CDF and pdf of the baseline distribution.

In this article, we present a new distribution which has its root from the Generalized Exponential Distribution and Kumarswamy distribution called the Odd Generalized Exponential-Kumaraswamy (OGE-K) distribution using the family proposed by Alzaatreh et-al 2013.

Odd Generalized Exponential Kumaraswamy distribution

In this section, we define new three-parameter distribution called Odd Generalized Exponential-Kumarsawamy (OGE-K) distribution with parameters a, b and λ.

A random variable X is said to have OGE-K distribution with parameters a, b and λ if its CDF is given by

$$F(x) = 1-\exp\left\{-\lambda\left(\left(1-x^{a}\right)^{b}-1\right)\right\} \tag{5}$$

and the corresponding pdf of X~OGE-K is

$$f(x) = ab\lambda x^{a-1}\left(1-x^{a}\right)^{-(b+1)}\exp\left\{-\lambda\left(\left(1-x^{a}\right)^{-b}-1\right)\right\} \tag{6}$$

It can easily be shown that the proposed distribution is a proper probability density function by working on equation(6).

The study examined the statistical properties of the proposed distribution as in the previous section which includes the Moment, Moment Generating Function, Characteristic Function, Quantile Function, Reliability Analysis and Order Statistics. Figures1 and 2 represent the graphical plots of the CDF and pdf OGE-K distribution

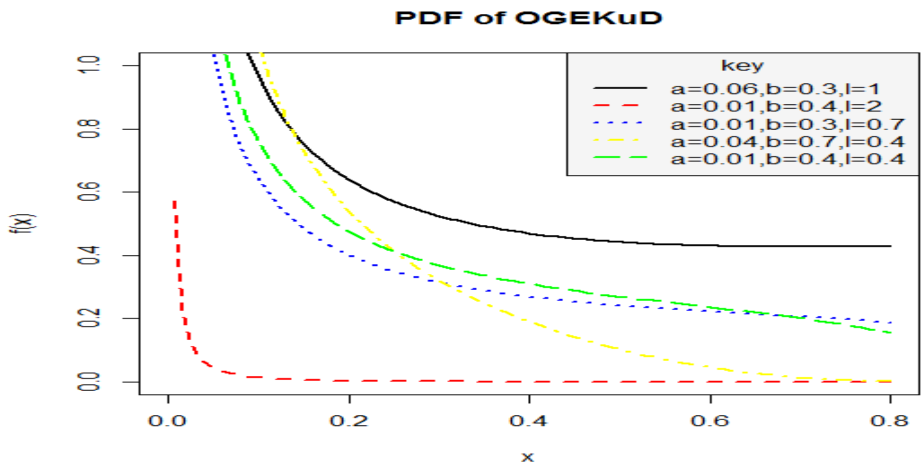

Figure 1: *PDF* of the *OGEKD* for different values of a, b, l where $l = \lambda$ the parameters

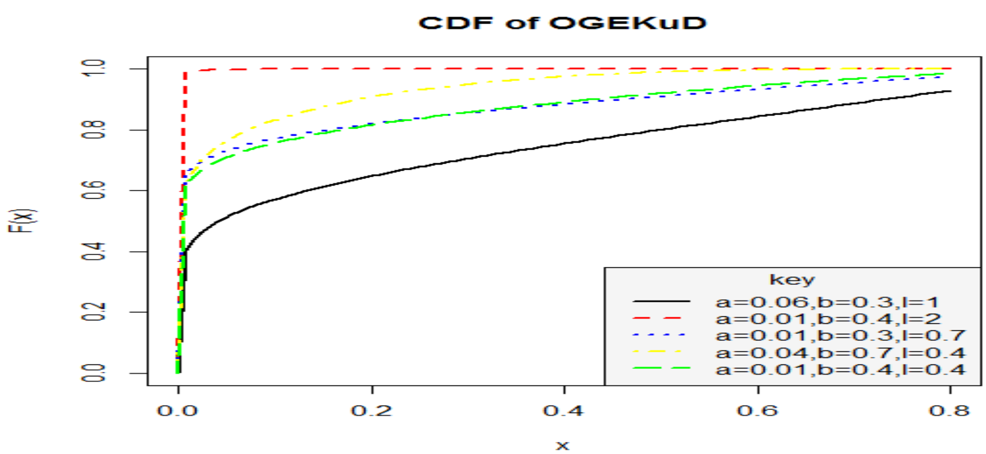

Figure2 *CDF of the OGEKD* for different values of a, b, l

LITERATURE REVIEW

Luguterah and Nasiru (2017) proposed a new distribution called the Odd-generalized Exponential Linear Exponential distribution and derived some of its mathematical properties which include the Moments, and order statistics. In addition, a real data set revealed that the model can be used as a better fit than its sub-models. In a similar way Rosaiah *et al.* (2016) proposed a new life time model, called the Odd-generalized Exponential Log-Logistic distribution(OGELLD). Some of its properties were derived and some structural properties of the new distribution were studied. Method of maximum likelihood was used in estimating the parameters and the Fisher's information matrix was derived. The proposed model was illustrated by a real data set and it was found useful.

Maiti and Premanik (2015) studied a new probability distribution called Odds Generalized Exponential - Exponential Distribution as a particular case of T-X family of distributions proposed by Alzaatreh *et al.* (2013). The structural and reliability properties of this distribution have been studied and inference on parameters were made. The proposed distribution was compared with some standard distributions with two parameters through simulation study and the superiority of the proposed distribution was established. The appropriateness of fitting the odds generalized exponential - exponential distribution has also been established by analyzing a real-life data set.

PROPERTIES

The Moment

In this subsection, we will derive the r^{th} moments of the OGE-K distribution as an infinite series expansion.

Let X denote a continuous random variable, the n^{th} moment of X is given by;

$$\mu_n^{'} = E\left[X^n\right] = \int_0^\infty x^n f(x)dx$$

where *f(x)* the *pdf* of the *OGEKD* is as given in equation (6)

$$f(x) = ab\lambda x^{a-1}\left(1-x^a\right)^{-(b+1)} \exp\left\{-\lambda\left(\left(1-x^a\right)^{-b} - 1\right)\right\} \tag{7}$$

By expanding the exponential term in (7) using power series, we obtain:

$$\exp\left\{-\lambda\left[\frac{1-\left(1-x^a\right)^b}{1-\left(1-\left(1-x^a\right)^b\right)}\right]\right\} = \sum_{k=0}^{\infty}\frac{(-1)^k(\lambda)^k}{k!}\left[\frac{1-\left(1-x^a\right)^b}{1-\left(1-\left(1-x^a\right)^b\right)}\right]^k$$

$$\exp\left\{-\lambda\left[\frac{1-\left(1-x^a\right)^b}{1-\left(1-\left(1-x^a\right)^b\right)}\right]\right\} = \sum_{k=0}^{\infty}\frac{(-1)^k(\lambda)^k}{k!}\frac{\left(1-\left(1-x^a\right)^b\right)^k}{\left(1-x^a\right)^{bk}} \tag{8}$$

Making use of the result in (7) above, equation (8) becomes

$$f(x) = ab\lambda x^{a-1}\left(1-x^a\right)^{-(b+1)}\sum_{k=0}^{\infty}\frac{(-1)^k(\lambda)^k}{k!}\frac{\left(1-\left(1-x^a\right)^b\right)^k}{\left(1-x^a\right)^{bk}}$$

$$f(x) = \sum_{k=0}^{\infty}\frac{(-1)^k(\lambda)^{k+1}}{k!}ab x^{a-1}\left(1-x^a\right)^{bk-(b+1)}\left(1-\left(1-x^a\right)^b\right)^k \tag{9}$$

using the binomial theorem, we can write the last term from the (9)

$$\left[1-\left(1-x^a\right)^b\right]^k = \sum_{j=0}^{k}(-1)^j\binom{k}{j}\left(1-x^a\right)^{bj} \tag{10}$$

Again, making use of the expansion in (10) a, (9) can be written as (11)

$$f(x) = \sum_{k=0}^{\infty}\frac{(-1)^k(\lambda)^{k+1}}{k!}ab x^{a-1}\left(1-x^a\right)^{bk-(b+1)}\sum_{j=0}^{k}(-1)^j\binom{k}{j}\left(1-x^a\right)^{bj}$$

(11)

$$f(x) = \sum_{j=0}^{k}\sum_{k=0}^{\infty}\frac{(-1)^{j+k}(\lambda)^{k+1}}{k!}\binom{k}{j}ab x^{a-1}\left(1-x^a\right)^{bj-bk-b-1}$$

$$f(x) = W_{j,k}ab x^{a-1}\left(1-x^a\right)^{b(j-k-1)-1} \tag{12}$$

where

$$W_{j,k}=\sum_{j=0}^{k}\sum_{k=0}^{\infty}\frac{(-1)^{j+k}(\lambda)^{k+1}}{k!}\binom{k}{j}$$

Hence,

$$\mu_n^{'}=E\left[X^n\right]=\int_0^1 x^n f(x)dx=\int_0^1 W_{j,k}ab\,x^{a-1}\left(1-x^a\right)^{b(j-k-1)-1}dx$$

$$\mu_n^{'}=E\left[X^n\right]=\int_0^1 x^n f(x)dx=W_{j,k}ab\int_0^1 x^{a-1}\left(1-x^a\right)^{b(j-k-1)-1}dx \tag{13}$$

for the Kumaraswamy distribution;

$$E\left[X^r\right]=\int_0^1 x^n f(x)dx=ab\int_0^1 x^{n+a-1}\left(1-x^a\right)^{b-1}dx=B\lambda\left(\tfrac{n}{a}+1,b\right)$$

this implies that

$$\mu_n^{'}=E\left[X^n\right]=\int_0^1 x^n f(x)dx=W_{j,k}ab\int_0^1 x^{a-1}\left(1-x^a\right)^{b(j-k-1)-1}dx$$

$$\mu_n^{'}=E\left[X^n\right]=W_{j,k}bB\left(\tfrac{n}{a}+1,b\left(j-k-1\right)\right) \tag{14}$$

This completes the proof.

The Mean

The mean of the *OGEKD* can be obtained from the n^{th} moment of the distribution when *n=1* as follows:

$$\mu_n^{'}=E\left[X^n\right]=W_{j,k}bB\left(\tfrac{n}{a}+1,b\left(j-k-1\right)\right)$$

$$\mu_1^{'}=E\left[X\right]=W_{j,k}bB\left(\tfrac{1}{a}+1,b\left(j-k-1\right)\right) \tag{15}$$

Also the second moment of the *OGEKD* is obtained from the n^{th} moment of the distribution when *n=2* as

$$\mu_1^{'}=E\left[X^2\right]=W_{j,k}bB\left(\tfrac{2}{a}+1,b\left(j-k-1\right)\right) \tag{16}$$

The Variance

The n^{th} central moment or moment about the mean of *X*, say μ_n, can be obtained as

$$\mu_n=E\left[X-\mu_1^{'}\right]^n=\sum_{i=0}^{n}(-1)^i\binom{n}{i}\mu_1^{'i}\mu_{n-i}^{'} \tag{17}$$

The variance of *X* for *OGEKD* is obtained from the central moment when *n=2,* that is,

$$Var(X)=E[X^2]-\left\{E[X]\right\}^2$$

$$Var(X)=W_{j,k}bB\left(\tfrac{2+a}{a},b\left(j-k-1\right)\right)-\left\{W_{j,k}bB\left(\tfrac{1+a}{a},b\left(j-k-1\right)\right)\right\}^2 \tag{18}$$

Moment Generating Function

The moment generating function of a random variable *X* can be obtained by

$$M_x(t) = E\left[e^{tx}\right] = \int_0^\infty e^{tx} f(x)dx \tag{19}$$

Recall that by power series expansion,

$$e^{tx} = \sum_{n=0}^{\infty} \frac{(tx)^n}{n!} = \sum_{n=0}^{\infty} \frac{t^n}{n!} x^n \tag{20}$$

Using the result in equation (20) and simplifying the integral in (19), gives;

$$M_x(t) = E\left[e^{tx}\right] = \sum_{n=0}^{\infty} \frac{(tx)^n}{n!} = \sum_{n=0}^{\infty} \frac{t^n}{n!} \int_{-\infty}^{\infty} x^n f(x)dx$$

$$M_x(t) = E\left[e^{tx}\right] = \sum_{n=0}^{\infty} \frac{(tx)^n}{n!} = \sum_{n=0}^{\infty} \frac{t^n}{n!} \mu_n' \tag{21}$$

where *n* and *t* are constants, *t* is a real number and μ_n' denotes the n^{th} ordinary moment of *X* and can be obtained from equation (13) as stated previously.

Characteristic Function

The characteristics function of a random variable *X* is given by;

$$\varphi_x(t) = E\left[e^{itx}\right] = E\left[\cos(tx) + i\sin(tx)\right] = E\left[\cos(tx)\right] + E\left[i\sin(tx)\right] \tag{22}$$

Recall from power series expansion that

$$\cos(tx) = \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n}}{(2n)!} x^{2n}$$

$$E\left[\cos(tx)\right] = \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n}}{(2n)!} \mu_{2n}'$$

And also, that

$$\sin(tx) = \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n+1}}{(2n+1)!} x^{2n+1}$$

$$E\left[\sin(tx)\right] = \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n+1}}{(2n+1)!} \mu_{2n+1}'$$

Simple algebra and power series expansion proves that

$$\phi_x(t) = \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n}}{(2n)!} \mu_{2n}' + i\sum_{n=0}^{\infty} \frac{(-1)^n t^{2n+1}}{(2n+1)!} \mu_{2n+1}' \tag{22}$$

Where μ_{2n}' and $\mu_{2n+1}'$are the moments of *X* for *n*=2*n* and *n*=2*n*+1 respectively and can be obtained from μ_n' in equation (13)

Quantile Function

The Quantile function is obtained by inverting of the *cdf*

Let $Q(u) = F^{-1}(u)$ be the quantile function (qf) of $F(x)$ for $0 < u < 1$.
Taking $F(x)$ to be the *cdf* of the *OGEKD* and inverting it as above will give us the quantile function as follows.

$$F(x) = 1 - \exp\left\{-\lambda\left(\left(1-x^a\right)^{-b} - 1\right)\right\}$$

Inverting $F(x) = u$

$$F(x) = 1 - \exp\left\{-\lambda\left(\left(1-x^a\right)^{-b} - 1\right)\right\} = u \tag{23}$$

Simplifying equation (22) above, gives:

$$Q(u) = X_q = \sqrt[a]{\left\{1 - \left[1 + \left(\frac{1}{\lambda}\ln\left(\frac{1}{1-u}\right)\right)\right]^{-\frac{1}{b}}\right\}} \tag{24}$$

Reliability Analysis

Survival Function

Mathematically, the survival function is given by:

$$S(x) = 1 - F(x) \tag{25}$$

Considering that $F(x)$ is the *cdf* of the proposed *OGEKD*, substituting and simplifying, we obtain;

$$S(x) = \exp\left\{-\lambda\left(\left(1-x^a\right)^{-b} - 1\right)\right\} \tag{26}$$

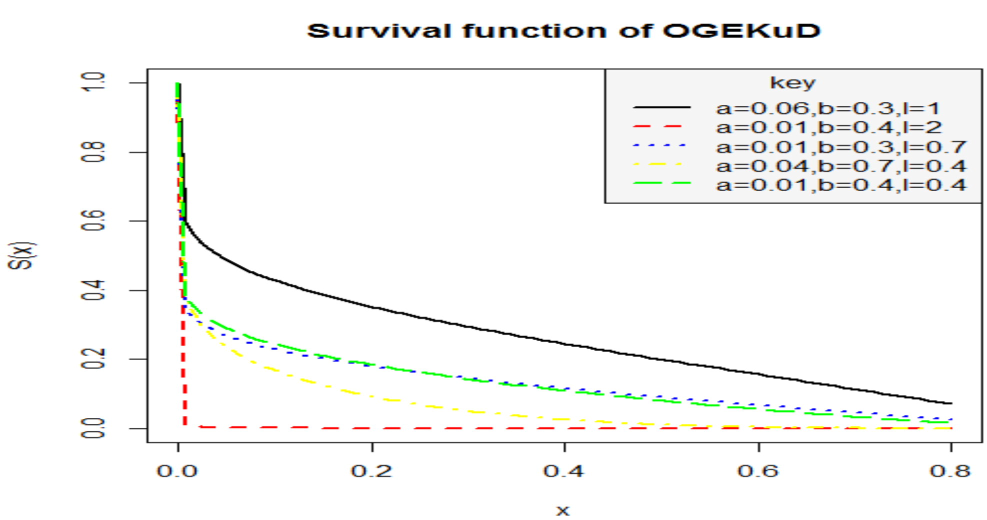

Figure3: the survival function of the *OGEKD* at different values of the parameters.

Hazard Function

The hazard function is defined as;

$$h(x)=\frac{f(x)}{1-F(x)}=\frac{f(x)}{S(x)} \tag{27}$$

Taking *f(x)* and *F(x)* to be the *pdf* and *cdf* of the proposed *OGEKD* given (5) and (6)

Substituting for *f(x)* and *F(x)* in (26) and simplifying gives

$$h(x)=ab\lambda x^{a-1}\left(1-x^{a}\right)^{-(b+1)} \tag{28}$$

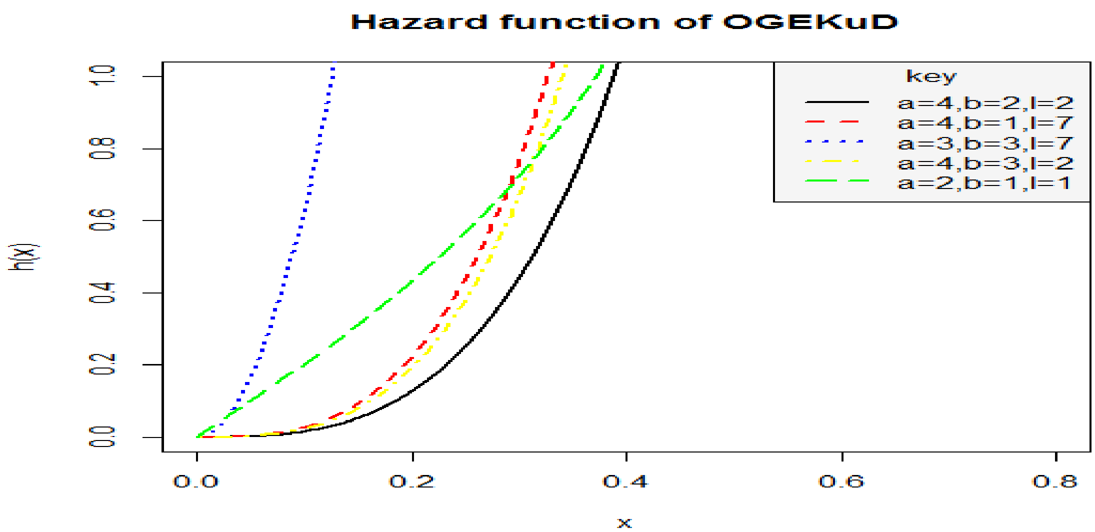

Figure4 the hazard function of the *OGEKD* at different values of the parameters.

Order Statistics

Suppose $X_1, X_2, \ldots\ldots, X_n$ is a random sample from a distribution with *pdf*, *f(x)*, and let $X_{1:n}, X_{2:n}, \ldots\ldots, X_{i:n}$ denote the corresponding order statistic obtained from this sample. The *pdf*, $f_{i:n}(x)$ of the i^{th} order statistic can be defined as;

$$f_{i:n}(x)=\frac{n!}{(i-1)!(n-i)!}f(x)F(x)^{i-1}\left[1-F(x)\right]^{n-i} \tag{29}$$

where *f(x)* and *F(x)* are the *pdf* and *cdf* of the *OGEKD* respectively.
Using (5) and (6) in (29), the *pdf* of the a^{th} order statistics $X_{a:n}$, can be expressed from (30) as;

$$f_{a:n}(x)=\frac{n!}{(a-1)!(n-a)!}\sum_{k=0}^{n-a}(-1)^{k}\binom{n-a}{k}\left[ab\lambda x^{a-1}\left(1-x^{a}\right)^{-(b+1)}\exp\left\{-\lambda\left(\left(1-x^{a}\right)^{-b}-1\right)\right\}\right]*\left[1-\exp\left\{-\lambda\left(\left(1-x^{a}\right)^{-b}-1\right)\right\}\right]^{a+k-1} \tag{30}$$

Hence, the *pdf* of the minimum order statistic $X_{(1)}$ and maximum order statistic $X_{(n)}$ of the *OGEKD* are given by;

$$f_{1:n}(x)=n\sum_{k=0}^{n-1}(-1)^{k}\binom{n-1}{k}\left[ab\lambda x^{a-1}\left(1-x^{a}\right)^{-(b+1)}\exp\left\{-\lambda\left(\left(1-x^{a}\right)^{-b}-1\right)\right\}\right]*\left[1-\exp\left\{-\lambda\left(\left(1-x^{a}\right)^{-b}-1\right)\right\}\right]^{k} \tag{31}$$

and

$$f_{n:n}(x)=n\left[ab\lambda x^{a-1}\left(1-x^{a}\right)^{-(b+1)}\exp\left\{-\lambda\left(\left(1-x^{a}\right)^{-b}-1\right)\right\}\right]\left[1-\exp\left\{-\lambda\left(\left(1-x^{a}\right)^{-b}-1\right)\right\}\right]^{n-1} \quad (32)$$

Respectively.

Parameter Estimation

Let $X_1, \ldots, X_n$ be a sample of size 'n' independently and identically distributed random variables from the *OGEKD* with unknown parameters *α, b* and *λ* defined previously. The *pdf* of the *OGEKD* is given as

$$f(x)=ab\lambda x^{a-1}\left(1-x^{a}\right)^{-(b+1)}\exp\left\{-\lambda\left(\left(1-x^{a}\right)^{-b}-1\right)\right\}$$

The likelihood function is given by;

$$L\left(X_1, X_2, \ldots\ldots, X_n / a,b,\lambda\right)=\left(ab\lambda\right)^{n}\sum_{i=1}^{n}x_i^{\,a-1}\exp\left\{-\lambda\sum_{i=1}^{n}\left(\left(1-x_i^{\,a}\right)^{-b}-1\right)\right\} \quad (33)$$

Let the log-likelihood function, $l\left(\theta\right)=\log L\left(X_1, X_2, \ldots\ldots, X_n / a,b,\lambda\right)$, therefore

$$l=n\log a+n\log b+n\log\lambda+(a-1)\sum_{i=1}^{n}\log x_i-\lambda\sum_{i=1}^{n}\left(\left(1-x_i^{\,a}\right)^{-b}-1\right) \quad (34)$$

Differentiating *l* partially with respect to *a*, *b* and λ respectively gives;

$$\frac{\partial l}{\partial a}=\frac{n}{a}+\sum_{i=1}^{n}\log x_i-b\lambda\sum_{i=1}^{n}\left\{\left(1-x_i^{\,a}\right)^{b}x^{a}\log x\right\} \quad (35)$$

$$\frac{\partial l}{\partial b}=\frac{n}{b}+\lambda\sum_{i=1}^{n}\left\{\left(1-x_i^{\,a}\right)^{-b}\log\left(1-x_i^{\,a}\right)\right\} \quad (36)$$

$$\frac{\partial l}{\partial \lambda}=\frac{n}{\lambda}-\sum_{i=1}^{n}\left\{\left(1-x_i^{\,a}\right)^{-b}-1\right\} \quad (37)$$

Now, solving the equations $\frac{dl}{da}=0,\ \frac{\partial l}{\partial b}=0,$ and $\frac{\partial l}{\partial \lambda}=0$ will give the maximum likelihood estimates (*MLEs*), $\hat{a},\hat{b}, and\,\hat{\lambda}$ of parameters $a,b, and\,\lambda$ respectively.

APPLICATION

Two data-set are used to demonstrate the proposed distribution is flexible and better to fit lifetime data in this section. The first data is flood data with 20 observations obtained from Dumonceaux and Antle (1973). This data is as follows: 0.265 0.269 0.297 0.315 0.324 0.338 0.379 0.379 0.392 0.402 0.412 0.416 0.41 0.423 0.449 0.484 0.494 0.613 0.65 0.740 and the second data set is on shape measurements of 48 rock samples from a petroleum reservoir. This data was extracted from BP research, image analysis by Ronit Katz, u Oxford. This data is given as follows: 0.0903296 0.189651 0.228595 0.200071 0.280887 0.311646 0.176969 0.464125 0.148622 0.164127 0.231623 0.144810 0.179455 0.276016 0.438712 0.420477 0.183312 0.203654 0.172567 0.113852 0.191802 0.19753 0.163586 0.200744 0.117063 0.162394 0.153481 0.291029 0.133083 0.326635 0.253832 0.262651 0.122417 0.150944 0.204314

0.240077 0.225214 0.154192 0.328641 0.182453
0.167045 0.148141 0.262727 0.161865 0.341273
0.276016 0.230081 0.200447.

Table 4. 1: Summary of the two data-set

Parameters	Data set I	Data set II
n	20	48
Minimum	0.265	0.0903
Q_1	0.3345	0.1623
Median	0.4070	0.1988
Q_3	0.4578	0.2627
Mean	0.4232	0.2181
Maximum	0.7400	0.4641
Variance	0.0157	0.0069
Skewness	1.0677	1.1694
Kurtosis	0.5999	1.1099

In order to compare the models above with the proposed OGEK, we consider criteria like log likelihood (LL), Akaike Information Criterion (AIC), Consistent Akaike Information Criterion (CAIC) and Bayesian information criterion (BIC) for the data sets. The better distribution corresponds to smaller LL, AIC, AICC and BIC values of these statistics.

Table1 and 2 lists the MLEs, and the statistics and p values for the flood's data and data on shape measurements of 48 rock samples from a petroleum reservoir respectively. The tables indicate that the OGEK distribution has the lowest values for the AIC, BIC and CAIC statistics among the fitted models, and therefore it could be chosen as the best model.

Table 1: Performance of the distribution using data-set I

Distributions	**Parameter estimates**	**ll= (minus log-likelihood value)**	***AIC***	***CAIC***	***BIC***	**Ranks of model's performance**
OGEKD (proposed)	$\hat{a}$=4.9107 $\hat{b}$=-0.0454 $\hat{\lambda}$=-612.5154	-12240.38	-24474.76	-24476.86	-24473.26	**1**
TKD (khan et-al 2016)	$\hat{a}$=3.0703 $\hat{b}$=1.3365 $\hat{\lambda}$=31.2329	-52.4474	-98.8948	-100.9917	-97.3948	**2**

Distributions	**Parameter estimates**	**ll= (minus log-likelihood value)**	***AIC***	***CAIC***	***BIC***	**Ranks of model's performance**
EKD (Lemonte et-al 2013)	$\hat{a}$=1.5527 $\hat{b}$=6.6302 $\hat{\gamma}$=4.7419	-14.6387	-23.2774	-25.3743	-21.7774	**3**
KD (kumaraswamy 1980)	$\hat{a}$=3.2412 $\hat{b}$=10.5889	-12.8416	-21.6832	-23.0811	-20.9773	**5**
KKD (El-Sherpieny and Ahmed 2014)	$\hat{a}$=1.3732 $\hat{b}$=5.7143 $\hat{\alpha}$=5.5816 $\hat{\beta}$=1.0939	-14.6783	-21.3566	-24.1525	-18.6899	**4**

Table 2: Performance of the distribution using data set II.

Distributions	Parameter estimates	-ll=(minus log-likelihood value)	*AIC*	*CAIC*	*BIC*	Ranks of model's performance
OGEKD (proposed)	$\hat{a}$=3.3205 $\hat{b}$=-0.4713 $\hat{\lambda}$=-705.6697	-33875.29	-67744.58	-67745.54	-67744.03	**1**
TKD (khan et-al 2016)	$\hat{a}$=1.9668 $\hat{b}$=2.6799 $\hat{\lambda}$=27.4026	-142.9759	-279.9518	-280.9081	-279.4063	**2**
EKD (Lemonte et-al 2013)	$\hat{a}$=0.9364 $\hat{b}$=6.3994 $\hat{\gamma}$=3.8207	-49.9166	-93.8332	-94.7895	-93.2877	**4**
KD (kumaraswamy 1980)	$\hat{a}$=1.8978 $\hat{b}$=12.5398	-47.3652	-90.7304	-91.3679	-90.4637	**5**
KKD (El-Sherpieny and Ahmed 2014)	$\hat{a}$=1.1446 $\hat{b}$=4.9263 $\hat{\alpha}$=3.5243 $\hat{\beta}$=3.8411	-54.6854	-101.3708	-102.6458	-100.4406	**3**

CONCLUSION

A new distribution has been proposed. Some mathematical and statistical properties of the proposed distribution have been studied appropriately. The derivations of some expressions for its moments, moment generating function, characteristics function, survival function, hazard function, quantile function and ordered statistics has been done appropriately. We also estimated the parameters of the proposed distribution via the method of maximum likelihood estimation technique. An application of the OGEK distribution to a real data set indicates that this distribution outperforms both the Kumaraswamy and other generalized distributions.

RECOMMENDATIONS

The findings of this research recommend that the proposed distribution should be used to model positively skewed data sets with higher peak. It was also found that the distribution can be used to model age dependent events, systems, components or random variables.

REFERENCES

Adepoju, K. A. and Chukwu, O. I. (2015). Maximum Likelihood Estimation of the kumaraswamy Exponential Distribution with Applications. *Journal of Modern Applied Statistical Methods. 14, (1).*

Alzaghal, A., Famoye. F. and Carl L. (2013). Exponentiated *T-X* Family of Distributions with Some Applications. *International Journal of Statistics and Probability, 2 (1927-7032).*

Alzaatreh, A., Lee, C., & Famoye, F. (2013). A new method for generating families of continuous distributions. *Metron*, *71*(1), 63-79.

Kumaraswamy. P. (19880). A generalized probability density function for double-bounded random processes, *Journal of Hydrology* 46, pp. 79–88.

Kumaraswamy P. (1976) Sinepower probability density function. *Journal of Hydrology 31:181–184*

Kumaraswamy P. (1978) Extended sine power probability density function. *Journal of Hydrology 37:81–89*

Luguterah, A. and Nasiru, S. (2017). The Odd Generalized Exponential Generalized Linear Exponential Distribution. *Journal of Statistics Applications & Probability,6(1),139-148.*

Maiti, S. S. and Pramanik, S. (2015). Odds Generalized Exponential – Exponential Distribution. *Journal of Data Science 13, (733-754).*

Oguntunde, P. E., Odetunmibi, O. A., Okagbue, H. I., Babatunde, O. S. and Ugwoke, P. O. (2015). The Kumaraswamy-Power Distribution: A Generalization of the Power distribution. *International Journal of Mathematical Analysis 9, 637-645.*

Rosaiah, K., Rao, G. S., Sivakumar D. C. U., and Kalyani, K (2016). The Odd Generalized Exponential Log Logistic Distribution International. *Journal of Mathematics and Statistics Invention (IJMSI). 4, (5), 21-29.*

