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A decomposition approach for magnetohydrodynamics stagnation point flow over an inclined shrinking/stretching sheet with suction/injection

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Abstract

In this paper, the approximate solution to Magnetohydrodynamics Stagnation Point Flow over an inclined Shrinking/Stretching Sheet with Suction/injection was analyzed via the Adomian Decomposition. The governing partial differential equations (PDEs) were reduced with the help of similarity variables to non linear coupled ordinary differential equations (ODEs). The effects of various pertinent parameters were presented numerically and graphically. Numerical comparisons were carried out with the existing literature and a good agreement was established. The angle of inclination was found to enhance the velocity profile.

Keywords: Angle of inclination; stagnation point; magnetohydrodynamics; adomian decomposition method.

1 Introduction

Magnetohydrodynamic flows with or without the movement of heat in an electrically conducting fluids have attracted a large interest in the context of metallurgical fluid dynamics, aerothermodynamics, astronautics, geophysics, nuclear engineering and applied mathematics. Carrier and Greenspan (1960) considered unsteady hydromagnetic flows past a semi-infinite flat plate moving impulsively in its own plane. Gupta (1960) considered unsteady magneto-convection under buoyancy forces. Singer (1965) carried out further study on unsteady free convection heat transfer with magnetohydrodynamic effects in a channel regime. Pop (1969) works on transient buoyancy-driven convective hydromagnetics from a vertical surface. Tokis (1986) implored the Laplace transforms to analyze the three dimensional free-convection hydromagnetic flows near an infinite vertical plate moving in a rotating fluid when the plate temperature undergoes a thermal transient. The influence of oscillatory pressure gradient on transiently rotating hydromagnetic flow was reported by Ghosh (1993).

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Abd-El Aziz (2006) carried out a study on the thermal radiation flux effects on unsteady Magnetohydrodynamics micropolar fluid convection. Ogulu and Prakash (2006) in their work, presented an analytical solutions for variable suction and radiation effects on dissipative-free convective, optically-thin, Magnetohydrodynamic flow using a differential approximation to describe the radiative flux. Recent studies involving thermal radiation and transient hydromagnetic convection with specie transfer and viscous heating can be found in the analyses of Prasad et al. (2006) and Zueco (2007). In many geophysical and metallurgical flows, porous medium can arise also. Classically, the Darcian model is used to showcase the bulk effects of porous materials on flow dynamics and is valid for Reynolds numbers based on the pore radius. Chamkha (1996) works on the transient-free convection Magnetohydrodynamic boundary layer flow in a fluid-saturated porous medium channel, and later Chamkha (2001) extended the study to consider the influence of temperature-dependent properties and inertial effects on the convection regime. B´eg et al. (2005) presented perturbation solutions for the transient oscillatory hydromagnetic convection in a Darcian porous media with present of heat source. Chaudhary and Jain (2008) carried out the influence of oscillating temperature on Magnetohydrodynamic convection heat transfer past a vertical plane in a Darcian porous medium. Lately, Variational iteration method was applied for squeezing MHD Nano fluid flow in a rotating channel with the lower stretching porous surface, see Shahmohamadi and Rashidi (2016) for example. More extensive works as contained in the works of Mishra and Bhatti (2017), Rashidi et al. (2014), Sheikholeslami and Bhatti (2017), Abbas et al. (2017) and Bhatti and Rashidi (2016).

The Adomian decomposition method was introduced by the American mathematician, George Adomian. It is based on the search for a solution in the form of a series and on decomposing the nonlinear operator into a series in which the terms are calculated recursively using Adomian polynomials (Adomian 1994). There are many merit of this technique over classical techniques. It avoids perturbation in order to find solutions of given nonlinear equation. This method provides an accurate approximation of the solution. As a main advantage of this Method over traditional numerical methods, the decomposition procedure of Adomian does not require discretization of the solution. Therefore, unlike other numerical methods, this method does not result in any large system of linear or nonlinear equations. Thus, it is not affected by computation round off errors and the solution is found without taking a long time and a large amount of computer memory.

This study is a new advancement in the literature in which a decomposition approach for Magnetohydrodynamics stagnation point flow over an inclined shrinking/stretching sheet with suction/injection is presented.

2 Problem formulation

Considering a steady two-dimensional incompressible, non rotational flow of a Magnetohydrodynamics fluid, near a stagnation point over a permeable shrinking/stretching sheet coinciding with the plane at z=0. Following the work of Bhatti *et al.* (2018) in an inclined plane at angle θ, the governing equations for continuity and momentum equation can be written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial P}{\partial x}+\mu\frac{\partial^2 u}{\partial z^2}-\sigma B_0^2\left(u-u_0\right)+\rho gSin\theta\,, \tag{2}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial P}{\partial y}+\mu\frac{\partial^2 v}{\partial z^2}-\sigma B_0^2\left(v-v_0\right)+\rho gSin\theta\,, \tag{3}$$

$$\rho\left(u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial P}{\partial z}+\mu\frac{\partial^2 w}{\partial z^2}\,, \tag{4}$$

subject to the boundary condition:

$$\left.\begin{array}{l} u=v=w=0,\ z\prec 0 \\ u=bx,\ \ v=cx,\ \ w=-S,\ \ z=0 \\ u_0=ax,\ \ v=0,\ \ w=-az,\ \ z\to\infty \end{array}\right\} \tag{5}$$

$P=P_0-\dfrac{1}{2}a^2\rho x^2-\dfrac{1}{2}\rho w^2+\rho\upsilon\dfrac{\partial w}{\partial x}$, where P_0 is the stagnation pressure.

The velocity along the x, y and z axes are respectively u, v and w ρ is the density, μ is the viscosity, a is the strength of the stagnation flow, b is the stretching rate ($b\prec 0$), and $-c$ is the location of the shrinking origin, υ is the kinematic viscousity, σ is the electrical conductivity, B_0 external magnetic field and S is the suction.

The similarity variables are defined as follows:

$$\eta=\sqrt{\frac{a}{v}}z\,,\qquad u=axf^{\prime}\,,\ v=cxh\,,\ \text{and}\ \ w=-\sqrt{av}\,f\left(\eta\right) \tag{7}$$

where η, $f\left(\eta\right)$, and $h\left(\eta\right)$ are the non-dimensional distance, velocity and function *h*.

Introducing the transformation in (7) into (2) to (4), we obtained the simplified form as

$$\left.\begin{array}{l} f^{\prime\prime\prime}+ff^{\prime\prime}-f^{\prime 2}+1-M\left(f^{\prime}-1\right)+KS\operatorname{in}\theta=0 \\ h^{\prime\prime}-f^{\prime}h+fh^{\prime}-Mh+KSin\theta=0 \end{array}\right\}$$

(8)

with corresponding boundary conditions:

$$\left.\begin{array}{l} \eta=0:\ f\left(0\right)=k,\ \ f^{\prime}\left(0\right)=\alpha \\ \eta\to\infty:\ f^{\prime}\left(\infty\right)=1,\ \ h\left(\infty\right)=0 \end{array}\right\}$$

(9)

in which : $M=\frac{\sigma B_0^2}{\rho a}$, $K=\frac{\rho g}{a^2 x}$, $\alpha=\frac{b}{a}$ $k=\frac{S}{\sqrt{av}}$ are the Hartmann number, gravitational Parameter, shrinking/stretching parameter, and Suction/injection parameter respectively.

3 Implementation of improved Adomian Decomposition Method

We start by introducing the improved Adomian decomposition method to get the solution by letting

$\frac{d^3}{d\eta^3}=L\left[f(\eta)\right]$ and $\frac{d^2}{d\eta^2}=L\left[h(\eta)\right]$ so that equations (8) becomes

$$\left.\begin{array}{l} L_1\left[f(\eta)\right]=L_1^{-1}\left[-ff^{//}+f^{/2}-1+M\left(f^{/}-1\right)-KS\,\mathrm{in}\,\theta\right] \\ L_2\left[h(\eta)\right]=L_2^{-1}\left[f^{/}h-fh^{/}+Mh-KSin\theta\right] \end{array}\right\}$$

(10)

where $L_1^{-1}=\iiint(\bullet)\,d\eta d\eta d\eta$ and $L_2^{-1}=\iint(\bullet)\,d\eta d\eta$

(11)

Discretizing equation (10), we obtain

$$\left.\begin{array}{l} f_n(\eta)=L_1^{-1}\left[-\sum_{k=0}^{n-1}\left(f[k](\eta)\cdot f^{/}[n-1-k](\eta)\right)+\sum_{k=0}^{n-1}\left(f^{/}[k](\eta)\cdot f^{/}[n-1-k](\eta)\right)+M\sum_{k=0}^{n-1}\left(f^{/}[k](\eta)\right)\right] \\ h_n(\eta)=L_2^{-1}\left[-\sum_{k=0}^{n-1}\left(f[k](\eta)\cdot h^{/}[n-1-k](\eta)\right)+\sum_{k=0}^{n-1}\left(h[k](\eta)\cdot f^{/}[n-1-k](\eta)\right)+M\sum_{k=0}^{n-1}\left(h[k](\eta)\right)\right] \end{array}\right\} \quad (12)$$

Where
$$\left.\begin{array}{l} f_0(\eta)=k+\frac{\eta^2 f^{//}(0)}{2}+\alpha\eta-L_3^{-1}\left[1+M+KSin(\theta)\right] \\ h_0(\eta)=1+\eta h^{/}(0)-L_2^{-1}\left[KSin(\theta)\right] \end{array}\right\} \quad (13)$$

Using equation (13) as the starting points in (12), we obtained the final solution as:

$$\left.\begin{array}{l} f(\eta)=\sum_{n=1}^{4} f_n(\eta) \\ h(\eta)=\sum_{n=1}^{4} h_n(\eta) \end{array}\right\}$$

(14)

$\alpha \succ 0$	$f^{//}(0)$	$f^{//}(0)$	$f^{//}(0)$	$-h^{/}(0)$	$-h^{/}(0)$	$-h^{/}(0)$
	Wang (2008)	Bhatti et al (2018)	Present Results	Wang (2008)	Bhatti et al (2018)	Present Results
0	1.232588	1.232587765	1.232760858	0.811301	0.81130132	0.8122505037
0.1	1.14656	1.14656100	1.146776576	0.86345	0.863451660	0.8632814244
0.2	1.05113	1.051129994	1.051615715	0.91330	0.91330283	0.9103133911
0.3	-	0.94681611	0.9478629348	-	0.96111587	0.9529744453
0.5	0.71330	0.71329495	0.7166611172	1.05239	1.05145843	1.025327202
1	0	0	0	1.25331	1.25331413	1.202451078
2	-1.88731	-1.88730667	-1.986101147	1.58957	1.58956678	1.075360359
5	-10.2647	-10.2647493	-9.397856609	2.33810	2.33809899	1.965189020

Table 1: Comparison of results between Numerical and Analytical for $\alpha \geq 0$ with $M = k = K = \theta = 0$

$\alpha \succ 0$	$f^{//}(0)$	$f^{//}(0)$	$f^{//}(0)$	$-h^{/}(0)$	$-h^{/}(0)$	$-h^{/}(0)$
	Wang (2008)	Bhatti et al (2018)	Present Results	Wang (2008)	Bhatti et al (2018)	Present Results
-0.25	1.40224	1.40224081	1.402780907	0.66857	0.66857275	0.6697136814
-0.5	1.49567	1.4956676	1.496430342	0.50145	0.50144758	0.504494669
-0.75	1.48930	1.48929824	1.491565016	0.29376	0.29376251	0.2977485389
-1	1.32882	1.32881688	1.333008938	0	0	0
-1.15	1.08223	1.08223117	1.071447454	-0.29799	-0.29799548	-0.258003298
-1.2465	0.55430	0.58428167	0.6967789475	-0.99904	-0.94776590	-0.558263842

Table 2: Comparison of results between Numerical and Analytical for $\alpha \prec 0$ with $M = k = K = \theta = 0$

4 Results and discussion

In order to establish the effectiveness of the present technique, the nonlinear coupled ordinary differential equations (8) alongside the boundary conditions (11) has been solved by the improved Adomian Decomposition Method as seen in Section 3.0 and results have been compared with the existing results of Wang (2008) and Bhatti et al(2018) for different values of α in Table 1 and 2 above. There is a good agreement between the current results and the existing results.

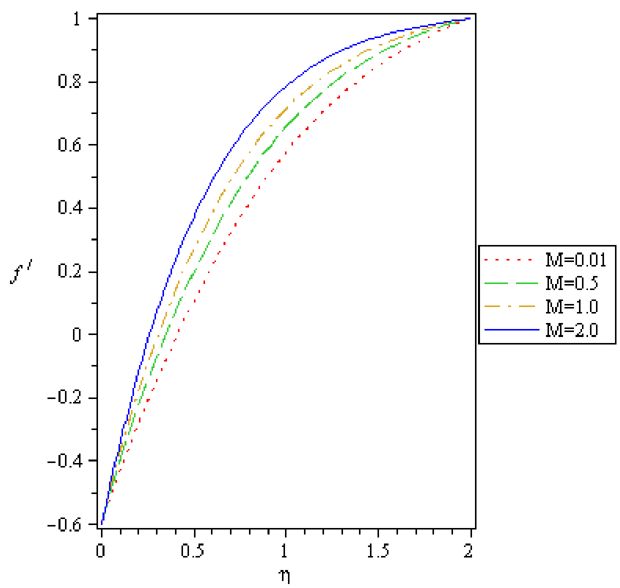

Figure 1: Variation of Hartmann number (M) on $f'(\eta)$ for α=-0.6

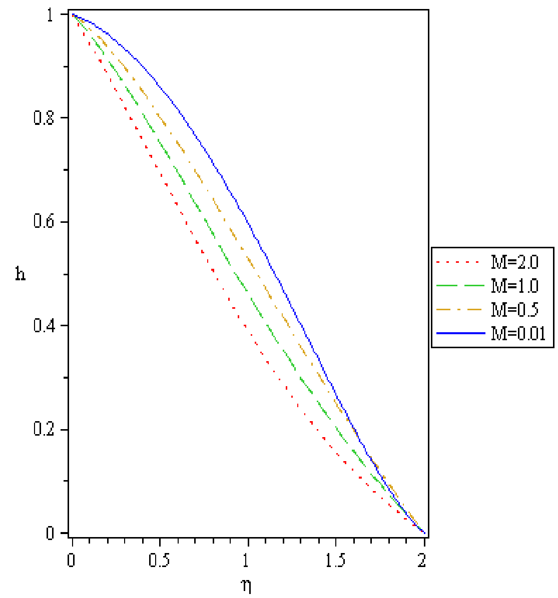

Figure 2: Variation of Hartmann number (M) 0n h(η) for α=-0.6

Figures 1 and 2 displays the variations of Hartmann number on velocity f' and *h* on the shrinking sheet and it was observed that the boundary thickness increases for f' as Hartmann number increases while it reduces for *h* as a result of the drag like forces from the Hartmann number. Figures 3 and 4 showcases the effects of Hartmann number on velocity and function *h* on the stretching sheet. Same effects were noticed as in Figures 1 and 2 respectively. Figures 5 and 6 presents the variation of shrinking parameter on velocity f' and function *h*. The shrinking parameter is observed to be a reducing agent on the velocity and an increasing agent on function *h*.

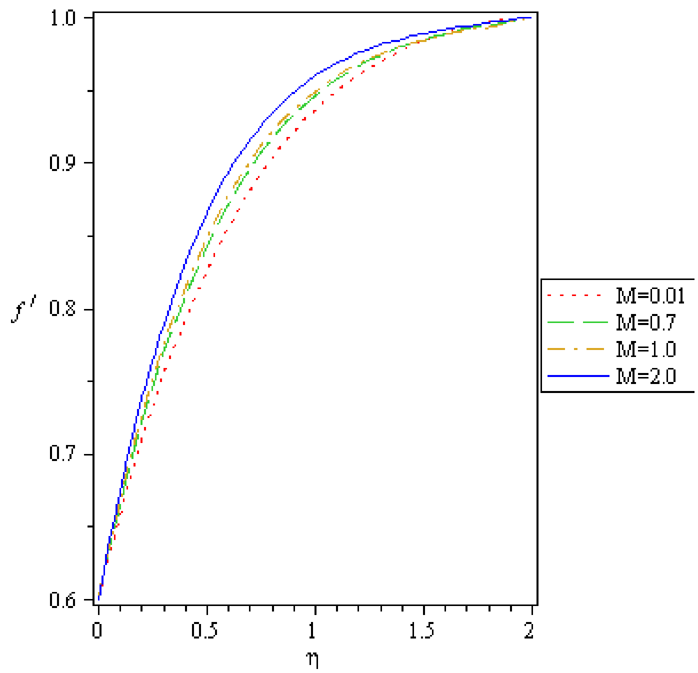

Figure 3: Variation of Hartmann number (M) on f' for α=0.6

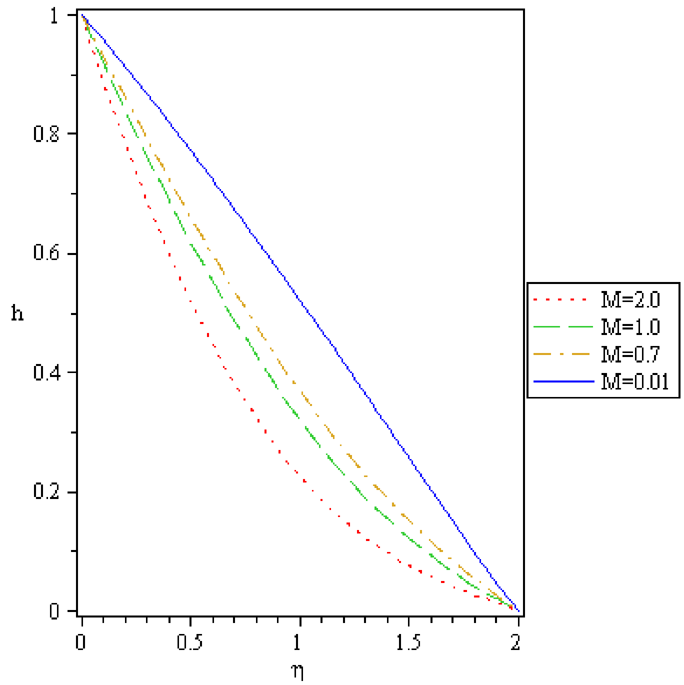

Figure 4: Variation of Hartmann number (M) on h for α=0.6

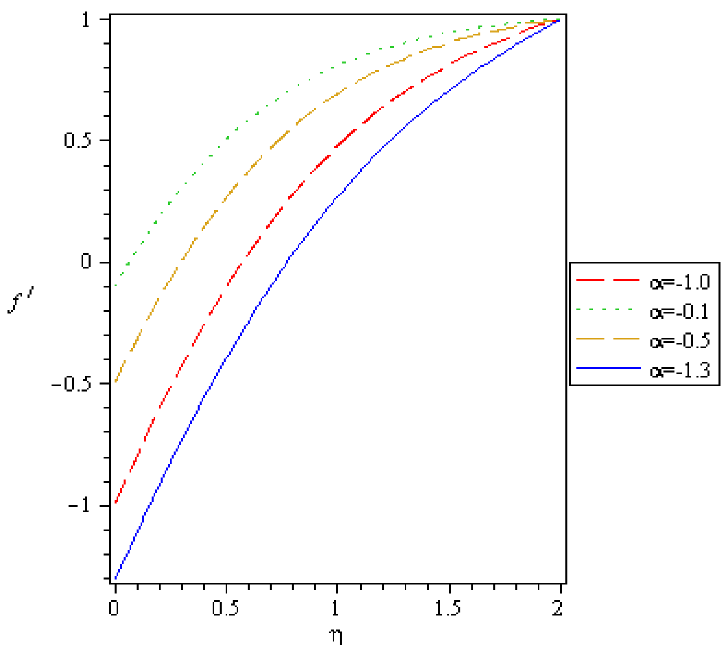

Figure 5: Variation of shrinking parameter (α) on f'

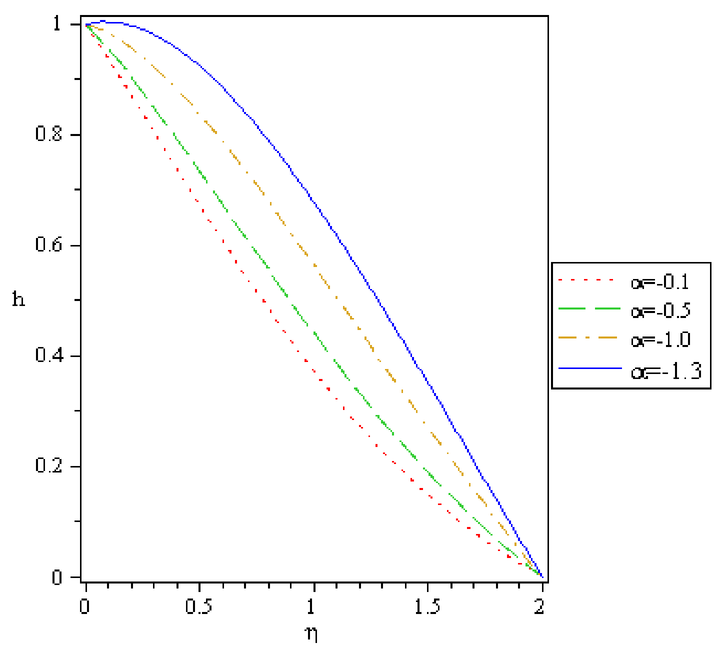

Figure 6: Variation of shrinking parameter (α) on h

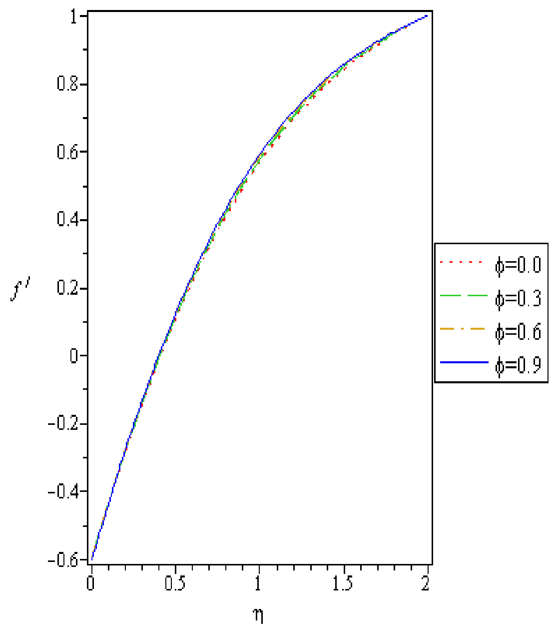

Figure 7: variaon of angle of inclination (φ) on f' for α=-0.6

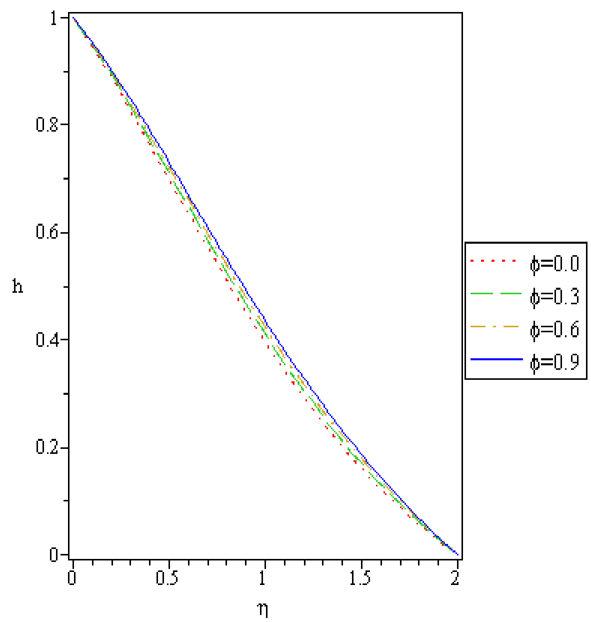

Figure 8: Variation of angle of inclination (φ) on h for α=-0.6

Figures 7 to 10 shows the variation of angle of inclination on the velocity f' and function h on the shrinking sheet and stretching sheet respectively. The angle of inclination is observed to be an increasing agent on the velocity and function h on both sheets, which is, as a result of gravity. It is also worthy to mention that the boundary layers for both the velocity and function h are observed to be thicker on the stretching sheet than shrinking sheet since stretching sheet is in favor gravity.

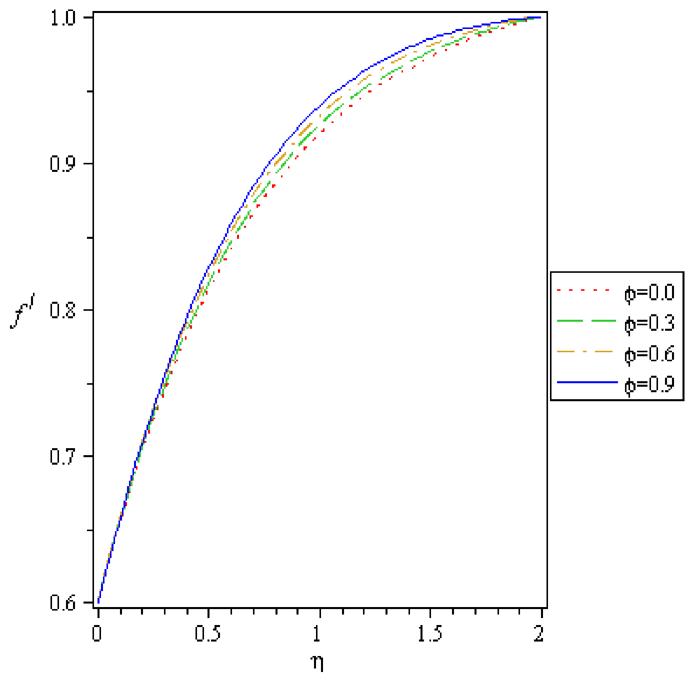

Figure 9: Variation of angle of inclination on f' for α=0.6

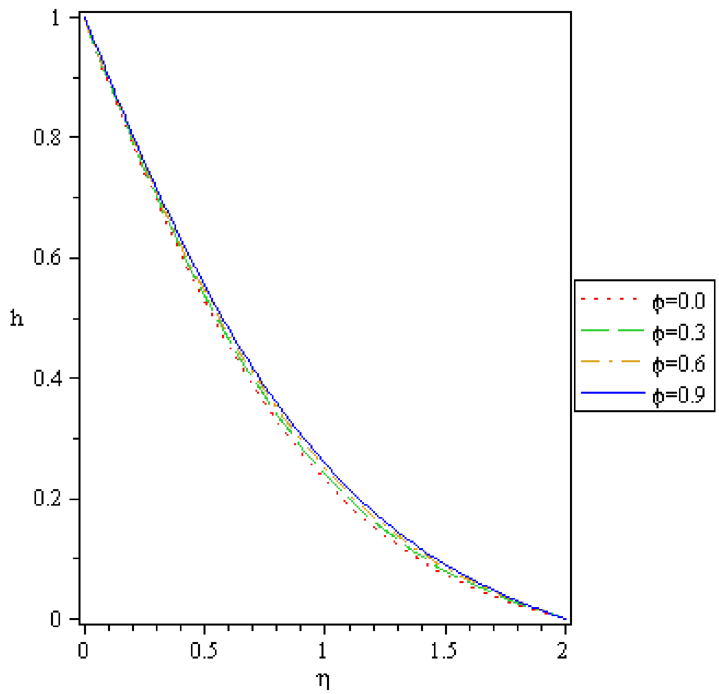

Figure 10: Variation of angle of inclination on h for α=0.6

Conclusion

This present work considered the decomposition approach for magnetohydrodynamics stagnation point flow over an inclined shrinking/stretching sheet with suction/injection by considering the work of Bhatti et al. (2018) in an inclined sheet. The PDE formulated in rectangular system was reduced to ODE via some similarity variables. The non linear coupled ODE depends on some physical parameters such as Hartmann number *(M)* , shrinking/ stretching parameter (α) and gravitational parameter K. the following observation were made:-

1. The graphs presented in this work clearly satisfy the boundary conditions, which imply that the problem is well posed.
2. The larger values of the dimensionless distance is choosing to be at $\eta = 2$.
3. The results presented in this work were compared with the results of the existing literatures as seen in Table 1 to 2 and a good agreement was established.
4. All the parameters varied on velocity and function *h* has same effects on both stretching and shrinking sheet, but the boundary layers are generally thicker on the stretching sheet.
5. All the results presented in this work conformed with reality which further depict that the problem is well posed and the efficiency of the method.
6. Unlike Wang (2008) and Bhatti et al (2018), the present method is simpler, requires no linearization of the original problem, and provide results on all points of η.

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