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Semi Analytical Method for Solving Lymphatic Filariasis Epidemic Model

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ABSTRACT: In this paper, we present a deterministic model on the transmission dynamics of Lymphatic Filariasis. Non-Standard Finite Difference Method (NSFDM) is employed to attempt the solution of the model. The validity of the NSFDM in solving the model is established by using the computer in-built classical fourth-order Runge-Kutta method. The comparism between Non-Standard Finite Difference Method solution and Runge-Kutta (RK4) were performed which were found to be efficient, accurate and rapidly convergence.

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Lymphatic filariasis (LF) commonly known as elephantiasis is a painful and profoundly disfiguring disease that has a major social and economic impact in Asia, Africa, the Western pacific and parts of the Americas (Ottesen & Ramachandran, 1995). It is one of the leading causes of permanent and long-term disability in the world (WHO, 1995). About one billion people in 80 different countries are known to be at risk of this disease (WHO, 2012). Globally, the disease is known to affect about 120 million people in 73 endemic countries, is a debilitating disease, is one of the most prevalent and yet one of the most neglected tropical diseases with serious economic and social consequences (WHO, 1992), (Remme *et al.,* 1993). Lymphatic filariasis affects women, men and children of all ages. It is a mosquito-borne disease caused by tissue dwelling nematodes of Brugia malayi, Brugia timori, and Wuchereria bancrofti species (Weil & Ramzy, 2007) and is estimated to affect about 120 million people worldwide with 40 million suffering from serious incapacitation and disfigurement and 1.23 billion people are at risk of infection in 58 countries worldwide. Currently, one-third of the people infected with LF live in India; one-third are in Africa and one-third are in South Asia, the Pacific, and the Americas (Michael & Bundy, 1997).

There are so many human problems that can be shown as mathematical model in nonlinear ordinary differential equations (NL ODEs), such as epidemiology problems. SIRS epidemic model is one of them which the population have temporary immunity to the disease so they can be the susceptible population again. SIRS epidemic model can be shown as continuous model as a NL ODEs. This continuous system has several main properties related to its solution and equilibrium point stability. However, the exact solution is difficult to be found analytically because, as general, NL ODEs has complicated form. Therefore, numerical scheme has an important role to approximate differential equation solution which is difficult to solve analytically. Numerical approximation that often used is Euler method and 4th order Runge Kutta (RK4). However, on several cases, Euler method has disadvantages because the discretization model is not dynamically consistence with the continuous model. Also, equilibrium stability of discrete model of RK4 is restricted to chosen step size on its numerical simulation. Therefore, Mickens develop non-standard finite difference (NSFD) method that, hopefully, obtain scheme which is consistent with the continuous model.

MATERIALS AND METHODS

We develop a mathematical model to study the transmission dynamics of Lymphatic Filariasis incorporating relevant features such as the classes undergoing treatment, vector control (using bed-net and insecticide) and drug administration to Susceptible class, the infected class with symptoms and without symptoms. The Human population of size $N_h(t)$ is subdivided based on lymphatic filariasis status into the following subpopulations: Susceptible human without treatment ($S_1(t)$), Susceptible human

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undergoing treatment ($S_2(t)$), Infected human but not showing signs of elephantiasis ($I_1(t)$), Infected and displaying elephantiasis symptoms ($I_2(t)$) , human undergoing treatment from Infected human (not showing signs of elephantiasis)($T_1(t)$) and. human undergoing treatment from Infected human with signs of elephantiasis ($T_2(t)$).Thus, the total human population is given by

$$N_h(t)=S_1(t)+S_2(t)+I_1(t)+I_2(t)+T_1(t)+T_2(t) \quad (1)$$

The mosquito population is divided into the following subgroups: non-carrier vector (mosquitoes) ($V_1(t)$) and carrier vector (mosquitoes) ($V_2(t)$), so the total mosquito population is given by

$$N_v(t)=V_1(t)+V_2(t) \quad (2)$$

The mosquitoes and human beings are recruited into their susceptible corresponding populations at rates Λ_v and Λ_h respectively. Mosquitoes experience natural death at a rate μ_v and death by insecticide at a rate δ which is proportional to the number in each mosquito class. Similarly, human beings experience natural death at a rate μ_h , which is proportional to the number in each human class.

The mosquito ingests microfilariae when bitting a human who is infected with filariasis (elephantiasis causing nematodes) at rate β_v , with force of infection

$$\frac{\beta_v(\theta_h I_1(t)+I_2(t))}{N_v(t)} \quad (3)$$

Where, β_v is the average number of mosquito bites which cause transmission of disease from infected human to susceptible mosquito and $\theta_h \in (0,1)$ accounts for reduced number of microfilariae in the blood stream of individuals infected but not showing elephantiasis symptoms.

Upon getting infected, non-carrier vector (mosquitoes) enters the carrier class $V_2(t)$. Microfilariae pass through mosquito gut into hemocoel and develop into filariform juveniles. Filariform juveniles escape from mosquitoes proboscis when the insect is feeding and then penetrate wound structure of a human being at a rate β_h with force of infection

$$\frac{\beta_h V_2(t)}{N_h(t)} \quad (4)$$

Where, β_h is the average number of mosquito bites which cause transmission of disease from carrier of parasite (mosquito) to susceptible human per mosquito. Thus, humans are infected at a rate β_v following a bite by mosquito to move into the exposed class $I_1(t)$. Individuals in $I_1(t)$ progress to the stage of showing filariasis symptoms $I_2(t)$ at rate ρ . However, some progress to the $I_2(t)$ as a result of secondary infection at a rate β_v . Individuals in $S_2(t)$, $I_1(t)-class$ and $I_2(t)-class$ are treated using Diethycarbamzime (DEC) and albendazole drugs at a rate τ to move into the classes undergoing treatment, since recovery from filariasis is not permanent. With filariasis, there is no disease-induced death. The schematic representation of the model is given in figure 1

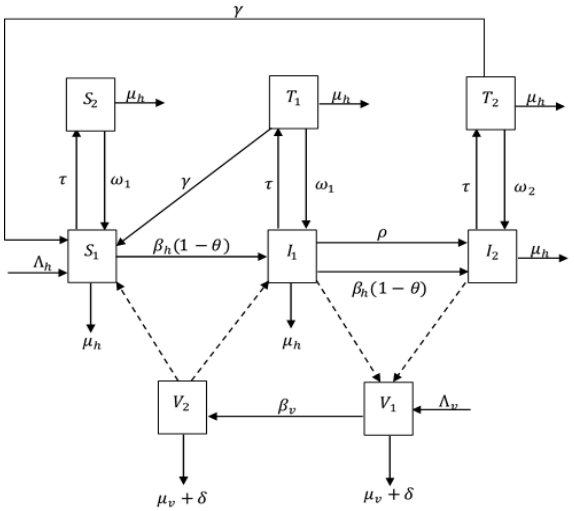

Fig 1 Schematic Diagram of the Model

Based on the model development description above and the schematic diagram in figure 1, the following model equations were derived:

$$\frac{dS_1}{dt}=\Lambda_h-\frac{\beta_h(1-\theta)}{N_h}V_2S_1-k_1S_1+(T_1+T_2)\gamma+\omega_1S_2 \quad (5)$$

$$\frac{dS_2}{dt}=\tau S_1-k_2S_2 \quad (6)$$

$$\frac{dI_1}{dt}=\frac{\beta_h(1-\theta)}{N_h}V_2S_1-\frac{\beta_h(1-\theta)}{N_h}V_2I_1-k_3I_1+\omega_1T_1 \quad 7)$$

$$\frac{dI_2}{dt} = \frac{\beta_h(1-\theta)}{N_h} V_2 I_1 + \rho I_1 - k_1 I_2 + \omega_2 T_2 \quad (8)$$

$$\frac{dT_1}{dt} = \tau I_1 - k_4 T_1 \quad (9)$$

$$\frac{dT_2}{dt} = \tau I_2 - k_5 T_2 \quad (10)$$

$$\frac{dV_1}{dt} = \Lambda_v - \frac{\beta_v(\theta_h I_1 + I_2)}{N_v} V_1 - k_6 V_1 \quad (11)$$

$$\frac{dV_2}{dt} = \frac{\beta_v(\theta_h I_1 + I_2)}{N_v} V_1 - k_6 V_2 \quad (12)$$

And summing (5) – (10) and (11) – (12) gives

$$\frac{dN_h}{dt} = \Lambda_h - \mu_h N_h \quad (13)$$

$$\frac{dN_v}{dt} = \Lambda_v - (\mu_v + \delta) N_v \quad (14)$$

Where,

$$k_1 = (\mu_h + \tau) \quad (15)$$

$$k_2 = (\mu_h + \omega_1) \quad (16)$$

$$k_3 = (\mu_h + \tau + \rho) \quad (17)$$

$$k_4 = (\mu_h + \gamma + \omega_1) \quad (18)$$

$$k_5 = (\mu_h + \gamma + \omega_2) \quad (19)$$

$$k_6 = (\mu_v + \delta) \quad (20)$$

Table 1: Notation and definition of variables and parameter

Symbol	Description
$S_1(t)$	Susceptible human that are not taking drugs at time t
$S_2(t)$	Susceptible human undergoing treatment at time t
$I_1(t)$	Infected human but not showing signs of elephantiasis symptoms at time t
$I_2(t)$	Infected human displaying elephantiasis symptoms at time t
$T_1(t)$	Human population undergoing treatment from infected individuals without symptoms at time t
$T_2(t)$	Human population undergoing treatment from infected individuals with signs of elephantiasis at time t
$V_1(t)$	Non-carrier vectors (mosquitoes) at time t
$V_2(t)$	Carrier vectors (mosquitoes) at time t
$N_h(t)$	Total human population at time t
$N_v(t)$	Total vector population at time t
Λ_h	Recruitment rate of human population
Λ_v	Recruitment rate of mosquito population
β_h	The rate at which the mosquitoes ingests microfilaria when biting a human who is infected.
β_v	The infected rate of human population
μ_h	Natural death rate for the human population
μ_v	Natural death rate for the mosquito population
δ	Vector (mosquitoes) death rate using insecticide
ρ	Rate of progression of human from $I_1(t)$ -class to $I_2(t)$ - class.
θ	Proportion of the susceptible population using mosquito net and Insecticide
γ	Healing rate or the rate at which those treated lose their immunity with time
τ	Treatment rate for the populations undergoing treatment
θ_h	This accounts for reduced number of microfilariae in the blood stream of Individuals infected but not showing elephantiasis symptoms
ω_1	Rate at which individuals in $S_2(t)$ and $T_1(t)$ stop taking drugs
ω_2	Rate at which individuals in $T_2(t)$ stop taking drugs

Positivity of the Solutions: Since the model monitors both human and vector population, we need to show that all the state variables remain non-negative for all times.

$$\Omega = \left\{ \begin{array}{l} (S_1, S_2, I_1, I_2, T_1, T_2) \in \Re_+^6 : S_1(0) > 0, \\ S_2(0) > 0, I_1(0) > 0, I_2(0) > 0, \\ T_1(0) > 0, T_2(0) > 0 \\ S_1 + S_2 + I_1 + I_2 + T_1 + T_2 \le \frac{R_h}{\mu_h} \\ (V_1, V_2) \in \Re_+^2 : V_1(0) > 0, V_2(0) > 0 \\ V_1 + V_2 \le \frac{R_v}{k_6} \end{array} \right\}$$

Theorem 1: Let then the solutions of $\{S_1(t), S_2(t), I_1(t), I_2(t), T_1(t), T_2(t), V_1(t), V_2(t)\}$ of the system (5) – (12) are positive for all $t \ge 0$

Proof:

As applied in Wiah *et al*. (2014)

From (5), we have

$$\frac{dS_1}{dt} = R_h - \frac{\beta_v(1-\theta)}{N_h} V_2 S_1 - k_1 S_1 + (T_1 + T_2)\gamma + \omega_1 S_2$$

$$\frac{dS_1}{dt} \ge -k_1 S_1$$

$$\frac{dS_1}{S_1} \ge -k_1 dt$$

$$\int \frac{dS_1}{S_1} \ge \int -k_1 dt$$

$$S_1(t) \ge S_1(0) e^{-k_1 t} \ge 0 \qquad (21)$$

from (6), we have

$$\frac{dS_2}{dt} = \tau S_1 - k_2 S_2$$

$$\frac{dS_1}{dt} \ge -k_2 S_2$$

$$\frac{dS_2}{S_2} \ge -k_2 dt \qquad (22)$$

$$\int \frac{dS_2}{S_2} \ge \int -k_2 dt$$

$$S_2(t) \ge S_2(0) e^{-k_2 t} \ge 0$$

Similarly, we have

$$I_2(t) \ge I_2(0) e^{-k_1 t} \ge 0 \qquad (24)$$

$$T_1(t) \ge T_1(0) e^{-k_4 t} \ge 0 \qquad (25)$$

$$T_2(t) \ge T_2(0) e^{-k_5 t} \ge 0 \qquad (26)$$

$$V_1(t) \ge V_1(0) e^{-k_6 t} \ge 0 \qquad (27)$$

$$V_2(t) \ge V_2(0) e^{-k_6 t} \ge 0 \qquad (28)$$

Hence, the solutions of $\{S_1(1), S_2(t), I_1(t), I_2(t), T_1(t), T_2(t), V_1(1), V_2(t)\}$ of the system (5) – (12) are positive for all $t \ge 0$

Non-Standard Finite Difference Method: The solutions of the finite difference scheme. For the construction of the numerical scheme, discretizations of the system of equations are made based on the approximations of temporal derivatives by a generalized forward scheme of first order. Hence, if $f(t) \in C'(R)$, let us define its derivative as follows;

$$\frac{d f(t)}{dt} = \frac{f(t+h) - f(t)}{G(h)} + \mathrm{O}(G(h)), \ as \ h \to 0 \qquad (29)$$

Where $G(h)$ is a real-valued function on R. In our work, we will also make use of denominator functions which are little complex function of the time step size than the classical one

We apply Micken's scheme by replacing the step-size h by functions $G_i(h), \ i = 1, 2, ..., 10$ and use non-local approximation for the non-linear terms. Let us discretize the system of equations (5) – (12) and change the NSFD scheme to explicit form as follows;

$$S_1(k+1) = \frac{N_v(k)\left[T_2(k)G_1\gamma + T_1(k)G_1\gamma + S_2(k)G_1 w_1 + G_1\Lambda_h + S_1(k)\right]}{-\theta\beta_v V_2(k)G_1 + k_1 G_1 N_v(k) + \beta_v V_2(k)G_1 + N_v(k)} \qquad (30)$$

$$S_1(k+1)=\frac{\tau S_1(k+1)G_2+S_2(k)}{(G_2k_2+1)} \tag{31}$$

$$I_1(k+1)=\frac{\beta_v V_2(k)S_1(k+1)G_3-\beta_v V_2(k)S_1(k+1)G_3\theta+w_2T_1(k)N_v(k)G_3+I_1(k)N_v(k)}{k_3N_v(k)G_3-\beta_v V_2(k)\theta G_3+\beta_v V_2(k)G_3+N_v(k)} \tag{32}$$

$$I_2(k+1)=\frac{\rho I_1(k+1)N_v(k)G_4-\beta_v V_2(k)I_1(k+1)\theta G_4+w_2T_2(k)N_v(k)G_4+\beta_v V_2(k)I_1(k+1)G_4+I_2(k)N_v(k)}{N_v(k)(k_3G_4+1)} \tag{33}$$

$$T_1(k+1)=\frac{\tau I_1(k+1)G_5+T_1(k)}{(k_4G_5+1)} \tag{34}$$

$$T_2(k+1)=\frac{\tau I_2(k+1)G_6+T_2(k)}{(k_5G_6+1)} \tag{35}$$

$$V_1(k+1)=\frac{N_h(k)(\Lambda_v G_7+V_1(k))}{I_1(k+1)\theta_h\beta_h G_7+I_2(k+1)\beta_h G_7+k_6G_7N_h(k)+N_h(k)} \tag{36}$$

$$V_2(k+1)=\frac{\theta_h\beta_h G_8V_1(k+1)I_1(k+1)+\beta_h G_8V_1(k+1)I_2(k+1)+V_2(k)N_h(k)}{N_h(k)(k_6G_8+1)} \tag{37}$$

$$N_h(k+1)=\frac{\Lambda_h G_9+N_h(k)}{(\mu_h G_9+1)} \tag{38}$$

$$N_v(k+1)=\frac{\Lambda_v G_{10}+N_v(k)}{(k_6G_{10}+1)} \tag{39}$$

Where

$$G_1=\frac{e^{k_1h}-1}{k_1},\quad G_2=\frac{e^{k_2h}-1}{k_2},\quad G_3=\frac{e^{k_3h}-1}{k_3},\quad G_4=\frac{e^{k_1h}-1}{k_1},\quad G_5=\frac{e^{k_4h}-1}{k_4},\quad G_6=\frac{e^{k_5h}-1}{k_5},$$

$$G_7=\frac{e^{k_6h}-1}{k_6},\quad G_8=\frac{e^{k_6h}-1}{k_6},\quad G_9=\frac{e^{\mu_h h}-1}{\mu_h},\quad G_{10}=\frac{e^{k_6h}-1}{k_6}$$

are the denominator functions. With the initial conditions

$S_1(0)=100,00,000$, $S_2(0)=30,000,000$,

$I_1(0)=12,000,000$, $I_2(0)=6,840,000$,

$T_1(0)=8,000,000$, $T_2(0)=6,000,000$,

$V_1(0)=2,000,000,000$, $V_2(0)=960,000,000$

And the parameter values are;

$N_h = 177,155,754$, $N_v = 2,960,000,000$,

$\mu_h = 0.0189$, $\mu_v = 0.00013$,

$\Lambda_h = 3,348,245$, $\Lambda_v = 384,800$,

$\beta_h = 0.009926$, $\beta_v = 0.000249$,

$\rho = 0.00002797$, $\delta = 0.0017$,

$\theta_h = 0.25$, $\theta = 0.25$,

$\gamma = 0.1667$, $w_1 = 0.01$,

$w_2 = 0.0001$, $\tau = 0.125$

for

$k = 0, 1, 2, 3$

The computation of equations above was done using maple 2017 software

RESULTS AND DISCUSSION

We present the numerical simulation which demonstrates the analytical results for the proposed Lymphatic Filariasis (Elephantiasis) model. This is achieved by using some set of parameter values given above and whose source are mainly from literature and well as assumptions. The NSFDM is demonstrated against mapple in-buit fourth order Runge-Kutta Procedure for the solution of the model. Figure 2 to Figure 5 shows the combined plots of the solutions of $S_1(t)$, $S_2(t)$, $I_1(t)$, $I_2(t)$, $T_1(t)$, $T_2(t)$, $V_1(t)$ and $V_2(t)$ by NSFD and RK4.

The graphical representations are from the analytical solutions of the model equations. They are plotted using MAPLE software.

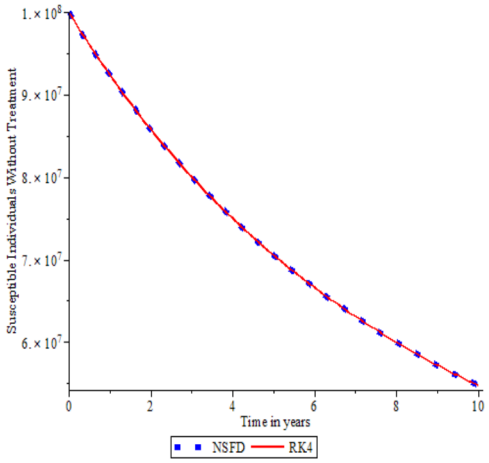

Fig 2: Comparison Solution of Susceptible population without treatment by NSFD and RK4

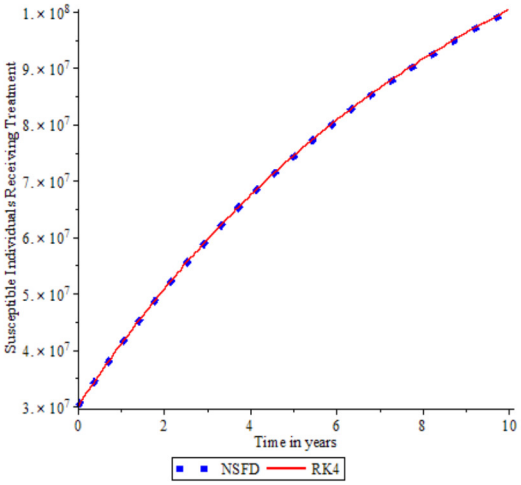

Fig 3: Comparison Solution of Susceptible Population receiving treatment by NSFD and RK4

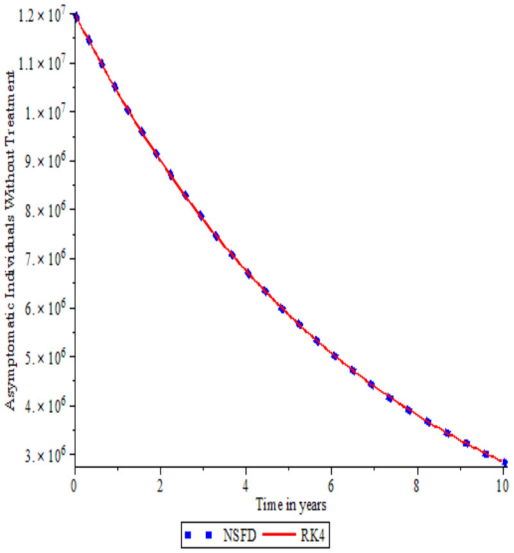

Fig 4: Comparison Solution of Infected Population without symptom by NSFD and RK4

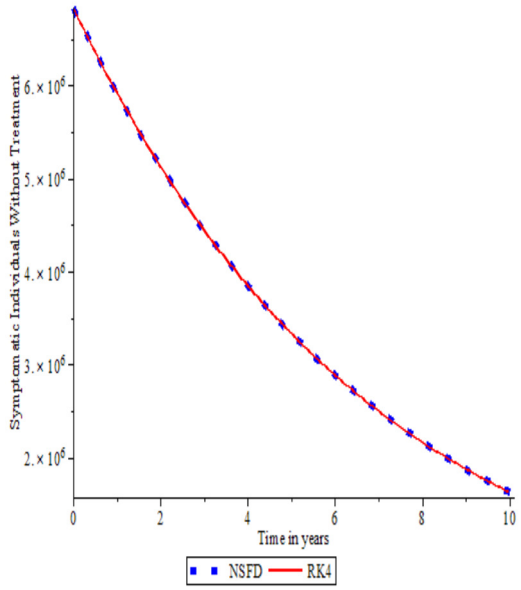

Fig 5: Comparison Solution of Infected Population with symptom by NSFD and RK4

Figure 2 to Figure 5 shows some of the combined plots of the solutions of $S_1(t)$, $S_2(t)$, $I_1(t)$, $I_2(t)$, $T_1(t)$, $T_2(t)$, $V_1(t)$ and $V_2(t)$ by NSFD and RK4. The solutions obtained by using Non-Standard Finite Difference Method with given initial conditions compared favorably with the solution obtained by using classical fourth-other Runge-Kuta method. The solutions of the two methods from our analysis follows the same pattern and behavior. This shows that Non-Standard Finite Difference Method is suitable and efficient to conduct the analysis of Lymphatic Filariasis model.

Conclusion: Non-Standard Finite Difference Method (NSFDM) has been successfully applied to solve for the solution of the Elephantiasis Model with given initial conditions. This method provides an explicit solution which is very useful for understanding and analyzing an epidemic model. Numerical simulations were carried out to compare the results obtained by Non-Standard Finite Difference Method with the classical fourth-order Runge-Kutta method. It can be concluded that this method is very powerful and efficient in obtaining numerical solutions for the analysis of modern epidemics.

REFERENCES

Bhunu, CP and Mushayabasa, S (2012). Transmission dynamics of Lymphatic Filariasis: A mathematical Approach. *International Scholarly Research Network Biomathematics*, vol.2012, article ID 930130.

Castillo-Chavez, C; Feng, Z and Huang, W (2002). On the computation of R_0 and its role on global stability. Mathematical approaches for Emerging & Reemerging Infectious Diseases. Models, Methods and Theory, the IMA Volumes in Mathematics and its Applications, 125, 229-250.

Castillo-Chavez, C and Song, B (2004). Dynamical models of tuberculosis and their applications. *Mathematical Biosciences and Engineering*, 1(2), 361-401.

Cuenco, KT; Halloran, ME; Louis-Charles, J and Lammie, PJ (2004): A family study of lymphedema of the leg in a lymphatic filariasis-endemic area. *American Journal of Tropical Medicine and Hygiene*, 70(2), 180-184.

Derrick, WR and Grossman, SI (1976). Elementary differential equations with applications, Philippines, Addison-Wesley Publishing Company. 468-489.

Diekmann, O and Heesterbeek, JAP (2000). Mathematical epidemiology of infectious diseases: Model building, analysis and interpretation, *Mathematical and Computational Biology*, 1-301, New York, John Wiley & Sons.

Dreyer, G; Noroes, J; Figueredo-Silva, J and Piessens, WF (2000): Pathogenesis of lymphatic disease in bancroftian filariasis: a clinical perspective, Parasitology *Today*,16(12), 544–548

Hethcote, W (2000). The mathematics of infectious disease. Society for Industrial and Applied Mathematics Review. 42(4), 599-653.

Melrose, WD (2002). Lymphatic Filariasis: new insights into an odd disease, *International journal for parasitology*, 32(8), 947-960.

Michael, E; Bundy, DAP and Grenfell, BT (1996). "Re-assessing the global prevalence and distribution of

Supriatna, AK; Serviana, H and Soewono, E (2009). A mathematical model to investigate the Long-Term Effects of the Lymphatic Filariasis Medical Treatment in Jati Sampurna, West Java. *Journal of Mathematics and Fundamental Science* 41(1), 1-14.

Tan, JZG (2003). The elimination of lymphatic filariasis: a strategy for poverty alleviation and sustainable development-perspectives from the

Philippines, *Filaria Journal*, .2(12), 35-50.

Wiah, EN ; Otoo, H; Nabubie, IB and Mohammed, HR (2014): Nonlinear Dynamics and Chaos in HIV/AIDS Epidemic Model with Treatment. *Journal of Applied mathematics*. 4(3), 86-96.

World Health Organization (2005), Global Programme to Eliminate Lymphatic Filariasis-Progress Report for 2004, Weekly Epidemiology Record., 80,202-212.

World Health Organization, (1992). Lymphatic filariasis: the disease and its control, Fifth Report of the World Health Organization Technical Report Series 821-871, World Health Organization Expert committee on Filariasis.

World Health Organization (2015). WHO Fact sheet. Lymphatic filariasis 102, Geneval, Switzerland. Retrieved from. Retrieved www.who.int/lf/publication/global report. . Assessed 18/08/2015.