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Analytical Simulation of Cholera Dynamics with Controls

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Abstract

In this paper, an analytical simulation of cholera dynamics with control is presented. The model incorporates therapeutic treatment, water sanitation and Vaccination in curtailing the disease. We prove the existence and uniqueness of solution. The systems of equations were solved analytically using parameter-expanding method coupled with direct integration. The results are presented graphically and discussed. It shows clearly that improvement in treatment, water sanitation and Vaccination can eradicate cholera epidemic. It also observed that with proper combination of control measures the spread of cholera could be reduced.

Keywords: Cholera model, control strategies, simulation, dynamical systems

1. Introduction

Cholera is a contagious infectious disease that is characterized by extreme vomiting, profuse watery diarrhea and leg pain. It has been found that transmission transpires mostly via absorption of contaminated drinking water or food. Worldwide, almost every year there is an estimated 3-5 million cholera cases and 100, 000-130, 000 deaths due to cholera a year as of 2010 [1]. It has a very short incubation period which starts from a few hours to five days. The health of an infected person disintegrates rapidly and death may occur if treatment is not promptly given. Cholera was first discovered in the Indian subcontinent in 1817. The disease reaches all the way through Asian continent in the 1960s, getting in to Africa in 1970 and Latin America in 1991[2,3]. In many parts of Africa and Asia the disease is still endemic.

Cholera is a disastrous water-borne infectious disease that is caused by the bacterium vibrio cholera. It is a very serious problem in many developing countries due to inadequate access to safe drinking water supply, improper treatment of reservoirs and improper sanitation. In 2012, WHO reported 245, 393 cholera cases and 3034 death cases across 48 countries in which 67% cases occurred in African countries [4]. In 2005, Nigeria had 4, 477 cases and 174 deaths. There were reported cases of cholera in 2008 in Nigeria in which there were 429 deaths out of 6, 330 cases. Furthermore, 2, 304 cases were reported in Niger State in which 114 were death cases [5].

[6] evidenced that recent years have seen a strong trend of cholera outbreak in developing countries, such as in India (2007), Iraq (2008), Congo (2008), Zimbabwe (2008-2009), Haiti (2010), Kenya (2010) and Nigeria (2010).

In Nigeria, outbreaks of the disease have been taking place with ever-increasing occurrence ever since the earliest outbreak in recent times in 1970, [7,8]. In summary the United Nation (UN)

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unit, reports: "despite Nigeria's oil wealth, more than 70% of the country's 126 million people live below the poverty line and cholera outbreaks are common in poor urban areas which lack proper sanitation and clean drinking water" (UN Office for the Coordination of Humanitarian Affairs Integrated Regional Information Networks (IFIN) 2005).

In the last few decades, [9 - 16], designed mathematical models to explore the transmission dynamics and control of the disease.

The global asymptotic stability of the Disease Free Equilibrium and endemic equilibrium was not discussed in [2] but was discussed rigorously in [17].

This present work is based on the analytical solution of the equations describing cholera dynamics with control proposed by [2]. We establish the conditions for existence and uniqueness of the solution of models and provide an analytical solution via parameter-expanding method.

2. Model Formulation

Following [17], the equations describing cholera dynamics with control are:

$$\frac{dS}{dt} = PnH - (n+v)S - \frac{aBS}{k+B} \quad (1)$$

$$\frac{dI}{dt} = \frac{aBS}{k+B} - (r+u)I \quad (2)$$

$$\frac{dB}{dt} = eI - (m+w)B \quad (3)$$

$$\frac{dR}{dt} = (1-p)nH + (r-n+u)I - nR + vS \quad (4)$$

As initial condition based on our assumptions, we choose

$$S(0) = S_0, \quad I(0) = I_0, \quad B(0) = B_0, \quad R(0) = R_0 \quad (5)$$

The assumptions made in the above equations are:

- Vaccination is introduce to the Susceptible population at a rate of v,so that vS individuals per time are removed from the susceptible class and added to the recovered class.
- Therapeutic treatment is applied to infected people at a rate of u, so that uI individuals per time are removed from the infected class and added to the recovered class.
- Water sanitation leads to the death of vibrios at a rate of w.
- Another type of vaccination is applied to (some) newborns so that only a proportion $p(0 < p \le 1)$ of individuals entering the total population are susceptible.

Where

Table 1: Symbols used in the model

Symbol	Description
State Variables	
$S(t)$	susceptible individuals
$I(t)$	infected individuals
$B(t)$	cholera concentration in the water supply
$R(t)$	recovered individuals
Parameters	
H	total human population
n	natural human birth/death rate
a	constant rate of exposure with contaminated water
k	half saturation rate (the infectious dose in water sufficient to produce in 50% of those exposed)

r	rate at which people recover from cholera
e	contribution of each infected person to the population of cholera
m	net death rate of cholera
$\frac{aBS}{k+B}$	incidence which determines the rate of new infection

3.0 Method of Solution

3.1 Existence and Uniqueness of Solution

Here, we shall prove the existence and uniqueness of solution of the model following Derrick and Grossman [19].

For,convenience,let $S = x, I = y, B = z, R = q$

Theorem 1

Consider.

$$\frac{dx}{dt} = PnH - (n+v)x - \frac{axz}{k+z} \quad (6)$$

$$\frac{dy}{dt} = \frac{axz}{k+z} - (r+u)y \quad (7)$$

$$\frac{dz}{dt} = ey - (m+w)z \quad (8)$$

$$\frac{dq}{dt} = (1-p)nH + (r-n+u)y - nq + vx \quad (9)$$

with initial values

$x(0) = x_0$, $y(0) = y_0$, $z(0) = z_0$ $q(0) = q_0$. (10)

Let $\Omega = \{(X,t) : |X_1| \le b_0 \quad 0 \le t < a_0\}$,

where $\underline{X} = (x(t), y(t), z(t), q(t))$, $a, e, H, k, p, r, m, n, p, u, v$, and w are real positive constants and $a_0, b_0 < \infty$.

Then the system of equation (6) - (9) satisfying (10) has a unique solution.

Proof:

We rewrite the system of equation (6) – (9) in vector form as

$$\frac{d}{dt}\begin{pmatrix} x(t) \\ y(t) \\ z(t) \\ q(t) \end{pmatrix} = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \\ q(t) \end{pmatrix}'$$

That is

$$\begin{pmatrix} x(t) \\ y(t) \\ z(t) \\ q(t) \end{pmatrix}' = \begin{pmatrix} pnH - (n+v)x - \frac{axz}{k+z} \\ \frac{axz}{k+z} - (r+u)y \\ ey - (m+w)z \\ (1-p)nH - (r-n+u)y - nq + vx \end{pmatrix}$$

(11)

With the initial condition

$$\begin{pmatrix} x(t) \\ y(t) \\ z(t) \\ q(t) \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \\ q_0 \end{pmatrix}$$

We then define $f_j(t, x(t), y(t), z(t), q(t))$ as follows

$$f_1(t, x(t), y(t), z(t), q(t)) = pnH - (n+v)x - \frac{axz}{k+z}$$

$$f_2(t, x(t), y(t), z(t), q(t)) = \frac{axz}{k+z} - (r+u)y$$

$$f_3(t, x(t), y(t), z(t), q(t)) = ey - (m+w)z$$

$$f_4(t, x(t), y(t), z(t), q(t)) = (1-p)nH + (r-n+u)y - nq + vx$$

Since $\underline{X}$ is bounded, then $f_j(\underline{X}, t)$, $j = 1,....,4$ are define and continuous for all point $(\underline{X}, t)$, $j = 1,....,4$ in Ω then take their maximum in Ω. Let this maximum be defined by

$$M'_i = \sup_{(t,\underline{X})\varepsilon\Omega} \left| f_j(t, \underline{X}) \right|, \quad i, j = 1,....,4$$

Thus $f_j(\underline{X}, t)$ are define and continuous over Ω.

Then there exist at most an M', such that

$\left| f_j(t, \underline{X}) \le M' \right| \quad \forall \quad \delta = \min\left(a_0, \frac{b_o}{M'} \right)$ which imply $f_j(t, \underline{X})$ are continuous and bounded in Ω, then the system of equation (6) – (9) amd (10) has a solution in the interval $|t| < \delta$

Now

$$\left|\frac{\partial f_1}{\partial x}\right| = \left| -(n+v) - \frac{az}{k+z} \right| \le \left| (n+v) + \frac{az}{k+z} \right| = \beta_1 < \infty$$

(12)

$$\left|\frac{\partial f_1}{\partial y}\right| = 0 \quad (13)$$

$$\left|\frac{\partial f_1}{\partial z}\right| = \left| -\frac{axk}{(k+z)^2} \right| \le \left| \frac{axk}{(k+z)^2} \right| = \frac{axk}{(k+z)^2} = \beta_2 < \infty$$

(14)

$$\left|\frac{\partial f_1}{\partial q}\right| = 0$$

(15)

$$\left|\frac{\partial f_2}{\partial x}\right| = \left| \frac{az}{k+z} \right| = \frac{az}{k+z} = \beta_3 < \infty$$

(16)

$$\left|\frac{\partial f_2}{\partial y}\right| = \left| -(r+u) \right| \le \left| (r+u) \right| = \beta_4 < \infty$$

(17)

$$\left|\frac{\partial f_2}{\partial z}\right| = \left| \frac{axk}{(k+z)^2} \right| = \beta_5 < \infty$$

(18)

$$\left|\frac{\partial f_2}{\partial q}\right| = 0$$

(19)

$$\left|\frac{\partial f_3}{\partial x}\right| = 0$$

(20)

$$\left|\frac{\partial f_3}{\partial y}\right| = |e| = e < \infty$$

(21)

$$\left|\frac{\partial f_3}{\partial z}\right| = \left| -(m+w) \right| \le \left| (m+w) \right| = \beta_6 < \infty$$

(22)

$$\left|\frac{\partial f_3}{\partial q}\right| = 0$$

(23)

$$\left|\frac{\partial f_4}{\partial x}\right| = |v| = v < \infty$$

(24)

$$\left|\frac{\partial f_4}{\partial y}\right| = \left| (r-n+u) \right| \le \left| (r+n+u) \right| = \beta_7 < \infty$$

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(25)

$$\left|\frac{\partial f_4}{\partial z}\right| = 0$$

(26)

$$\left|\frac{\partial f_4}{\partial q}\right| = |-n| = n < \infty$$

(27)

By the condition of the theorem, e, n, v and β_k, $k = 1,.....,7$ are real and continuous in Ω, hence $\left|\frac{\partial f_i}{\partial X_j}\right|$ $i, j = 1,...,4$ are continuous and bounded. Hence the system of equations $(6)-(9)$ subject to (10) has a unique solution.

3.2 Solution by Parameter-expanding Method

Parameter-expanding method proposed by He and successfully applied to various engineering problems [18]. We apply parameter-expanding method to equations (6) – (9), where details can be found in [18]. Suppose the solution $x(t)$, $y(t)$, $z(t)$ and $q(t)$ in $(6)-(9)$ can be expressed as

$$\left.\begin{aligned} x(t) &= x_0(t) + ax_1(t) + a^2x_2(t) + h.o.t \\ y(t) &= y_0(t) + ay_1(t) + a^2y_2(t) + h.o.t \\ z(t) &= z_0(t) + az_1(t) + a^2z_2(t) + h.o.t \\ q(t) &= q_0(t) + aq_1(t) + a^2q_2(t) + h.o.t \end{aligned}\right\}, \qquad (28)$$

Where h.o.t read "higher order terms in a" and $S = x$, $I = y$, $B = z$, $R = q$. In our analysis, we are interested only in the first two terms.

Substituting (28) into (6) – (9) and processing, we obtain:

$$\frac{dx_0}{dt} = pnH - (n+v)x_0 \qquad x_0(0) = x_0 \qquad (29)$$

$$\frac{dy_0}{dt} = -(r+u)y_0 \qquad y_0(0) = y_0 \qquad (30)$$

$$\frac{dz_0}{dt} = ey_0 - (m+w)z_0 \qquad z_0(0) = z_0 \qquad (31)$$

$$\frac{dq_0}{dt} = (1-p)nH - (r-n+u)y_0 - nq_0 + vx_0 \qquad q_0(0) = q_0 \qquad (32)$$

$$\frac{dx_1}{dt} = -(n+v) - \frac{x_0 z_0}{(k+z_0)} \qquad x_1(0) = 0 \qquad (33)$$

$$\frac{dy_1}{dt} = \frac{x_0 z_0}{(k+z_0)} - (r+u)y_1 \qquad y_1(0) = 0 \qquad (34)$$

$$\frac{dz_1}{dt} = ey_1 - (m+w)z_1 \qquad z_1(0) = 0 \qquad (35)$$

$$\frac{dq_1}{dt} = (r-n+u)y_1 - nq_1 + vx_1 \qquad q_1(0) = 0 \qquad (36)$$

Solving equations (29) – (36) by direct integration, we obtain

$$x_0(t) = a_1 + a_2 e^{-bt} \qquad (37)$$

$$y_0(t) = y_0 e^{-a_3 t} \qquad (38)$$

$$z_0(t) = a_6 e^{-a_3 t} + a_7 e^{-a_5 t} \quad (39)$$

$$q_0(t) = b_4 e^{-nt} + b_5 e^{-a_3 t} + b_6 e^{-b_0 t} + b_7 \quad (40)$$

$$x_1(t) = p_1 e^{-b_0 t} + p_2 e^{-a_3 t} + p_3 e^{-a_5 t} + p_4 e^{-(a_3+b_0)t} + p_5 e^{-(a_5+b_0)t} - p_6 e^{-2a_3 t} - p_7 e^{-(a_3+a_5)t} - p_8 e^{-(2a_3+b_0)t} - p_9 e^{-(a_3+a_5-b_0)t} \quad (41)$$

$$y_1(t) = c_1 e^{-a_3 t} - c_2 e^{-(2a_5+b_0)t} - c_3 e^{-(a_3+a_5-b_0)t} - c_4 e^{-b_0 t} - c_5 e^{-2a_5 t} + c_6 e^{-(a_3+a_5)t} + c_7 e^{-2a_3 t} - c_8 e^{-(a_5+b_0)t} - c_9 e^{-(a_3+b_0)t} - c_{10} e^{-a_5 t} \quad (42)$$

$$z_1(t) = d_1 e^{-a_5 t} + d_2 e^{-(a_3+b_0)t} + d_3 e^{-(a_5+b_0)t} - d_4 e^{-2a_3 t} - d_5 e^{-(a_3+a_5)t} + d_6 e^{-2a_5 t} + d_7 e^{-b_0 t} + d_8 e^{-(a_3+a_5-b_0)t} + d_9 e^{-(2a_5+b_0)t} - d_{10} e^{-a_3 t} \quad (43)$$

(43)

$$q_1(t) = n_1 e^{-nt} + n_2 e^{-(a_3+a_5-b_0)t} + n_3 e^{-(2a_3+b_0)t} + n_4 e^{-(a_3+a_5)t} + n_5 e^{-2a_3 t} + n_6 e^{-(a_5+b_0)t} + n_7 e^{-(a_3+b_0)t} + n_8 e^{-a_5 t} - n_9 e^{-a_3 t} + n_{10} e^{-b_0 t} + n_{11} e^{-2a_5 t} + n_{12} e^{-(2a_5+b_0)t} \quad (44)$$

$$u = 0.5r, \quad v = 0.5n, \quad w = 0.5m,$$

$$a_0 = pnH, \quad b_0 = n + v, \quad a_1 = \frac{a_0}{b_0},$$

$$a_2 = x_0 - a_1 \quad a_3 = r + u, \quad a_5 = m + w,$$

$$a_6 = \frac{ey_0}{a_5 - a_3}, \quad a_7 = z_0 - a_6, \quad b_2 = r - n + u$$

$$b_3 = b_1 + va_1, \quad b_4 = q_0 - \frac{b_3}{n} - \frac{b_2 y_0}{n - a_3} - \frac{va_2}{n - b_0},$$

$$b_5 = \frac{b_2 y_0}{n - a_3}, \quad b_7 = \frac{b_3}{n}$$

$$p_1 = \left(\frac{a_2 a_7 a_7}{2a_3 k^2} + \frac{2a_2 a_6 a_7}{k^2(a_3+a_5)} + \frac{a_2 a_6 a_6}{2a_3 k^2} + \frac{a_1 a_7 a_7}{k^2(2a_3 - b_0)} + \frac{2a_1 a_6 a_7}{k^2(a_3+a_5-b_0)} + \frac{a_1 a_6 a_6}{k^2(2a_3-b_0)} - \frac{a_2 a_7}{ka_5} - \frac{a_2 a_6}{ka_3} - \frac{a_1 a_7}{k(a_5-b_0)} - \frac{a_1 a_6}{k(a_3-b_0)} \right)$$

$$p_2 = \frac{a_1 a_6}{k(a_3-b_0)}, \quad p_3 = \frac{a_1 a_7}{k(a_5-b_0)},$$

$$p_4 = \frac{a_2 a_6}{ka_3}, \quad p_5 = \frac{a_2 a_7}{ka_5},$$

$$p_6 = \frac{a_1 a_7 a_7}{k^2(2a_3-b_0)} + \frac{a_1 a_6 a_6}{k^2(2a_3-b_0)},$$

$$p_7 = \frac{2a_1 a_6 a_7}{k^2(a_3+a_5-b_0)}, \quad p_8 = \frac{a_2 a_6 a_6}{2a_3 k^2} + \frac{a_2 a_7 a_7}{2a_3 k^2},$$

$$p_9 = \frac{2a_2 a_6 a_7}{k^2(a_3+a_5)}$$

$$c_1 = \left(\frac{a_2 a_7 a_7}{k^2(b_0+2a_5-a_3)} + \frac{2a_2 a_6 a_7}{k^2(a_5-b_0)} + \frac{a_2 a_6 a_6}{k^2(a_3+b_0)} + \frac{a_1 a_7 a_7}{k^2(2a_5-a_3)} - \frac{2a_1 a_6 a_7}{k^2 a_5} - \frac{a_1 a_6 a_6}{k^2 a_3} + \frac{a_2 a_7}{k(a_5-a_3+b_0)} + \frac{a_2 a_6}{kb_0} + \frac{2a_1 a_7}{k(a_5-a_3)} + \frac{a_1 a_6}{k} \right)$$

$$c_2 = \frac{a_2 a_7 a_7}{k^2(b_0+2a_5-a_3)}, \quad c_3 = \frac{2a_2 a_6 a_7}{k^2(a_5-b_0)},$$

$$c_4 = \frac{a_2 a_6 a_6}{k^2(a_3+b_0)}, \quad c_5 = \frac{a_1 a_7 a_7}{k^2(2a_5-a_3)},$$

$$c_6 = \frac{2a_1 a_6 a_7}{k^2 a_5}, \quad c_7 = \frac{a_1 a_6 a_6}{k^2 a_3}, \quad c_8 = \frac{a_2 a_7}{k(a_5-a_3+b_0)},$$

$$c_9 = \frac{a_2 a_6}{kb_0}, \qquad c_{10} = \frac{2a_1 a_7}{k(a_5 - a_3)},$$

$$d_1 = \left(\begin{array}{l} \frac{ec_1}{a_3 - a_5} - \frac{ec_2}{a_5 + b_0} - \frac{ec_3}{a_3 - b_0} - \frac{ec_4}{b - a_5} - \\ \frac{ec_5}{a_5} + \frac{ec_6}{a_3} + \frac{ec_7}{a_3 - a_5} + \frac{ec_8}{b_0} - \frac{ec_9}{a_3 - a_5 + b_0} - ec_{10} \end{array} \right)$$

$$d_2 = \frac{ec_9}{a_3 - a_5 + b_0}, \quad d_3 = \frac{ec_8}{b_0}, \quad d_4 = \frac{ec_7}{2a_3 - a_5},$$

$$d_5 = \frac{ec_6}{a_3}, \quad d_6 = \frac{ec_5}{a_5}, \quad d_7 = \frac{ec_4}{b_0 - a_5},$$

$$d_8 = \frac{ec_3}{a_3 - b_0}, \quad d_9 = \frac{ec_2}{a_5 + b_0}, \quad d_{10} = \frac{ec_1}{a_3 - a_5}$$

$$n_1 = \left(\begin{array}{l} \frac{b_1 c_1}{a_3 - n} - \frac{b_1 c_2}{2a_5 + b_0 - n} - \frac{b_1 c_3}{a_5 + a_3 - b_0 - n} - \\ \frac{b_1 c_4}{b_0 - n} - \frac{b_1 c_5}{2a_5 - n} + \frac{b_1 c_6}{a_3 + a_5 - n} + \frac{b_1 c_7}{2a_3 - n} - \\ \frac{b_1 c_8}{a_5 - b_0 - n} - \frac{b_1 c_9}{a_3 + b_0 - n} - \frac{b_1 c_{10}}{a_5 - n} + \\ \frac{vp_1}{b_0 - n} + \frac{vp_2}{a_3 - n} + \frac{vp_3}{a_3 - n} + \frac{vp_4}{a_3 + b_0 - n} + \\ \frac{vp_5}{a_5 + b_0 - n} - \frac{vp_6}{2a_3 - n} - \frac{vp_7}{a_3 + a_5 - n} - \\ \frac{vp_8}{2a_3 + b_0 - n} - \frac{vp_9}{a_3 + a_5 - b_0 - n} \end{array} \right)$$

$$n_2 = \frac{vp_9 + b_1 c_3}{a_3 + a_5 - b_0 - n},$$

$$n_3 = \frac{vp_8}{2a_3 + b_0 - n}, \quad n_4 = \frac{vp_7 - b_1 c_6}{a_3 + a_5 - n},$$

$$n_5 = \frac{vp_6 - b_1 c_7}{2a_3 - n}, \quad n_6 = \frac{b_1 c_8 - vp_5}{a_5 + b_0 - n},$$

$$n_7 = \frac{b_1 c_9 - vp_4}{a_3 + b_0 - n}, \quad n_8 = \frac{b_1 c_{10}}{a_5 - n} - \frac{vp_3}{a_3 - n},$$

$$n_9 = \frac{b_1 c_1 - vp_2}{a_3 - n}, \quad n_{10} = \frac{b_1 c_4 - vp_1}{b_0 - n}, \quad n_{11} = \frac{b_1 c_5}{2a_5 - n},$$

$$n_{12} = \frac{b_1 c_2}{2a_5 + b_0 - n}$$

The computations were done using computer symbolic algebraic package MAPLE.

4. Results and Discussion

We have proved the existence of unique solution of the model
under certain conditions using Derrick and Grossman approa-
ch. The model equations (6) – (9) are solved analytically using parameter-expanding method and computed for the values of
$p = 0.9,\ H = 10000,\ e = 1,\ k = 5000, u = 0.05,\ v = 0.0005,$
$w = 0.015,\ a = 0.1,\ x_0 = 6000, y_0 = 100,\ z_0 = 10,\ q_0 = 0$

The population of Susceptible, Infected and Recovered individuals and the cholera concentration in water supply are depicted graphically in figures 1 – 6.

From **Figure 1**,we can conclude that with the increase in therapeutic treatment u ,the infected individuals are reduce per time and the recovered individuals increases due to treat-treatment.

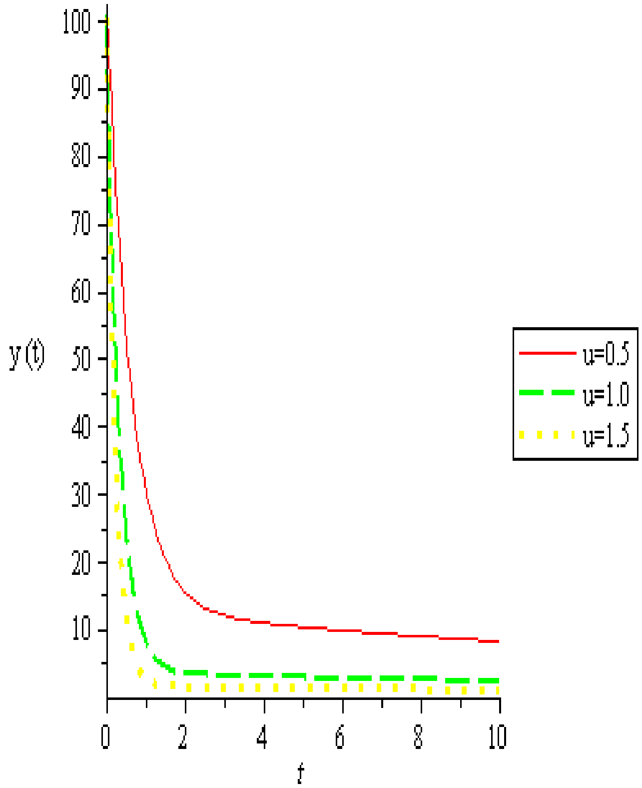

Figure1: y(t) versus time t for various values of u at $p = 0.9, H = 10000, e = 1, k = 5000, n = 0.0005, w = 0.015, a = 0.1, x_0 = 6000, y_0 = 100, z_0 = 10, q_0 = 0$

From **Figure 2**, we can conclude that with the increase in vaccination v, the susceptible individuals reduces per time .

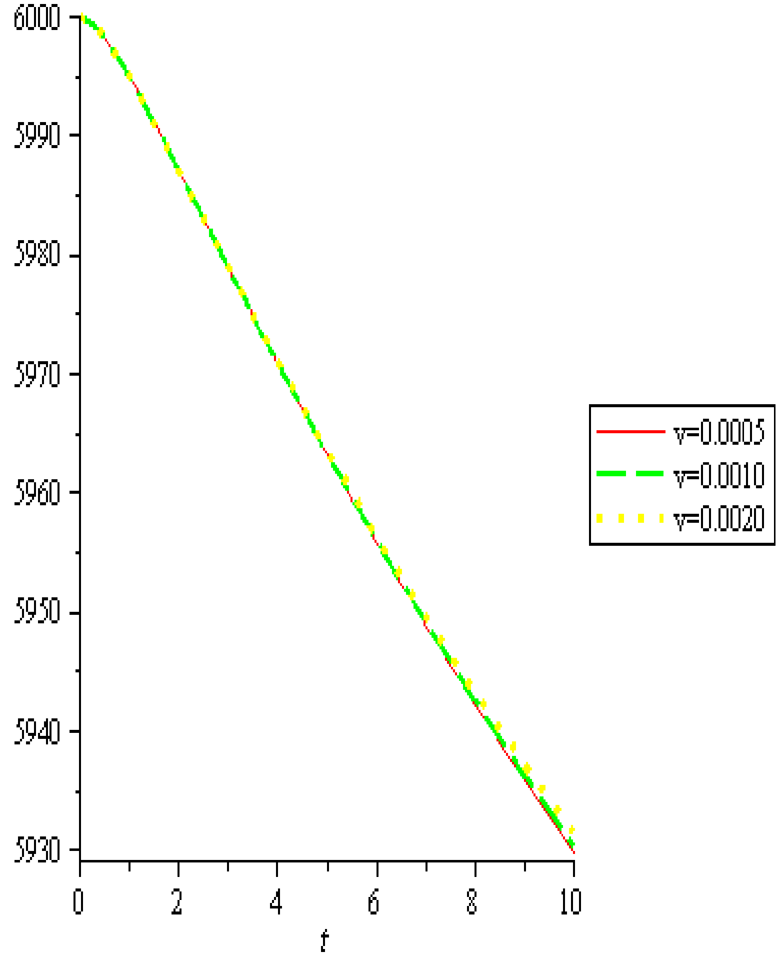

Figure 2: x(t) versus time t for various values of v at $p = 0.9, H = 10000, e = 1, k = 5000, u = 0.5\ v = 0.0005, w = 0.015, a = 0.1, x_0 = 6000, y_0 = 100, z_0 = 10, q_0 = 0$

From **Figure 3**, we can conclude that with the increase in vaccination v, the recover individuals increases per time.

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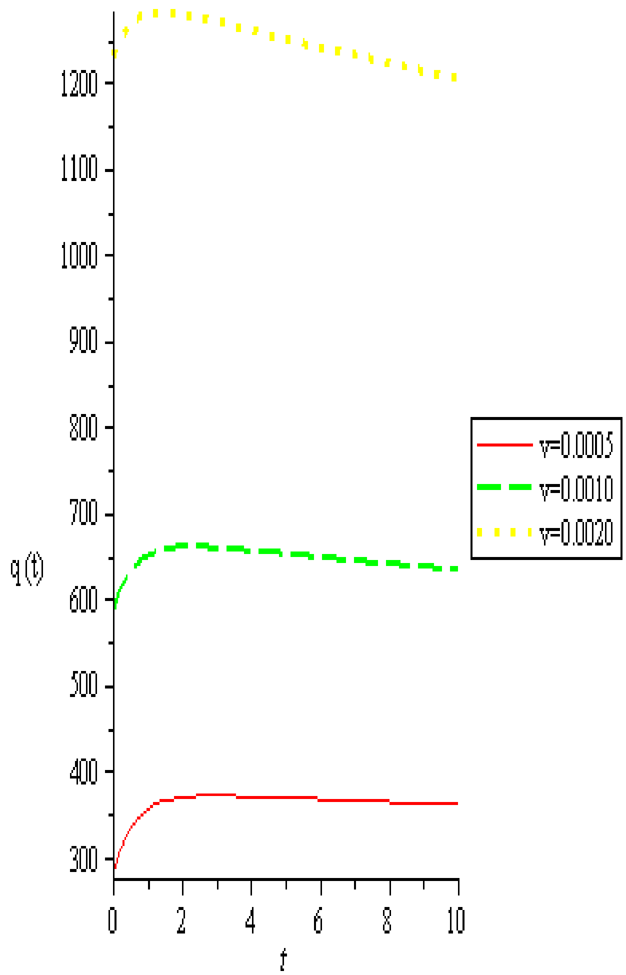

Figure 3: q(t) versus time t for various values of v at
$p = 0.9, H = 10000, e = 1, k = 5000, u = 0.5, w = 0.015, a = 0.1,$
$x_0 = 6000, y_0 = 100, z_0 = 10, q_0 = 0$

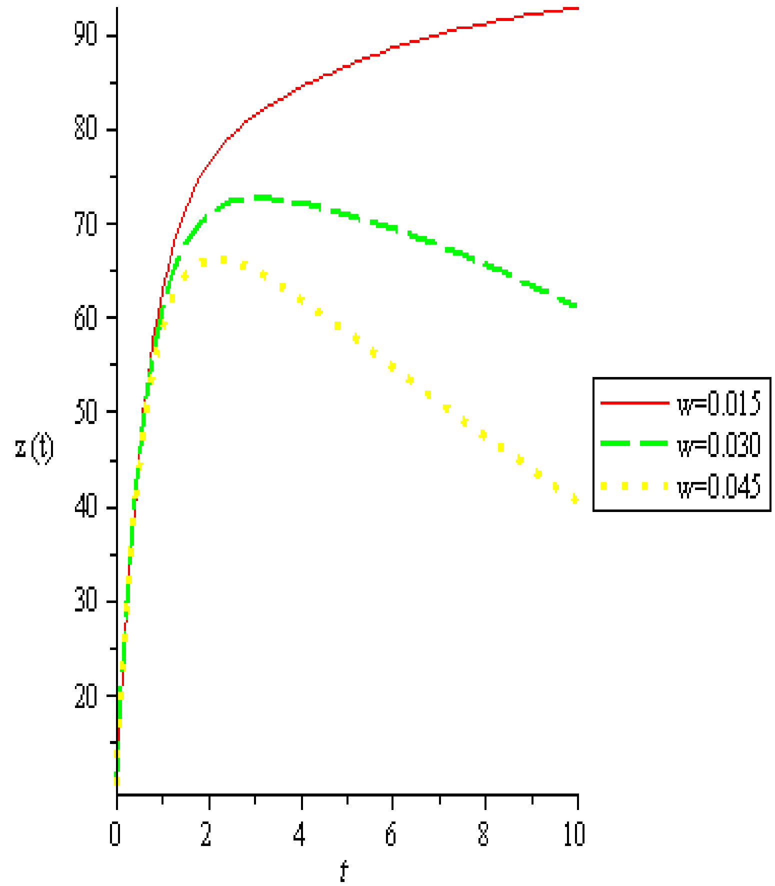

Figure 4: z(t) versus time t for various values of w at
$p = 0.9, H = 10000, e = 1, k = 5000, u = 0.5, v = 0.0005, a = 0.1,$
$x_0 = 6000, y_0 = 100, z_0 = 10, q_0 = 0$

From **Figure 4**: we can conclude that with the increase in water treatment rate w, the concentration of vibrio cholera in contaminated water reduces.

From **Figure 5,** we can conclude that with the increase in rate of exposure to contaminated water a, the susceptible individuals increases.

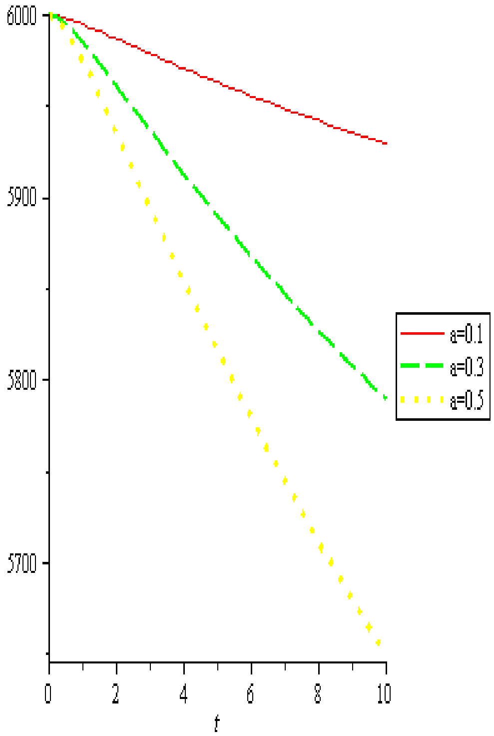

Figure 5: x(t) versus time t for various values of a at $p = 0.9, H = 10000, e = 1, k = 5000, u = 0.5, v = 0.0005, w = 0.015, x_0 = 6000, y_0 = 100, z_0 = 10, q_0 = 0$

From **Figure 6**, we can conclude that with the increase in rate of exposure to contaminated water a, the infected individuals increases.

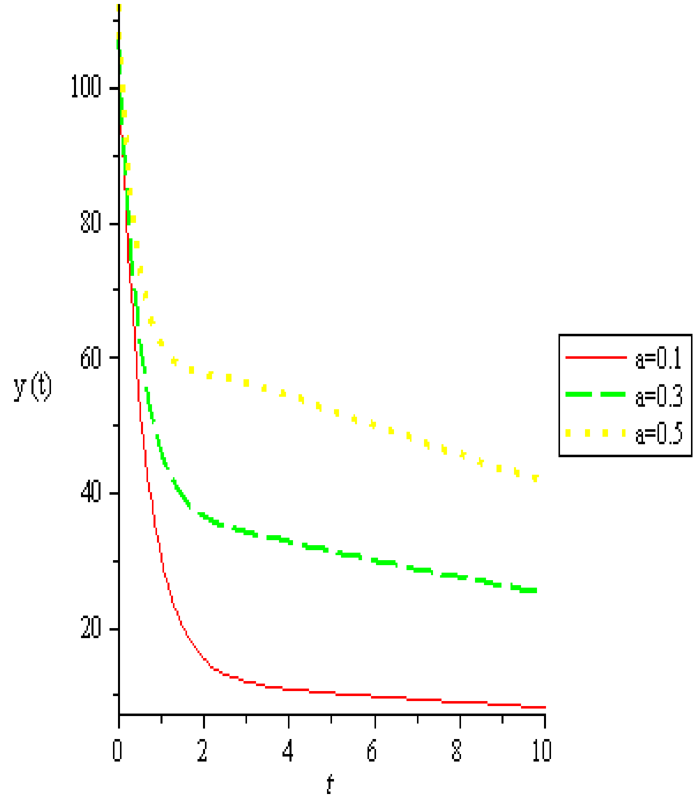

Figure 6: x(t) versus time t for various values of a at $p = 0.9, H = 10000, e = 1, k = 5000, u = 0.5, v = 0.0005, w = 0.015, x_0 = 6000, y_0 = 100, z_0 = 10, q_0 = 0$

5. CONCLUSION

From the studies made on this paper we concluded that 50% level of control measures is sufficient to effectively control the spread of cholera. The total number of the infected individuals decreases with the increase in level of the control measures stopping the disease from reaching an alarming level.

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