Gen. Math. Notes, Vol. 31, No. 2, December 2015, pp. 54-65
ISSN 2219-7184; Copyright © ICSRS Publication, 2015
www.i-csrs.org
Available free online at http://www.geman.in

Simulation of Heat and Mass Transfer in the Flow of an Incompressible Viscous Fluid Past an Infinite Vertical Plate

A.A. Mohammed[1], R.O. Olayiwola[2] and E.M. Yisa[3]

[1,2,3]Department of Mathematics and Statistics
Federal University of Technology, Minna, Nigeria
[1]E-mail: adamualhaj@yahoo.com
[2]E-mail: olayiwolarasaq@yahoo.co.uk
[3]E-mail: solomonyisa68@gmail.com

(Received: 7-8-15 / Accepted: 23-11-15)

Abstract

This paper presents an analytical method to describe the heat and mass transfer in the flow of an incompressible viscous fluid past an infinite vertical plate. The governing equations account for the viscous dissipation effect and mass transfer with chemical reaction of constant reaction rate. The coupled partial differential equations describing the phenomenon have been transformed using similarity transformation and solved analytically using iteration perturbation method. The results obtained are presented graphically. It is discovered that the heat transfer rate decreases due to increase of Prandtl number and Eckert number. Mass transfer rate decreases due to increase of Schmidt number and increases due to increase of reaction rate.

Keywords: *Heat and mass transfer, Incompressible fluid, Viscous dissipation, Chemical reaction, Similarity transformation, Iteration perturbation method.*

1 Introduction

Coupled heat and mass transfer problems in the presence of chemical reactions are of importance in many processes especially in industries and thus have received considerable amount of attention in recent times by many scholars. Examples of such processes can be found in drying, polymer, distribution of temperature and moisture over agricultural fields and groves of fruit trees. The study of the flow and heat transfer in fluid past a porous surface has attracted the interest of many scientific investigators in view of its applications in engineering practice, particularly in chemical industries, such as the cases of boundary layer control, transpiration cooling and gases diffusion. An extensive contribution on heat and mass transfer flow has been made by Khair and Bejan [1]. Doma et al. [2] examined the two-dimensional fluid flow past a rectangular plate with variable initial velocity. They investigate the motion of the time-independent flow of a viscous incompressible fluid. Hossain et al. [3] investigated the problem of natural convection flow along a vertical wavy surface with uniform surface temperature in the presence of heat generation/absorption. Chand et al. [4] investigated the hydro magnetic oscillatory flow through a porous medium bounded by two vertical porous plates with heat source and Soret effect. One plate of the channel is kept stationary while the other is moving with uniform velocity. Sharma and Singh [5] investigated the effects of variable thermal conductivity and heat source/sink on flow of a viscous incompressible electrically conducting fluid in the presence of uniform transverse magnetic field and variable free stream near a stagnation point on a non-conducting stretching sheet. The objective of this paper is to obtain an analytical solution for describing the heat and mass transfer in the flow of an incompressible viscous fluid past an infinite vertical plate. To simulate the flow analytically, the viscous dissipation effect is retained and mass transfer with chemical reaction of constant reaction rate is considered.

2 Model Formulation

Consider a steady two-dimensional mass transfer flow of an incompressible viscous fluid past an infinite vertical plate. The plate is maintained at a constant temperature T_w and the concentration is maintained at a constant value C_w. Introducing a Cartesian coordinate system, x-axis is chosen along the plate in the direction of flow and y-axis normal to it. The temperature of uniform flow is T_∞ and the concentration of uniform flow is C_∞. The viscous dissipation effect is retained and mass transfer with chemical reaction of constant reaction rate is considered. With the above assumptions the system of governing equations to be solved is:

Continuity Equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum Equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\upsilon\frac{\partial^2 u}{\partial y^2} \tag{2}$$

Energy Equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

Species Equation

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-\sigma C \tag{4}$$

Boundary Conditions

The boundary conditions at the, i.e., $y=0$ are given by the no-slip velocity condition. Thus

$$u(x,0)=0, \qquad v(x,0)=0, \qquad T(x,0)=T_w, \qquad C(x,0)=C_w \tag{5}$$

At the edge of the boundary layer, the viscous flow inside the boundary layer is required to smoothly transition into the in viscid flow outside the boundary layer.

$$u(y\to\infty)\to U_e(x), \qquad T(y\to\infty)\to T_\infty, \qquad C(y\to\infty)=C_\infty, \tag{6}$$

where the subscripts w and e represents the condition at the wall and edge of the boundary layer respectively. υ is the kinematic viscosity, t is the time, ρ is the fluid density, u and v are the components of velocity along x and y directions respectively, T is the temperature of the fluid, C is the species concentration, c_p is the specific heat capacity at constant pressure, σ is the reaction rate, k is the thermal conductivity, D is the diffusion coefficient.

3 Method of Solution

3.1 Variable Transformation

For the similarity transformations and the corresponding similar solutions, the incompressible stream function can be defined by:

$$\frac{\partial \psi}{\partial y} = u \tag{7}$$

$$\frac{\partial \psi}{\partial x} = -v \tag{8}$$

Equation (7) and (8) automatically satisfy the continuity equation (1). Then, the momentum equation (2), species equation (4) and energy equation (3) become:

$$\frac{\partial \psi}{\partial y}\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial y}\right) - \frac{\partial \psi}{\partial x}\frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial y}\right) = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \upsilon \frac{\partial^2}{\partial y^2}\left(\frac{\partial \psi}{\partial y}\right) \tag{9}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - \sigma C \tag{10}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{\upsilon}{c_p}\left(\frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial y}\right)\right)^2 \tag{11}$$

The dependent variable transformations are introduced as follows:

$$\psi(x, y) = U\sqrt{\frac{(2-\beta)\upsilon}{U}}\, x^{\frac{1}{2-\beta}} f(\eta) \tag{12}$$

$$u(x, y) = U_e(x) f'(\eta) = U x^{\frac{\beta}{2-\beta}} f'(\eta) \tag{13}$$

$$v(x, y) = -U\sqrt{\frac{\upsilon}{(2-\beta)U}}\, x^{\frac{\beta-1}{2-\beta}} \left(f(\eta) + (\beta - 1) f'(\eta)\right) \tag{14}$$

$$C(x, y) = C(\eta) \tag{15}$$

$$T(x, y) = T(\eta) \tag{16}$$

Independent variable transformation is introduced as follows:

$$\eta = y\sqrt{\frac{U}{(2-\beta)\upsilon}}\, x^{\frac{\beta-1}{2-\beta}}, \tag{17}$$

where β is Falkner-Skan pressure gradient parameter.

Introducing equations (12) - (17) into the momentum equation (9), species equation (10) and energy equation (11) and using Euler's equation at the edge of the boundary layer, i.e., $\frac{\partial p}{\partial x} = -\rho U_e \frac{dU_e}{dx}$, results in:

$$\frac{1}{(2-\beta)} x^{\frac{3\beta-2}{2-\beta}} f'^2 + \frac{\beta-1}{2-\beta} \eta x^{\frac{3\beta-2}{2-\beta}} f'f'' - \frac{1}{(2-\beta)} x^{\frac{3\beta-2}{2-\beta}} ff'' - \frac{(\beta-1)}{(2-\beta)} \eta x^{\frac{3\beta-2}{2-\beta}} f'f'' =$$
$$\frac{1}{(2-\beta)} x^{\frac{3\beta-2}{2-\beta}} + \frac{1}{(2-\beta)} x^{\frac{3\beta-2}{2-\beta}} f''' \tag{18}$$

$$\frac{\beta-1}{2-\beta} \eta x^{\frac{2\beta-2}{2-\beta}} f'C' - \frac{1}{2-\beta} x^{\frac{2\beta-2}{2-\beta}} f C' - \frac{\beta-1}{2-\beta} \eta x^{\frac{2\beta-2}{2-\beta}} f'C' = D \frac{1}{(2-\beta)\upsilon} x^{\frac{2\beta-2}{2-\beta}} C'' - \sigma C \tag{19}$$

$$\frac{\beta-1}{2-\beta} \eta\, x^{\frac{2\beta-2}{2-\beta}} f'T' - \frac{1}{2-\beta} x^{\frac{2\beta-2}{2-\beta}} f T' - \frac{\beta-1}{2-\beta} \eta\, x^{\frac{2\beta-2}{2-\beta}} f'T' =$$
$$\frac{k}{\rho c_p} \frac{1}{(2-\beta)\upsilon} x^{\frac{2\beta-2}{2-\beta}} T'' + \frac{\mu U^2}{(2-\beta)\rho c_p \upsilon} x^{\frac{4\beta-4}{2-\beta}} f''^2 \tag{20}$$

In the above analysis, we consider an important special case, $\beta = 1$, corresponding with stagnation point flow and by introducing the dimensionless temperature and species concentration:

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \qquad \phi(\eta) = \frac{C(\eta) - C_\infty}{C_w - C_\infty} \tag{21}$$

Results in:

$$f''' + ff'' - f'^2 + 1 = 0 \tag{22}$$

$$\theta'' + \Pr Ecf''^2 + \Pr f\theta' = 0 \tag{23}$$

$$\phi'' + Scf\phi' - Sc\sigma(\phi + \sigma_1) = 0 \tag{24}$$

Together with boundary conditions

$$\left.\begin{array}{ll} f(0) = f'(0) = 0, & f'(\eta \to \infty) = 1 \\ \theta(0) = 1, & \theta(\eta \to \infty) = 0 \\ \phi(0) = 1, & \phi(\eta \to \infty) = 0 \end{array}\right\}, \tag{25}$$

Where

$\Pr = \dfrac{\mu c_p}{k}$: Prandtl number, $Sc = \dfrac{\upsilon}{D}$: Schmidt number,

$Ec = \dfrac{{U_e}^2}{Cp(T_w - T_\infty)}$: Eckert number, $\sigma_1 = \left(\dfrac{C_\infty}{C_w - C_\infty}\right)$.

3.2 Solution by Iteration Perturbation Method

We solve equations (22) – (25) using iteration perturbation method (where details can be found in [6]).

Now we begin with the initial approximate solution (where details can be found in [6]):

$$f_0(\eta) = \eta - \frac{1}{b}\left(1 - e^{-b\eta}\right), \tag{26}$$

where b is an unknown constant.

Equations (22) – (24) can be approximated by the following equations:

$$f''' + \left(\eta - \frac{1}{b}\left(1 - e^{-b\eta}\right)\right) f'' + 1 - f'^2 = 0 \tag{27}$$

$$\theta'' + \Pr Ecf''^2 + \Pr\left(\eta - \frac{1}{b}\left(1 - e^{-b\eta}\right)\right)\theta' = 0 \tag{28}$$

$$\phi'' + Sc\left(\eta - \frac{1}{b}\left(1 - e^{-b\eta}\right)\right)\phi' - Sc\sigma(\phi + \sigma_1) = 0 \tag{29}$$

We rewrite equations (40) - (42) in the form:

$$f''' + bf'' + \left(\eta - \frac{1}{b}\left(1 - e^{-b\eta}\right) - b\right) f'' + 1 - f'^2 = 0 \tag{30}$$

$$\theta'' + \Pr Ecf''^2 + \Pr\theta' + \Pr\left(\eta - \frac{1}{b}\left(1 - e^{-b\eta}\right) - 1\right)\theta' = 0 \tag{31}$$

$$\phi'' + +Sc\phi' + Sc\left(\eta - \frac{1}{b}\left(1 - e^{-b\eta}\right) - 1\right)\phi' - Sc\sigma(\phi + \sigma_1) = 0 \tag{32}$$

Let $1 =\in \alpha$, $\sigma =\in \gamma$ and embed an artificial parameter $\in$ in equations (30) – (31) as follows:

$$f''' + bf'' + \in \left(\eta - \frac{1}{b}\left(1 - e^{-b\eta}\right) - b\right) f'' + \in \alpha - \in f'^2 = 0 \tag{33}$$

$$\theta'' + \Pr Ecf''^2 + \Pr \theta' + \in \Pr\left(\eta - \frac{1}{b}\left(1 - e^{-b\eta}\right) - 1\right)\theta' = 0 \tag{34}$$

$$\phi'' + +Sc\phi' + \in Sc\left(\eta - \frac{1}{b}\left(1 - e^{-b\eta}\right) - 1\right)\phi' - \in Sc\gamma(\phi + \sigma_1) = 0 \tag{35}$$

Suppose that the solution of equations (33) – (35) can be expressed as:

$$\left.\begin{array}{l} f(\eta) = f_0(\eta) + \in f_1(\eta) + \ldots \\ \theta(\eta) = \theta_0(\eta) + \in \theta_1(\eta) + \ldots \\ \phi(\eta) = \phi_0(\eta) + \in \phi_1(\eta) + \ldots \end{array}\right\} \tag{36}$$

Substituting (36) into (33) – (35) and processing, we obtain

$$f_0''' + bf_0'' = 0, \qquad f_0(0) = 0, \qquad f_0'(0) = 0, \qquad f_0'(\eta \to \infty) = 1 \tag{37}$$

$$\theta_0'' + \Pr \theta_0' + \Pr Ecf_0''^2 = 0, \qquad \theta_0(0) = 1, \qquad \theta_0(\eta \to \infty) = 0 \tag{38}$$

$$\phi_0'' + Sc\phi_0' = 0, \qquad \phi_0(0) = 1, \qquad \phi_0(\eta \to \infty) = 0 \tag{39}$$

$$f_1''' + bf_1'' + \left(\eta - \frac{1}{b}\left(1 - e^{-b\eta}\right) - b\right) f_0'' + \alpha - f_0'^2 = 0, \quad f_1(0) = 0, \quad f_1'(0) = 0, \quad f_1'(\eta \to \infty) = 1 \tag{40}$$

$$\theta_1'' + \Pr \theta_1' + 2\Pr Ecf_0''f_1'' + \Pr\left(\eta - \frac{1}{b}\left(1 - e^{-b\eta}\right) - 1\right)\theta_0' = 0, \quad \theta_0(0) = 1, \quad \theta_0(\eta \to \infty) = 0 \tag{41}$$

$$\phi_1'' + Sc\phi_1' + Sc\left(\eta - \frac{1}{b}\left(1 - e^{-b\eta}\right) - 1\right)\phi_0' - Sc\gamma(\phi_0 + \sigma_1) = 0, \quad \phi_1(0) = 1, \quad \phi_1(\eta \to \infty) = 0 \tag{42}$$

Seeking direct integration, we obtain the solution of equations (37) - (42) as

$$f(\eta)=\left(\eta-\frac{1}{b}\left(1-e^{-b\eta}\right)\right)+\in\left(\begin{array}{l}\frac{\eta^2 e^{-b\eta}}{2b}+\frac{e^{-b\eta}}{b^3}-\frac{e^{-b\eta}}{b}+\frac{3\eta e^{-b\eta}}{b^2}+\frac{6e^{-2b\eta}}{b^3}-\frac{e^{-2b\eta}}{2b^3}+\\ \frac{\left(b^2-4\right)e^{-b\eta}}{b^3}-\frac{5}{2b^3}-\eta e^{-b\eta}+\eta\end{array}\right)$$

(43)

$$\theta(\eta)=a_0e^{-\Pr\eta}-a_1e^{-2b\eta}+a_2e^{-\Pr\eta}+\in\left(\begin{array}{l}\left(\frac{h_4}{\Pr^2}-\frac{h_1}{\Pr}-\frac{h_{11}}{\Pr}\right)e^{-\Pr\eta}+\left(\frac{h_8}{2b}-\frac{h_2}{2b}-\frac{h_7}{2b^2}\right)\eta e^{-2b\eta}\\ +\left(\frac{h_8}{4b^2}-\frac{h_2}{4b^2}+\frac{h_2}{2b}-\frac{h_5}{2b}-\frac{h_7}{4b^3}-\frac{h_9}{2b}\right)e^{-2b\eta}\\ -\frac{q_2}{\Pr}\eta e^{-\Pr\eta}-\frac{h_7}{2b}\eta^2e^{-2b\eta}+\frac{h_6}{3b}e^{-3b\eta}+\\ \frac{h_{10}}{b+\Pr}e^{-(b+\Pr)\eta}-\frac{h_{12}}{\Pr}e^{-\Pr\eta}\end{array}\right)$$

(44)

$$\phi(\eta)=-\frac{e^{-Sc\eta}}{Sc}e^{-Sc\eta}+\in\left(\eta^2e^{-Sc\eta}-2\eta e^{-Sc\eta}-\left(\frac{1}{b}+1\right)\eta e^{-Sc\eta}-\frac{Sce^{-b\eta}}{b^2\left(b+Sc\right)}+\frac{\gamma\eta\, e^{-Sc\eta}}{Sc}\right)$$

(45)

Where,

$$a_0=\frac{b^2Ec}{2b-\Pr},\qquad a_1=\frac{b^2\Pr Ec}{2b(2b-\Pr)},\qquad a_2=\frac{bEc+2}{2},\qquad q_0=(a_0+a_2)\Pr^2,$$

$$q_1=2a_1b\Pr,\qquad q_2=\left(1+\frac{1}{b}\right)(a_0+a_2)\Pr^2,\quad q_3=\left(1+\frac{1}{b}\right)2a_1b\Pr^2,\quad q_4=\left(\frac{a_0+a_2}{b}\right)\Pr^2,$$

$$q_5=2a_1\Pr,\qquad q_6=b^2\Pr Ec,\qquad q_7=4\Pr Ec,\qquad q_8=4(b^2+1)\Pr Ec,$$

$$q_9=2\Pr Ec(b^3-5b),\quad h_1=\frac{q_0}{2},\qquad h_2=\frac{(q_9-q_1)}{\Pr-2b},\qquad h_5=\frac{(q_3-q_8)}{\Pr-2b},$$

$$h_6=\frac{(q_5+q_7)}{\Pr-3b},\qquad h_7=\frac{q_6}{\Pr-2b},\qquad h_8=\frac{q_6}{(\Pr-2b)^2},\qquad h_9=\frac{q_6}{(\Pr-2b)^3},$$

$$h_{10}=\frac{q_4}{b},\qquad h_{11}=\frac{q_9-q_1}{\Pr-2b}-\frac{q_3-q_8}{\Pr-2b}+\frac{q_5+q_7}{\Pr-3b}-\frac{q_6}{(\Pr-2b)^3}+\frac{q_4}{b},$$

$$h_{12}=-h_1-\frac{h_2\Pr}{4b^2}+\frac{h_3\Pr}{2b}+\frac{h_4}{\Pr}-\frac{h_5\Pr}{2b}+\frac{h_6\Pr}{3b}-\frac{h_7\Pr}{4b^3}+\frac{h_8\Pr}{4b^2}-\frac{h_9\Pr}{2b}+\frac{h_{10}\Pr}{b+\Pr}-h_{11}$$

The computations were done using computer symbolic algebraic package MAPLE.

4 Results and Discussion

The systems of partial differential equations describing the mass transfer flow of an incompressible viscous fluid past an infinite vertical plate are solved analytically using a similarity transformation and iteration perturbation method. The numerical values of Skin Friction, Nusselt Number and Sherwood Number are arranged in Table 1 below for various values of the parameters involved. Analytical solutions of equations (22) - (25) are computed for the values of $\Pr = 0.71, 0.85, 1.00,\quad Sc = 0.22, 0.62, 0.78,\quad Ec = 0.001, 0.030, 0.050$ $\sigma = 200, 400, 600,\quad \in = 0.01,\quad b = 2.3062$.

The following figures explain the fluid temperature and species concentration distribution against different dimensionless parameters.

From figure 1, we can conclude that with the increase of Prandtlnumber (Pr), temperature decreases.

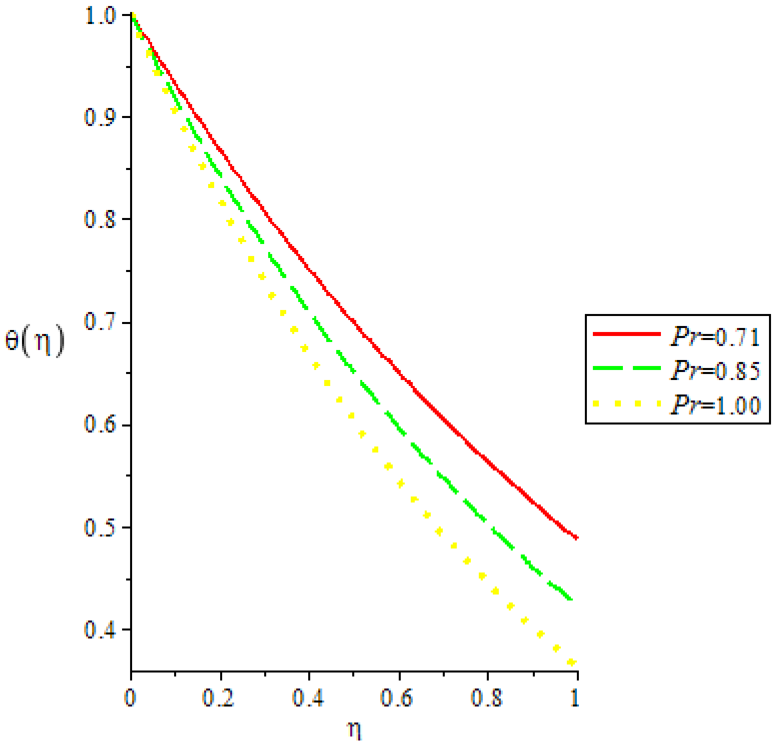

Figure 1: Variation of Temperature with Pr

From figure 2, we can conclude that with the increase of Schmidt number (Sc), species concentration decreases.

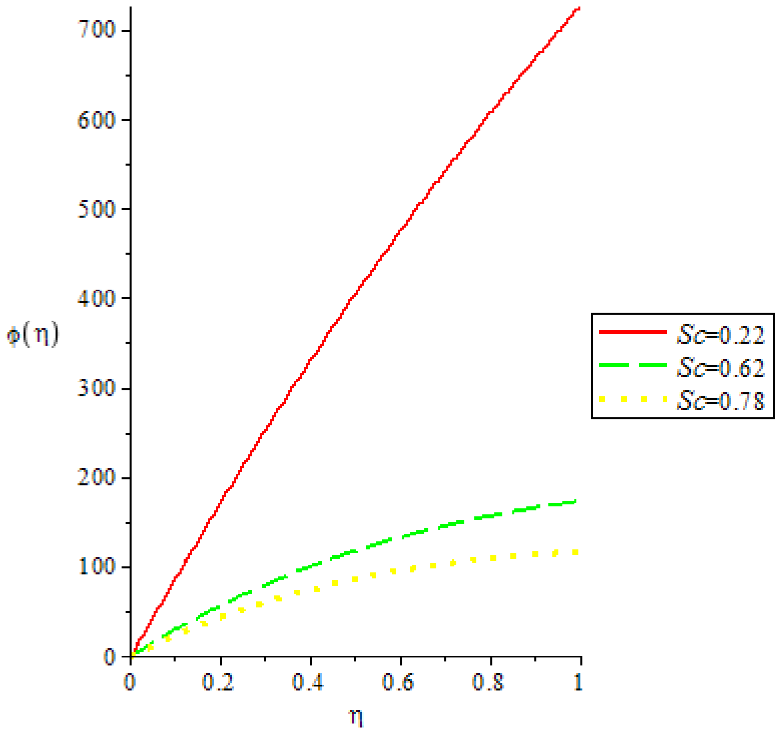

Figure 2: Variation of Concentration with *Sc*

From figure 3, we can conclude that with the increase of Eckert number (Ec), temperature increases.

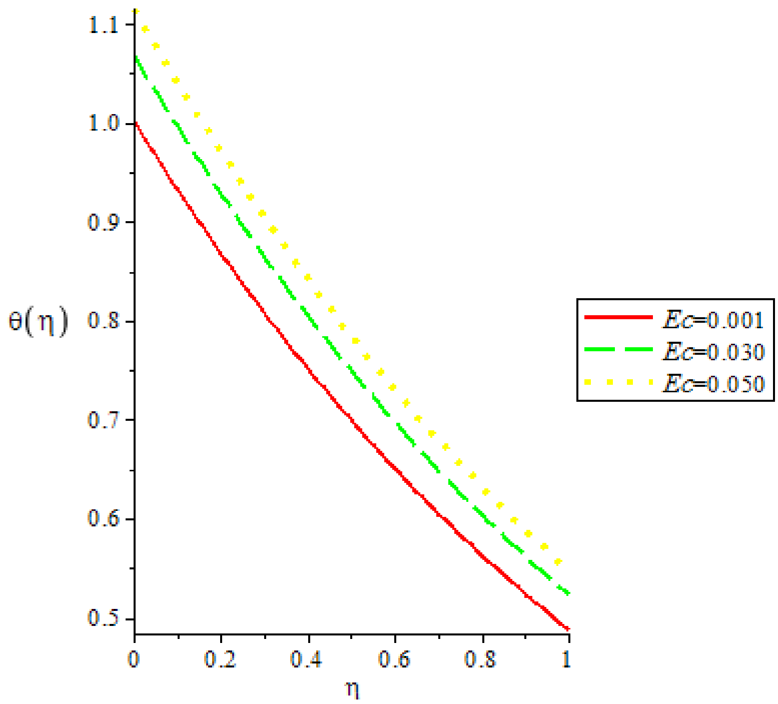

Figure 3: Variation of Temperature with *Ec*

From figure 4, we can conclude that with the increase of reaction rate (σ), species concentration increases.

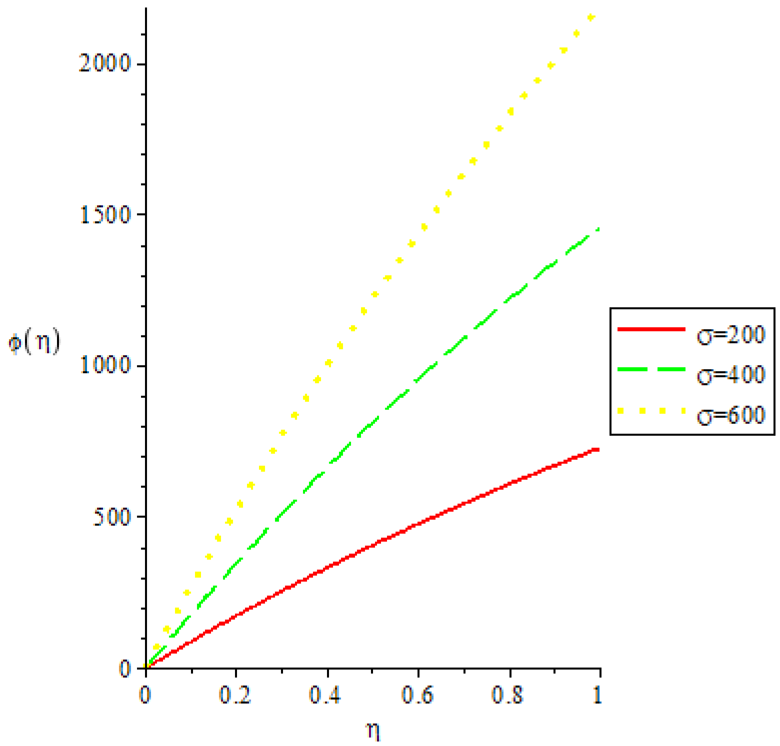

Figure 4: Variation of Concentration with σ

Table 1: The numerical values of skin friction, rate of heat and mass transfer

σ	Ec	Pr	Sc	$f''(0)$	$\theta'(0)$	$\phi'(0)$
200	0.001	0.71	0.22	2.413029923	-0.7217070810	910.0569506
200	0.001	0.85	0.22	2.413029923	-0.8641426876	910.0569506
200	0.001	1.00	0.22	2.413029923	-1.016785310	910.0569506
200	0.001	0.71	0.62	2.413029923	-0.7217070810	323.5472278
200	0.001	0.71	0.78	2.413029923	-0.7217070810	257.3770162
200	0.030	0.71	0.22	2.413029923	-0.7450149972	910.0569506
200	0.050	0.71	0.22	2.413029923	-0.7610894222	910.0569506
400	0.001	0.71	0.22	2.413029923	-0.7217070810	1819.147859
600	0.001	0.71	0.22	2.413029923	-0.7217070810	2728.238768

5 Conclusion

From the studies made on this paper we conclude as under.

1. Prandtl number decreases the fluid temperature.
2. Schmidt number decreases the species concentration.
3. Eckert number enhances the fluid temperature.
4. Reaction rate enhances the species concentration.
5. Heat transfer rate decreases due to increase of Prandtl number and Eckert number.
6. Mass transfer rate decreases due to increase of Schmidt number.
7. Mass transfer rate increases due to increase of reaction rate.

References

[1] K.R. Khair and A. Bejan, Mass transfer to natural convection boundary layer flow driven heat transfer, *Research Journal of Mathematics and Statistics*, 4(3) (2012), 63-69.

[2] S.B. Doma, I.H. El-Sirafy and A.H. El-Sharif, Two-dimensional fluid past a rectangle plate with variable initial velocity, *Alexandria Journal of Mathematics*, 1(2010), 1-22.

[3] M.A. Hossain, M.M. Molla and L.S. Yao, Natural convection flow along a vertical wavy surface with uniform surface temperature in the presence of heat generation/absorption, *International Journal of Thermal Science*, 43(2) (2004), 157-163, www.researchgate.net.

[4] K. Chand, R. Kumar and S. Sharma, Hydromagnetic oscillatory flow through a porous medium bounded by two vertical porous plate with heat source and soret effect, *Advance in Applied Science Research*, 3(4) (2012), 2169-2178, www.pelaglaresarchlibrary.com.

[5] P.R. Sharma and G. Singh, Unsteady MHD free convection flow and heat transfer along a vertical porous plate with variable suction and internal heat generation, *International Journal of Applied Mathematics*, 4(2008), 1-8.

[6] J.H. He, Some asymptotic methods for strongly nonlinear equations, *Int. J. Modern Phys.*, B 20(10) (2006), 1141-1199.