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A Framework for Feeder Bus Route Design Network Problem (FBRNDP)

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ABSTRACT

Metropolitan areas have in recent decades tried to improve the sustainability of transportation systems by developing efficient multi-modal public transit services. These systems are often designed independently and can lead to problems such as; duplicated lines or routes, increased out of vehicle times especially at transfer points, in-vehicle travel times, and a complicated fare structure, etc. Improving these multimodal systems will require an integrated transit system which in its simplest form consists of a coordinated operational design of the main lines (rapid rail, rapid bus systems) and a number of feeder routes connecting transfer stations. This work describes the basic components of FBRNDP ranging from the design of problem space and the objective function to solution methods. Furthermore, a flow of work is proposed for the initial route generation methods for feeder bus routes and also a systematic framework for the design of FBRNDP.

Keywords: Feeder bus routes, heuristics, metaheuristics, network, transportation systems.

1. INTRODUCTION

Planning a transportation system has four main elements carried out in sequence; they include transit route design, setting of transit frequencies, timetabling, bus and driver scheduling and together they are all known as the "*transit network design problem*-TNDP". A sub problem for TNDP relating to route networks is termed as *"transit route network design problem-TRNDP"*. It is associated with the design of a network of transit pathways and their associated frequencies so as to achieve some stated objectives, which are restricted by some operational bounds. Specific to bus routes is BRNDP which perhaps is the first and most important step in planning a bus transit services as highlighted by (Ceder and Wilson, 1986). A look at Figure 1(a) below consist of stops or demand locations for passengers in a transit network. BRNDP by implication is a way of connecting these demand locations in such a way that, most of the passengers if not all can connect at least from one demand location to another while ensuring that the objective function is optimized taking into consideration the perspectives imposed by the planners and operators. On the other hand, another type exists which involves a system that collects passengers in certain demand positions; usually residential areas, and connects them to a transfer point where they can continue their journey on the main service, in other words, it is the design of a feeder bus system which provides access to an existing main trunk line public transport system. It is called Feeder Bus Route network design Problem (FBRNDP) and it differs from the first type in that, there is a main trunk line which could either be rail system, or rapid bus system connected to another the bus/ paratransit systems which carries the passengers from designated areas such as bus stops of a locality to the different stations on the main line (Kuan *et al*. 2006). See Figure 1 below for an apt description of both cases. Developing an efficient public transit service to take into account sustainability will require a multi-modal transportation service so as to tap into different advantages it carries. Depending on the characteristics of the modes like speed, capacity, etc. this system is mainly classified into main lines and feeder lines. These multimodal systems or intermodal systems are often designed independently and can lead to increase in travel times both out of vehicle times at transfer points and in-vehicle times, duplicated lines, and complicated fare structures, etc. making the public transit system(PTs) less attractive especially from dissatisfied users. Improving these kind of systems can be very complex. It may be of benefit especially to passenger's comfort and convenience and reduction of both users and operating costs if these systems are integrated and coordinated. The aforementioned problems can be relaxed if a combined system between major public transport modes can be achieved through; ease of traveler's mobility between modes, clarity of service areas between each mode, reduction of duplicated services, joint fare structure, adjustment and coordination of schedules between modes, amongst other reasons. This type of transportation system creates a huge challenge and difficulties in the efficient and sustainable design of public transportation systems. Although a lot of researches have been conducted in this regards, there is a need to provide background information on the systematic and integrated approaches for designing an efficient multi-modal public transit network. This work identifies the components and states clearly a systematic and a consistent step in the design FBRNDP.

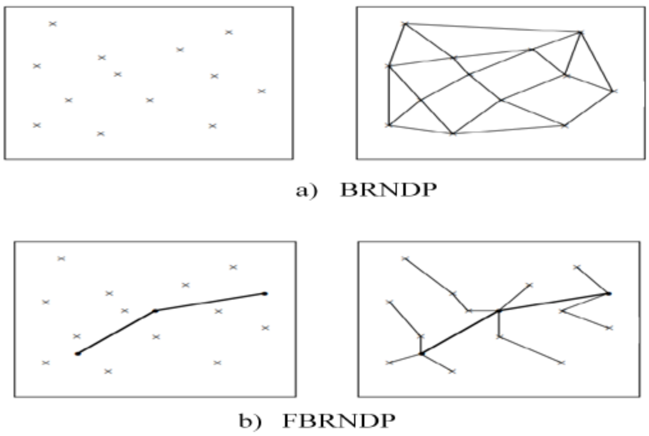

Figure1: this figure shows the difference between a) BRNDP and b) FBRNDP

2. COMPONENTS OF FBRNDP

This section describes the various components necessary for the design of FBRNDP. It is basically divided into 4 components, each closely related to one another. They are described in the following subsections. Figure 2 below expresses how FBRNDP can be grouped into various parts for the design and improvement feeder bus route network.

2.1. 1 Representation of FBRNDP

In TNDP, there is a need for the representation of network problem in mathematical terms so as to solve them. A transit network is mainly comprised of routes and stops which is represented by a graph with interconnecting nodes and arcs. This representation normally varies from one planner to the other because of the data availability, level of details used, and method of analysis. According to (Owais, 2015), street network representation can be classified into two basic levels.

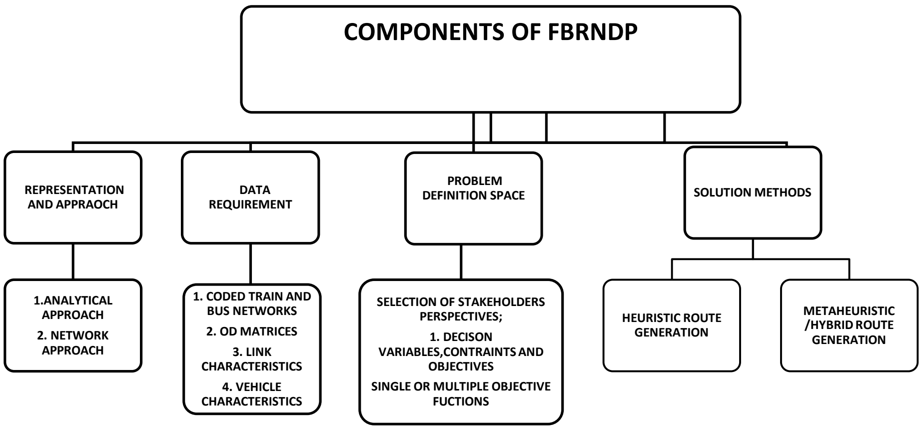

Figure 2: The components of FBRNDP

The first one aggregates demand at the zonal level and the second one identifies potential demand at each bus stop i.e. node level. Although, it may not be feasible to consider all bus stops, so consideration is given according to nodal demand and perhaps limits of walking distance from those nodes. It is worthy of note that, the representation of the transit network differs from original road networks which they are derived from. As stated earlier not all stops nor links can be fully represented and therefore, a distinction between road networks and transit networks is pertinent. Street networks refer to all roads and intersections in the study area while transit networks include transit route and stops or stations. See figure 4 below. Considering the FBRNDP, the transportation network is built around a trunk line that forms the backbone of the network. This is usually a primary transportation service (e.g. a rail line or metro line or metro bus etc.) and to increase the service area, secondary transportation services (e.g. tram lines, bus lines, paratransit, etc.) are added. Figure 3 shows the different feeder lines; evolving from linear minimal networks as a mainline to the addition of feeders and to addition detours to form Maeander networks. Others are demand responsive and flexible feeders. To increase the efficiency of a feeder bus system, one must consider the opposing perspectives of the user, operators, planners and even non-users. While the users' perspective of a good feeder system is more of coverage and access in the service area, low access cost and so on, the operators' perspectives are to lower operation costs usually by keeping total route length within certain limits. Other perspectives are the non-users or the environment which these systems may affect. To put into perspective, feeder bus network design a problem FBNDP as whole according to literature (Byrne & Vuchic, 1972; Kuah & Perl, 1988; Kuah & Perl, 1989) is said to be composed mainly of feeder bus routes determination (stations and route structure) and operating frequency.

2.1.2 Approaches to FBRNDP

According to (Almasi,2015), there are two main approaches when trying to model FBRNDP namely; analytical and network approaches. The analytical approaches use actual road networks therefore, the shape and geometry of the road becomes a very important. Also, the demand in the service area should be adequately represented in the demand function. The objective function is defined using a set of continuous design variables such as feeder bus route positions, mainline station spacing and locations, and service frequencies. This implies that the optimal relationships between various components of the feeder bus network problem can be found using extreme conditions on the objective function. So many researchers employ this analytical approaches;(Kuah and Perl, 1988; Chien and Schonfeld, (1998); Chien and Yang, (2000); Chien *et al*., 2001; Chowdhury *et al*., 2002).

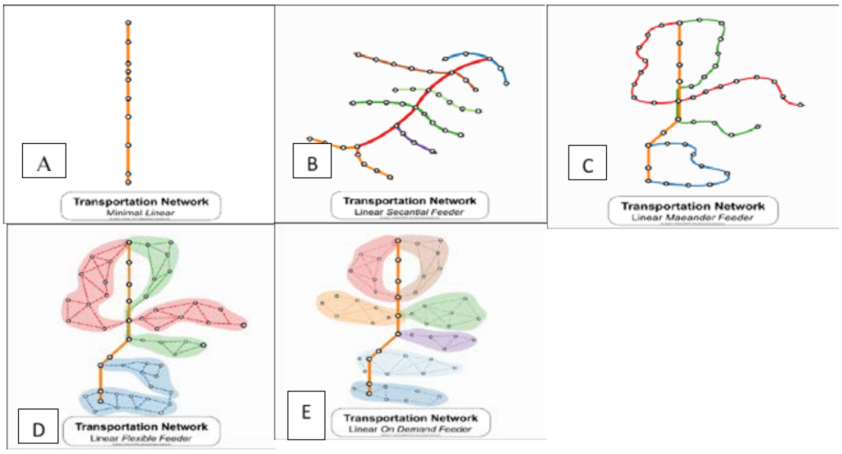

Figure 3: It shows the evolving services around the main line using different feeder systems

The disadvantage of analytic approaches is that they are only able to handle small and regular size networks. As the number of streets in the network increases so does the number of possible solutions increases which makes the search for an optimal solution from all possible solution critical, intractable and thus, analytical approaches are mainly used for theoretical purposes (Kuan *et.al*, 2004). Network approaches, on the other hand, tend to avoid the complexities of analytical approaches by considering the stops or stations and links as the service area. Transit routes are represented by a series of connected nodes while transit links can be represented by travel times or the travel distance between these connected nodes. As expressed in the previous section, demand can be taken at the nodes or aggregated in zone centroids and the entire network can be represented as the number of trips between all pairs of nodes as the origin-destination matrix. This approach can handle much larger size, irregular and real networks than the analytical approach. The network approaches have been used by many researchers (Kuah and Perl 1989; Martins and Pato 1998; Shrivastav and Dhingra 2001, Kuan *et al*. 2004, 2006; Mohaymany and Gholami 2010). See figure 4 below which shows the representation of transit networks as both analytical and network approaches.

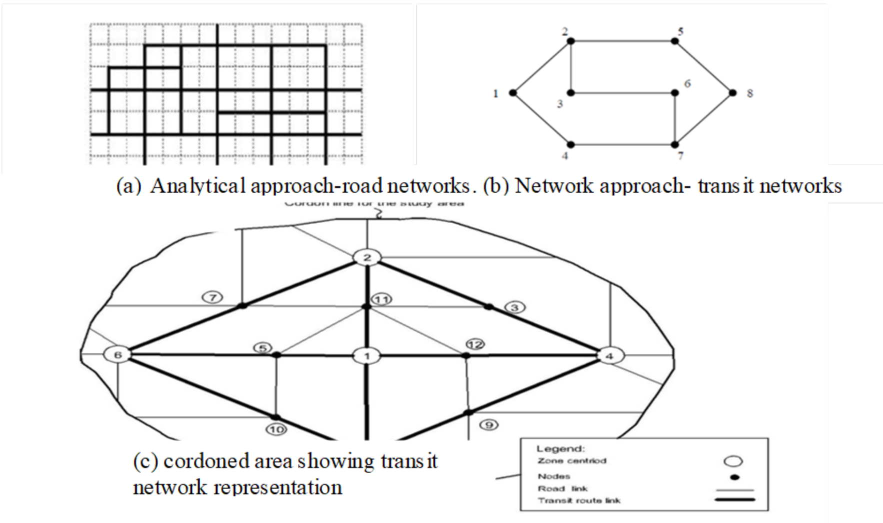

Figure 4: Representation of Transit Networks

2.2 Data Requirements

Different objectives of FBRNDP require different data: be it data which provides the demography, land use properties, socio-economic, surveys /smart card data extractions, etc. These together can be used to obtain details such as the behavioral pattern of travel, and the distribution of trips amongst traffic zones in the study area. They can also help in identifying potential zones to be served by regular feeder service. Kuah 1989, used four basic data for his analysis which includes; coordinates of the station and stop location, demand and stop density in the service area and also operating cost of both rail and bus transit which was derived from literature. Kuan 2004, also uses the same benchmark study and same data but introduces a second set of random data consisting of problem size and structure which was used to compare the performance of the different metaheuristics used. While on the other hand, other researchers like Shrivastava and O'Mahony; 2007 uses traffic surveys with coded networks to create potential O-D matrix which was then used to design the feeder bus route. Tabassuma, 2016 uses 3 sources of data; to include demography of the study area, socio-economic and land use characteristics so as to identify areas served by the feeder services and to obtain travel patterns. The researcher also used field surveys to obtain current travel patterns for main trunk users and assessment of current feeder modes used by to and from the main trunk line. Since the main is fixed and stop locations are fixed, a list of nodes with their coordinates, a list of links connecting the nodes, demand matrix based on travel behavior questionnaires or smart card data extractions and cost parameters which are based on ridership and financial reports may be required. As stated earlier the type of data requirements depends on the level of details required and type of objective to be met.

2.3 Problem Definition Space

This section constitutes all the complexities associated with trying to model a seemingly real situation in FBRNDP. It describes the demand pattern and characteristics, decision variables and constraints, stake holder's perspectives and objective functions.

2.3.1 Characteristics and Pattern of Demand

Demand can be classified based on its characteristics as "fixed-in elastic" and "variable-elastic"; this classification implies the effect of demand on the performance and services provided by PT networks. By inelastic we mean demand is not changing with performance or quality of service and vice versa for elastic. The elastic demand is a closer model to the real world and more objective according to literature (Lee and Vuchic, 2005). However, for simplification purposes, many researchers tend to use fixed demand (Fan Machemehl, 2006b). Moreover, for most developing cities, the issue of captive users is prevalent and the effect of variable demand is most likely minimized. That is to say that demand may be fixed for systems where passengers' preferences do not change with service quality or price of the service. However, variable demand can come to the fore when sharing or intense competition amongst the different modes of public transport, and increasing demand for mobility. These factors are important in the modeling of urban transportation, especially where different modes of transportation are present (Jakimavicius & Burinskiene, 2009).

Travel demand pattern is directly related to urban structure and spatial distribution of human activities. The decision between many-to-many(M-to-M) and many-to-one(M-to-1) demand patterns is one based on modeling approaches and prevalent conditions. An example is a case of public transportation networks which are designed towards commuting trips to and from the centers of activities i.e. many trip origins going to a single destination. It is expected that M-to-1 may be more suitable in this case. However, M-to-M patterns may be most useful in situations where transit services are designed to serve study areas with many important activity centers and or passengers having varying trip purposes. For FBRNDP both types of travel demand patterns, are relevant as mentioned above depending on the circumstance warranting their usage. The M-to-1 demand pattern is presented in many publications (Kuah and Perl, 1988, 1989; Chien and Schonfeld, 1998; Chien and Yang, 2000; Kuan, 2004; Kuan et al., 2004, 2006; Xiong et al., 2013), etc. It is often than not more related to FBRNDP, which transports passengers to and fro a common destination e.g. central business district (CBD) or a transfer station. The M-to-M differs from the M-to-1 for FBNDP in that the set of destinations includes the entire set of rail stations. Here, the demand at each bus stop is a multidimensional quantity and often, not simply a sum of M-to-1 FBNDPs, because, under it, a single feeder-bus route usually serves demands to multiple destinations making it difficult. First, the design of the feeder-bus network should take into account not only the linking to alternative rail stations but also alternative connections to rail lines. Depending on which rail line is chosen for connection, passengers may or may not have to transfer between rail lines. Second, the optimal feeder-bus network may include some bus stops on more than a single feeder-bus route. This results in a significantly more complex feeder-bus network.

2.4 Stake Holders' Perspectives and Objective Function

Public transportation systems often play a social role by attempting to lower operating cost as much as possible and also making mobility more accessible for the community in general in an equitable and efficient manner. Objectives for designing daily operations of a public transportation system should encompass some factors known to affect various stakeholders. Most studies actually focus on both the service and economic efficiency when designing such a system. Often, these objectives are controversial since cutbacks in operating costs may require reductions in the quality of services. Defining objectives, constraints, decision variables forms the basic structure of any TNDP complexity; this determines the problem space and consequently the type of solution method that can be employed. Determining objectives may come under the following factors such as political, social, environmental and economic factors either as a single or combined(multiple)objectives. Transit agencies are normally responsible for the choice of factors to be considered based on the importance of these factors as it relates to their goals and taking into cognizance other relevant stakeholders. The design FBRNDP is also driven by different and mostly contradicting objectives. This is largely due to the different stakeholders coexisting in the transportation planning space. Worthy of note is the fact that transportation requires a multi-dimensional and multi-perspective and a typical example of an objective is the social welfare which appears to be the most commonly used, usually interpreted as the minimization of total cost comprising mainly of user and operator's costs. Operators in general, in maintaining an operationally efficient transportation system try to reduce the overall cost of operation in the form of route length, fleet size and consequently minimizing the size of the crew. A study by Kuah and Perl, 1989 minimizes the total bus operating costs by optimizing routing structures and operating frequency. Baaj and Mahmassani, 1991 in their publication stated that TNDP is inherently a multi-objective problem. This allows planners, operators, users and even non-users to interweave together to form a complex mix that is a semblance of reality. User costs for FBRNDP may include the in-vehicle time, out of vehicle times (waiting time, transfer time), fare for both feeder buses and main trunk line vehicles.

2.5 Decision Variables and Constraints

There are many decision variables used and the most common are routes and frequencies or equivalent headways (kepaptasoglu and karlaftis, 2009). Other specific decision variables may include the size of the zones, location of the stops, size of the vehicles and fare structure. Especially that modern transit system offers a variety of fare products making it difficult to represent in a single fare level. Constraints are the policies and goals of the planners with the interest of other stakeholders in mind. Availability of resources is a major concern for the operator of the transit services which reflects the operating cost. Factors such as fleet size, the capacity of the vehicles, operating frequencies along with route length can represent desired performance standards which can be regulated by constraints. Kuah and Perl, 1989 gave some examples of constraints that can be used in FBRNDP. Similarly, Martins and Pato 1998 used the same constraints but with additional frequency limit. The examples of the constraints used by both authors are; capacity of the route, maximum number of fleet, the maximum length of route length and feasibility of the route. The feasibility of route was further checked with other constraints like; every bus stop must belong to a single route, the feeder network may include stops in more than one single route, these routes should be connected to a single station, all buses are assumed to stop at all stops in their routes, and bus stops are only assigned to the railway station if a route that passes through that stop terminates at that station. Fleet size constraint was used when considering resource optimization, unsatisfied demand constraint (i.e. insufficient direct feeder service or long distances from bus stops) was used by Shrivastava and O 'Mahony, 2009b for more efficient feeder routes. Although the number of transfer can be a useful constraint, the users may decide to move to other alternatives especially if more than two transfers were required. Nes, 2002 suggested that the development of a network specific shape (e.g. triangular, rectangular, grid, radial, etc.) was common among some transit agencies. Also, a common approach is to consider some of these constraints as objective functions since constraints might represent operator's goals, objectives or target, or even performance standards (Martins and Pato 1998 and Mohaymany and Gholami, 2010). These authors minimized the total cost by using a frequency variable bound as a constraint and still uses it as an objective. Others achieve optimal headway range in specific lines and chosen areas by using headways for both buses and trains as objectives and apply limits on the same headways (Chowdhury *et.al*. 2002).

3. SOLUTION METHODS

Methodologies in solving transit route network design problem are mainly influenced by; Level of details in the design environment, Quality of anticipated solution and available computational power.

TRNDP is usually partitioned in a sequence of procedures so as to be manageable. Two major approaches;

1. Route generation and configuration;
2. Route construction and improvement,

(kepapstaglou and kralifits,2009)

3.1.1 Heuristic used in Initial Route Generation Methods

The first approach, are usually heuristic; they are based on experiences used in the past for solving a similar problem which can be employed to reduce the size of the solution search space but in general do not guarantee an optimal solution but a practical solution. They are widely used in transit planning since they utilize planners' knowledge but may not be transferable to other systems. It involves finding a set of candidate routes considered as an initial solution for route design stage i.e. shortest-path-based algorithms are used to generate some candidate routes under certain constraints. These constraints may include the maximum/minimum number of routes, length of the route, travel time limit, etc. Because of the flexibility and practicability of heuristic methods, it is a common approach among literature. Therefore, FBRNDP can be solved by applying heuristic algorithms to reduces the size of the large solution search space, by building initial routes followed by improvement of these initial routes using optimization technique. Kuah and Perl 1989 sequential building heuristic for building initial solutions which is adoption from the sequential saving approach for Multi-Depot Vehicle Routing Problem (MDVRP). In an expanded study by Martins and Pato,1998 they generated the initial solution from two-phase building method by applying the sequential savings heuristics. Shrivastav and Dhingra,2003 proposed Heuristic Feeder Route Generation Algorithm (HFRGA) based on the demand matrix of Hadi Baaj and Mahmassani. Also, metaheuristics like Genetic algorithm (GA) can be used to the initial population. The initial population can also be generated at random as suggested by Chien *et al.* 2001 but it might not be a good selection for generating initial routes. Therefore, Kuan et al.2004 employed the concept of delimiter, proposed by Breedam, 2000. Most of the studies follow this approach of generating initial routes using various kinds of heuristics and then try to improve the initial solution by using the initial solution as an input in some optimization techniques to obtain a better solution, improvements can be implemented on the routes. There are a lot of optimization methods to improve solutions.

3.1.2 Metaheuristics/Hybrid used in Direct Route construction and Improvement

An alternative to initial route generation and subsequent improvement is direct route construction by using some metaheuristic and hybrid methods. Meta-heuristic methods, unlike heuristic approaches, are able to generate local optimum solutions to combinatorial optimization problems where FBRNDP belongs Most common examples used by researchers are; Ant colony optimization (ACO), Exhaustive search(ES), Genetic algorithm(GA), Simulated annealing(SA), Tabu search (TS), (Kuah and Perl, 1996 , Martins and Pato, 1998, Chien *et al.* 2001, Kuan *et al.* 2004, Kuan *et al.* 2006 Shrivastava and O'Mahony, 2006). For FBRNDP, the initial sets of routes or solutions are developed and further improvement is required for a better solution by using nature-inspired and non-nature inspired algorithms. These algorithms have their strengths and weaknesses, a comparison between the algorithms was carried out by several authors (Chien *et al.* 2001; Kuan *et a*l. 2004; Kuan *et al.* 2006). Some results of the performance comparison indicated that the local optimum solutions derived using ES and GA are identical even though ES had a higher computational time than GA, especially for large or complicated networks. Neighborhood search methods like SA and TS are good methods but defining the neighborhood seems to be very complex and difficult as solutions might keep changing. Also, difficulties arise from GAs in finding suitable cross overs every time. ACO does not depend on neighborhoods but depends on the continuous iteration of the previous solution to generate better solutions thus making it dependent on the number of iterations. Having discussed the pros and cons of some methods, a composite of some of them have been tried in the pasts. Some studies use heuristic method to generate the potential routes and then subsequently an optimization techniques scheduling problems (Shrivastava and O'Mahony, 2007). In another work by the same author, the reverse was the case, they used a technique with two sub-models: routing was done by using a genetic algorithm, and a subsequent heuristic repair algorithm was then applied (Shrivastava and O'Mahony, 2009a). In another research (Shrivastava and O'Mahony,2009b) developed a hybrid algorithm (SOHFRGA) using a similar theme of first designing the bus routes using GA and using the developed heuristics for coordination problem. In a more recent work Ciaffi et.al.2012 also used hybrid metaheuristics comprising of GA and a heuristic. Hybrid methods hold a lot of promises in terms of tackling problems that used to be intractable and can exploit the advantages of different methods thereby opening the possibility of integration of potential solution methods that can be combined.

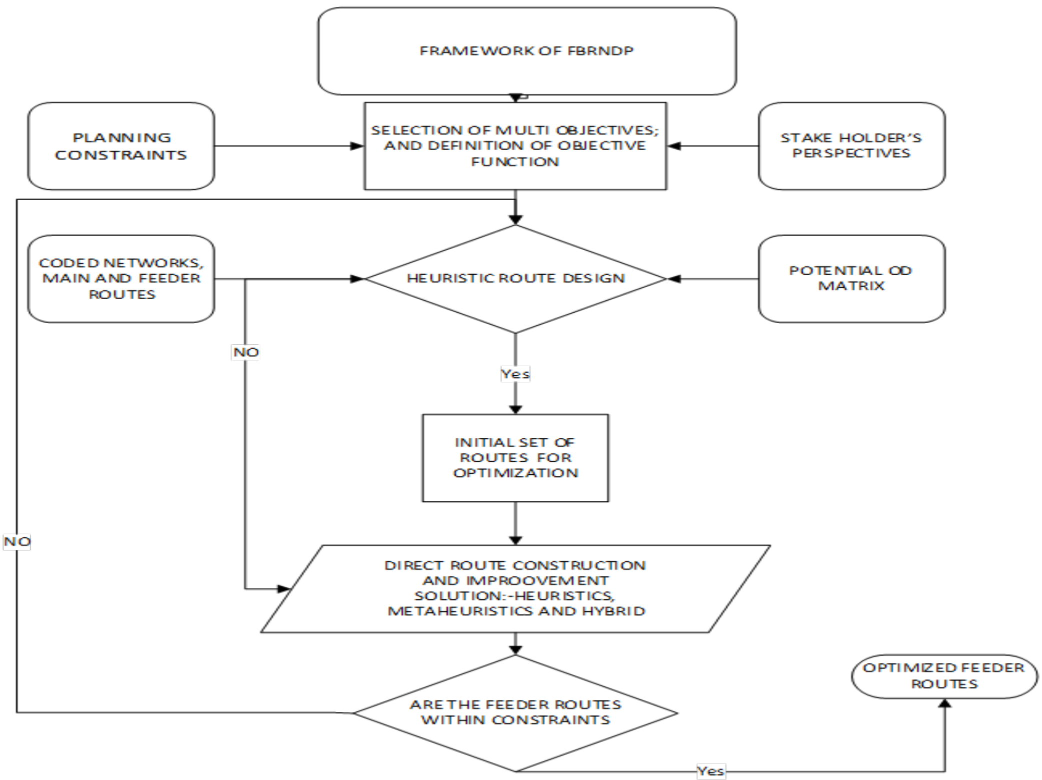

Figure 6: Proposed General Methodology for Design Feeder Bus Route

Figure 6shows a proposed methodology that can be used in designing FBRNDP. This flow of work takes into account the different components already discussed in the previous sections and presents the different approaches; heuristic route design and subsequent optimization of the initial route generated and direct route construction and improvement using metaheuristics.

4. CONCLUSION AND AREAS OF FURTHER RESEARCH

Attempts have been made of over decades to design a more efficient and yet sustainable transportation system by integrating the independent operations of multi-modal transportation through the coordination between main trunk lines and feeder services. The FBRNDP has a lot of potentials and researches are still ongoing because of many reasons such as; new policies by operators, level of details and the development of more sophisticated computing power. These reasons create new requirements, challenges, and potentials for planners and researchers. The FBNDP is a large and complex routing problem that can be only solved satisfactorily using a heuristic, metaheuristic and sometimes a combination of hybrid approaches. The object of the paper is to highlight the components of FBRNDP and a framework for designing feeder bus routes connected to the main truck line. See Figure 7 below for the suggested framework of FBRNDP. It comprises of three sequential steps starting at the selection of objectives taking into consideration stakeholders perspective as well local conditions such as emergencies, special services like owl services; then moving to state the decision variables and limitations or constraints. Together these two components form the target function and its limitations. The last part is the solution method which is either sequential or simultaneous.
Because of the ensuing challenges and continuous developments in the fields of mathematics and computers analysis, there are still areas of untapped opportunities for future research. Other areas that can be explored are listed below;

1. Building O-D matrix: A more detailed OD matrix at bus stop level might be possible with smartcard data. This is very important for a successful feeder route services where an accurate selection of route generation nodes is pertinent.

2. Feasibility and post evaluation: Aspects such as walking activities within transfer stations and capacity of the stations themselves can create serious concerns.

3. Modelling Aspects: conflicting perspectives of stake holders, t multimodal, makes route generation problem a multi objective The design of better metaheuristics can be explored as most of these solution methods have pros and cons leading to hybrid methods

4. Nature of problem: transit services are mainly planned and designed for normal daily operations but aspects like disasters management, seasonal/mega events or even emergencies for hospitals, owl services, etc. will change the nature of the problem and will open a new frontier in urban transit planning.

5. Access Modes: In most of the researches, railways, and feeder buses are the most researched even though different access modes or mode combinations exist. Modes like cycling, walking, can also be used to access main trunk lines station. Therefore, the inclusion of different modes or mode combinations can be looked into. The consideration of these access modes can help model FBRNDP to be much closer to the real world.

6. Fare: operators normally uses fare setting as a strategy for profit making while keeping in mind the total welfare in mind. This creates some sort of opposing objectives thereby leading to different fare strategies and products. Multimodal transportation also offers a different variety of fare products depending on the characteristics of the mode and the intent of the planners.

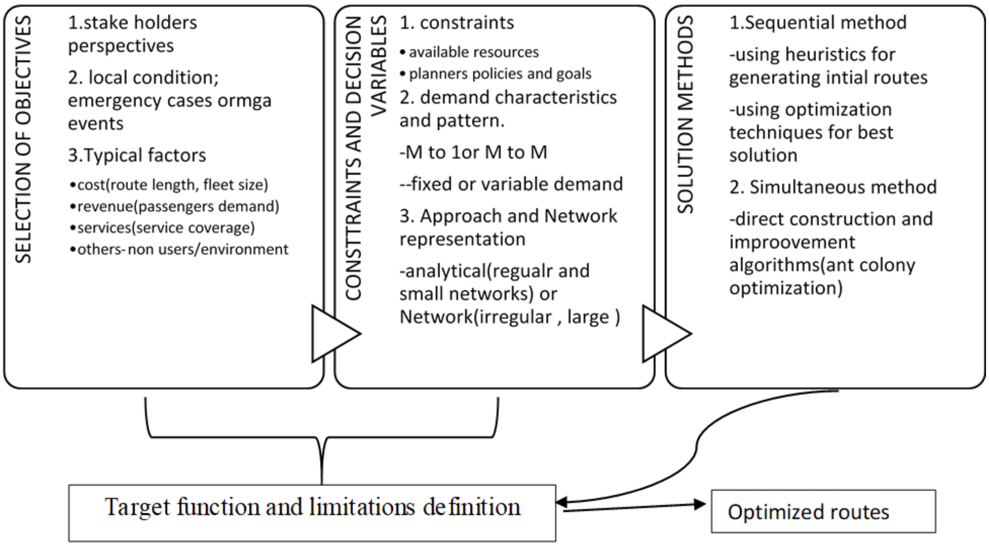

Figure 7: A Suggested framework for FBRNDP

REFERENCES

Almasi, M. (2015). Modelling and optimization of a transit services with feeder bus and rail system (Doctoral dissertation, University of Malaya).

Baaj, M. H., and Mahmassani H. S. (1991). AI-based approach for transit route system planning and design. Journal of Advanced Transportation, 25(2), 187–209.

Baaj M. H. and Mahmassani H. S. 1995. “Hybrid route generation heuristic algorithm for the design of transit networks," Transportation Research C, vol.3, no.1, pp.31–50.

Breedam, A.V. 2000 “Comparing descent heuristics and metaheuristics for the vehicle routing problem,” Computers and Operations Research, vol.28, no.4, pp.289–315.

Ceder A. and. Wilson N.H.M 1986. “Bus network design. Transportation Research B, vol.20, no.4, pp.331–344.

Chowdhury S. M. and Chien, S. I.-J. 2002. Intermodal transit system coordination. Transportation Planning and Technology, vol.25, no. 4, pp. 257–287.

Chien S. I.-J, 2005 “Optimization of headway, vehicle size and route choice for mínimum cost feeder service,” Transportación Planning and Technology, vol.28, no.5, pp.359–380.

Chien S. and Schonfeld, P. 1998. Joint optimization of a rail transit line and its feeder bus system. Journal of Advanced Transportation, vol.32, no.3, pp.253–284.
Chien S. and Yang Z. 2000. Optimal feeder bus routes on irregular street networks. Journal of Advanced Transportation, vol.34, no. 2, pp. 213–248, 2000.

Chien S., Yang Z., and Hou, E. 2001. Genetic algorithm approach for transit route planning and design. Journal of Transportation Engineering, vol.127, no.3, pp.200–207.

Epstein, D. 1994. “Finding the k shortest paths,” in Proceedings of the 35th IEEE Symposium on Foundations of Computer Science, pp. 154–165.

Gholami A. and Mohaymany, A. S. 2011. Economic conditions for minibus usage in a multimodal feeder network. Transportation Planning and Technology, vol.34, no.8, pp.839-856.

Fan, W., and Machemehl, R. 2004. Optimal transit route network design problem: Algorithm, implementations, and numerical results. Rep. SWUTC/04/167244-1, Center for Transportation Research, the University of Texas at Austin, Tex.

Jakimavicius M. and Burinskiene M. 2009. Assessment of Vilnius city development scenarios based on transport system modeling and multicriteria analysis. Journal of Civil Engineering and Management, vol.15, no.4, pp.361–368.

Kepaptasoglu, K., and Karlaftis, M. (2009). Transit route network design problem: Review. Journal of Transportation Engineering, 10.1061/(ASCE)0733-947X (2009) 135:8(491), 491–505.

Kuah G. K. and Perl J.1989. “The feeder-bus network-design problem,” Journal of the Operational Research Society, vol.40, no.8, pp. 751–767.

Kuan, S. 2004 Applying metaheuristics to feeder bus network design problem Master of engineering thesis, Engineering Department of Industrial and Systems Engineering, National University of Singapore.

Kuan S. N., Ong H. L., and Ng K. M., 2006. Solving the feeder bus network design problem by genetic algorithms and ant colony optimization. Advances in Engineering Software, vol.37, no.6, pp. 351–359.

Lee, Y.-J., and Vuchic, V. R. 2005. Transit network design with variable demand. Journal of Transportation Engineering, 1311, 1–10.

Lin J. & Wong Hu.2014. Optimization of a feeder-bus route design by using a multi-objective programming approach, Transportation Planning, and Technology, 37:5

Martins A.L. and Pato M. V. 1998. Search strategies for the feeder bus network design problem. European Journal of Operational Research, vol. 106, no. 2-3, pp. 425–440.

Mohaymany, A. S. and Gholami A. 2010. “Multimodal feeder network design problem: ant colony optimization approach,” Journal of Transportation Engineering, vol.136, no.4, pp. 323–331.

Nikolic M. and Teodorovic D., 2013. Transit network design by Bee Colony Optimization, Expert Systems with Applications, vol. 40, no.15, pp.5945–5955.

Nes, R V. 2002. Design of multimodal transport networks. (2002), the Ph.D. thesis of Delft University of Technology.

Owais, M. (2015). Investigating Optimal Bus Routes. Planning and Operation in Urban Areas. GRIN Verlag

Shrivastava P. and O’Mahony, M. 2009. “Modeling an Integrated Public Transportation System a Case Study in Dublin,” Ireland.

Shrivastava P. and O’Mahony, M., 2006. A model for the development of optimized feeder routes and coordinated schedules—A genetic algorithms approach. Transport Policy, vol.13, no.5, pp.413‑425.

Shrivastava P. and O’Mahony, M. 2007. Design of feeder route network using combined genetic algorithm and specialized repair heuristic. Journal of Public Transportation, vol.10, no.2, pp. 99–123.

Shrivastav P. and Dhingra S. L., 2001. Development of feeder routes for Suburban railway stations using a heuristic approach. Journal of Transportation Engineering, vol.127, no.4, pp.334‑341.

Tabassuma S., 2016. Feeder network design to access an existing bus rapid transit system in Lahore. Ph.D. dissertation Submitted to the Graduate School of Urban Innovation, Yokohama National University, Japan.