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Modeling And Analytical Simulation Of Heat And Mass Transfer In The Flow Of An Incompressible Viscous Fluid Past An Infinite Horizontal Wall

[1]A. A. Mohammed, [2]R. O. Olayiwola, [3]Bashir A. U.,[4]E. M. Yisa
[1,2]Department of Mathematics and Statistics,Federal University of Technology, Minna, Nigeria.
[3]Department of Science Education,Federal University of Technology, Minna, Nigeria.
[4]Department of Mathematics and Statistics,Federal University of Technology, Minna, Nigeria.

Abstract:*This paper presents an analytical method to describe the heat and mass transfer in the flow of an incompressible viscous fluid past an infinite horizontal wall. The governing equations account for the viscous dissipation effect and mass transfer with chemical reaction of constant reaction rate. The coupled partial differential equations describing the phenomenon have been solved analytically using variable seperation method and Fourier Sine transform. The results obtained are presented graphically. It is discovered that the Schmidt number enhances the fluid velocity and decreases the fluid temperature. Lewis number and Eckert number enhance the fluid temperature while reaction rate number decreases the species concentration.*
Keywords and phrases:*Heat and mass transfer, incompressible fluid, viscous dissipation, chemical reaction, variable seperation method, fourier sine transform.*

I. Introduction

In recent years, fluid flow and heat transfer problems have attracted the attention of a number of scholars because of their possible application in many branches of science and technology such as fibre and granular insulation, geo-thermal system, etc. The phenomena of heat and mass transfer are also very common in theory of stellar structure and observable effects are detectable on the solar structure. In nature and industrial application many transport processes exist where the heat and mass transfer takes place simultaneously as a result of combined effects of thermal diffusion and diffusion of chemical species.

The study of heat generation or absorption effects in moving fluids is important in view of several physical problems such as fluids undergoing exothermic or endothermic or transfer chemical reactions.

An extensive contribution on heat and mass transfer flow has been made by Khair and Bejan[1].Olajuwon and Dahimire[2] examined unsteady free convection heat and mass transfer in an MHD micropolar fluid in the presence of thermo diffusion and thermal radiation. They studied the effects of thermo-diffusion and thermal radiation on unsteady heat and mass transfer. The results show that the observed parameters have significance influence on the flow, heat and mass transfer. Uwanta and Omokhuale[3] studied viscoelastic fluid flow in a fixed plane with heat and mass transfer.

Recently, Ibrahim [4] examined the unsteady MHD convection heat and mass transfer past an infinite vertical plate embedded in a porous medium with radiation and chemical reaction under the influence of Dufour and Soret effects.

The objective of this paper is to obtain an analytical solution for describing the heat and mass transfer in the flow of an incompressible viscous fluid past an infinite horizontal wall. To simulate the flow analytically, the viscous dissipation effect is retained and mass transfer with chemical reaction of constant reaction rate is considered.We assume the wall suddenly start to move with constant velocity.

II. Model Formulation

Consider anunsteady two-dimensional mass transfer flow of an incompressible viscous fluid past an infinite horizontal wall. The wall which is maintained at a constant temperature T_w suddenly start to move with constant velocity U and the concentration is maintained at a constant value C_w. Introducing a Cartesian coordinate system, x -axis is chosen along the wall in the direction of flow and y -axis normal to it. Far above the wall, the temperature is assumed to be same as initial temperature of the fluid T_0 and the concentration assumed to be zero. We ignored pressure gradient and assume no body force. The viscous dissipation effect is retained and mass transfer with chemical reaction of constant reaction rate is considered. With the above assumptions the system of governing equations to be solved is:

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

Momentum equation

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \qquad (2)$$

Energy equation

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \qquad (3)$$

Species equation

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)+\sigma C \qquad (4)$$

This is a two-dimensional problem. Thus, the velocity vector

$$\underline{q}=(u,\,0), \qquad u=u(x,y,t), \qquad v=v(x,y,t) \qquad (5)$$

By symmetry and from continuity equation (1):

$$\nabla\cdot\underline{q}=0 \quad i.e., \quad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (6)$$

But $v=0$, so $\frac{\partial u}{\partial x}=0 \quad \Rightarrow u=u(y,t)$ (7)

Then, the system of governing equations reduce to

$$\frac{\partial u}{\partial t}=\upsilon\left(\frac{\partial^2 u}{\partial y^2}\right) \qquad (8)$$

$$\frac{\partial T}{\partial t}=\frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial y^2}\right)+\frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \qquad (9)$$

$$\frac{\partial C}{\partial t}=D\left(\frac{\partial^2 C}{\partial y^2}\right)+\sigma C \qquad (10)$$

withinitial and boundary conditions:

$$\left.\begin{array}{lll} u(y,0)=0, & u(0,t)=U, & u(\infty,t)=0 \\ T(y,0)=T_0, & T(0,t)=T_w, & T(\infty,t)=T_0 \\ C(y,0)=0, & C(0,t)=C_w, & C(\infty,t)=0 \end{array}\right\}, \qquad (11)$$

where the subscript w represents the condition at the wall. υ is the kinematic viscosity, t is the time, ρ is the fluid density, u and v are the components of velocity along x and y directions respectively, T is the temperature of the fluid, C is the species concentration, c_p is the specific heat capacity at constant pressure, σ is the reaction rate, k is the thermal conductivity, D is the diffusion coefficient.

III. Method Of Solution

3.1 Non-Dimensionalization

Here, we non-dimensionalized equations (8) – (11), using the following dimensionless variables:

$$t'=\frac{Dt}{L^2}, \qquad y'=\frac{y}{L}, \qquad \phi=\frac{C}{C_w}, \qquad \theta=\frac{T-T_0}{T_w-T_0}$$ (12) and we obtain

$$\frac{\partial u}{\partial t} = Sc\,\frac{\partial^2 u}{\partial y^2} \quad (13)$$

$$\frac{\partial \theta}{\partial t} = Le\,\frac{\partial^2 \theta}{\partial y^2} + Sc\;Ec\left(\frac{\partial u}{\partial y}\right)^2 \quad (14)$$

$\frac{\partial \phi}{\partial t} = \frac{\partial^2 \phi}{\partial x^2} + \sigma_1 \phi$ (15) together with initial and boundary conditions:

$$\left.\begin{array}{lll} u(y,0) = 0, & u(0,t) = 1, & u(\infty,t) = 0 \\ T(y,0) = 0, & T(0,t) = 1, & T(\infty,t) = 0 \\ C(y,0) = 0, & C(0,t) = 1, & C(\infty,t) = 0 \end{array}\right\}, \qquad (16)$$

where

$Sc = \frac{\upsilon}{D}$ = Schmidt number, $Le = \frac{k}{\rho CpD} = \frac{\alpha}{D} = \frac{Sc}{\Pr}$ = Lewis number, $Ec = \frac{U^2}{(T_w - T_0)c_p}$ = Eckert number

3.2 Analytical Solution

Here, we solve equations (13) and (14) by change of variable method and carry out necessary integration to obtain our solution. For equation (15), Fourier sine transform is employed to obtain the solution. For equations (13) and (14), we let

$$u(y,t) = f(\eta), \qquad \theta(y,t) = \theta(\eta) \text{ ,where } \eta = \frac{y}{2\sqrt{Sc\;t}} \qquad (17)$$

andobtain the solution of equation (13) as

$$u(y,t) = erfc\left(\frac{y}{2\sqrt{Sc\;t}}\right) \qquad (18)$$

and solution of equation (14) as

$$\theta(y,t) = \frac{\pi}{4}\sqrt{\frac{2}{a}}\,erf\left(\sqrt{\frac{a}{2}}\frac{y}{2\sqrt{Sc\;t}}\right) erf\left(\sqrt{2-\frac{a}{2}}\frac{y}{2\sqrt{Sc\;t}}\right) + c\;erf\left(\sqrt{\frac{a}{2}}\frac{y}{2\sqrt{Sc\;t}}\right) + 1, \qquad (19)$$

where,

$$a = \frac{2\,Sc}{Le}, \qquad b = \frac{4\,Sc\;Ec}{Le\;\pi}, \qquad c = \left(\frac{b\sqrt{\pi}}{2\sqrt{2-\frac{a}{2}}} - \sqrt{\frac{2a}{\pi}}\right)$$

For equation (15), we let

$\phi(y,t) = v(y,t)e^{\sigma_1 t}$ (20)

and obtain

$$\frac{\partial v}{\partial t} - \frac{\partial^2 v}{\partial x^2} = 0 \qquad (21)$$

$$v(y,0) = 0, \qquad v(0,t) = e^{-\sigma_1 t}, \qquad v(\infty,t) = 0$$

Using the Fourier Sine transform (see Myint-U and Debnath [5], p. 333 - 335), we obtain the solution of problem (21) in compact form as:

$$v(y,t) = \frac{2}{\pi}\int_0^\infty s\sin\;sy\;e^{-s^2 t}\int_0^t e^{(s^2-\sigma_1)\tau}\,d\tau\;ds \quad (22)$$

andwe simplify further the solution (22) as follows:

Taking

$$\int_0^\infty s \sin sy\, e^{-s^2 t} \int_0^t e^{(s^2-\sigma_1)\tau} d\tau\, ds \quad (23)$$

and integrating with respect to τ , we have

$$e^{-\sigma_1 t} \int_0^\infty \frac{\sin \sqrt{z^2+\sigma_1}\, y}{z} \left(1-e^{-z^2 t}\right) dz \quad (24)$$

where

$$z = \sqrt{s^2 - \sigma_1}$$

We want to compare (24) with the integral (see Abramowitz and Stegun[6], p. 78):

$$\int_0^\infty \frac{\sin zy}{z} dz = \frac{\pi}{2} \qquad (y > 0)\,(25)$$

andthe integral (see Myint-U and Debnath [5], p. 334):

$$\int_0^\infty e^{-a^2 y^2} \frac{\sin zy}{z} dz = \frac{\pi}{2} erf\left(\frac{y}{2a}\right) (26)$$

We let

$$a^2 = t, \qquad a = \sqrt{t} \qquad and \qquad y^2 = z^2 = s^2 - \sigma_1 \quad \Rightarrow y^2 + \sigma_1 = s^2$$

Then

$$e^{-\sigma_1 t} \int_0^\infty \frac{\sin \sqrt{z^2+\sigma_1}\, y}{z} \left(1-e^{-z^2 t}\right) dz = \frac{\pi}{2} e^{-\sigma_1 t}\left(1 - erf\left(\frac{\sqrt{y^2+\sigma_1}}{2\sqrt{t}}\right)\right) = \frac{\pi}{2} e^{-\sigma_1 t} erfc\left(\frac{\sqrt{y^2+\sigma_1}}{2\sqrt{t}}\right) (27)$$

Then, we have

$$v(y,t) = e^{-\sigma_1 t}\left(1 - erf\left(\frac{\sqrt{y^2+\sigma_1}}{2\sqrt{t}}\right)\right) = e^{-\sigma_1 t} erfc\left(\frac{\sqrt{y^2+\sigma_1}}{2\sqrt{t}}\right) (28)$$

Thus,

$$\phi(y,t) = \left(1 - erf\left(\frac{\sqrt{y^2+\sigma_1}}{2\sqrt{t}}\right)\right) = erfc\left(\frac{\sqrt{y^2+\sigma_1}}{2\sqrt{t}}\right) (29)$$

The computations were done using computer symbolic algebraic package MAPLE.

IV. Results And Discussion

The systems of partial differential equations describing the mass transfer flow of an incompressible viscous fluid past an infinite horizontalwall are solved analytically using a change of variable method and Fourier sine transform. Analytical solutions of equations (13) - (16) are computed for the values of $Le = 0.5, 0.8, 1.0,\ Sc = 0.22, 0.62, 0.78,\ Ec = 0.1, 0.3, 0.5\ \sigma_1 = 2, 4, 6$. The following figures explain the fluid velocity, fluid temperature and species concentration distribution against different dimensionless parameters.

From figures 1, 2 and 3, we can conclude that with the increase of Schmidtnumber (Sc), velocity increases.

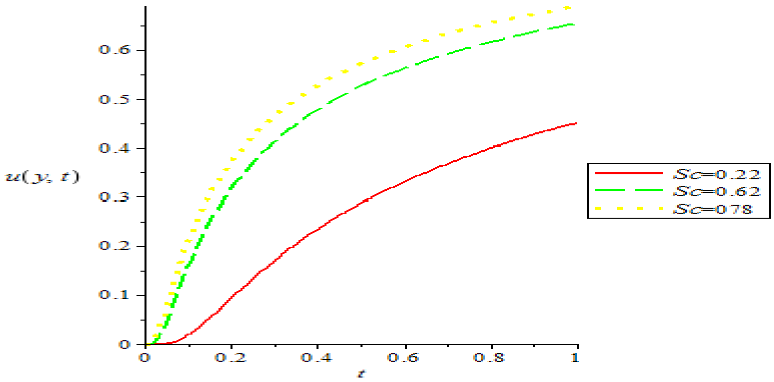

Figure 1: Variation of velocity with Sc

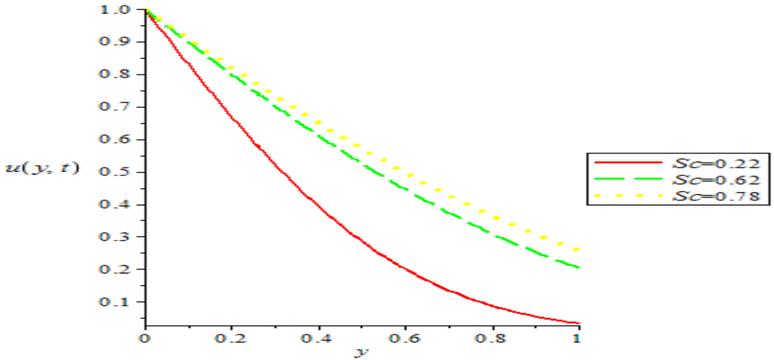

Figure 2: Variation of velocity with Sc

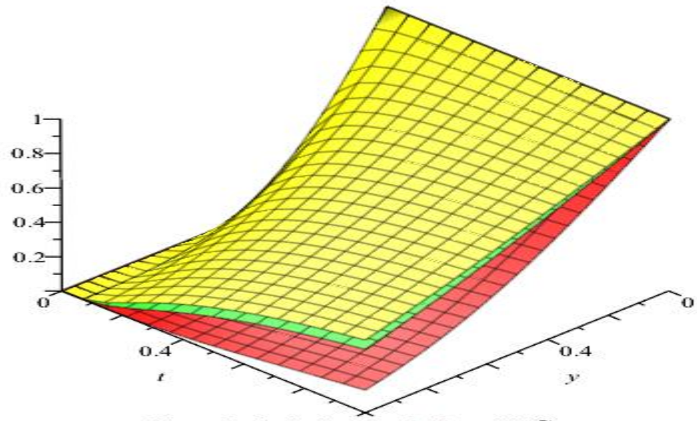

Figure 3: Variation of velocity with Sc

From figures4, 5 and 6, we can conclude that with the increase of Schmidtnumber (Sc), temperature decreases.

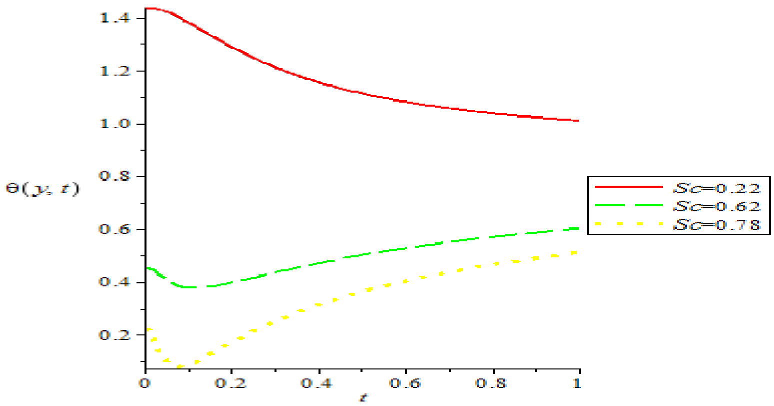

Figure 4: Variation of temperature with Sc

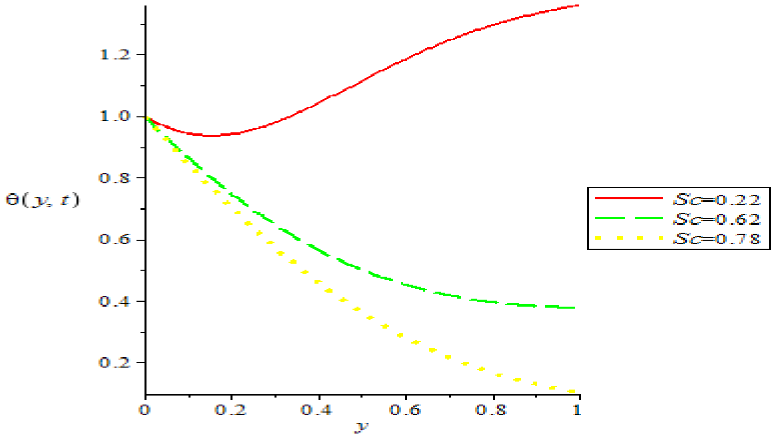

Figure 5: Variation of temperature with Sc

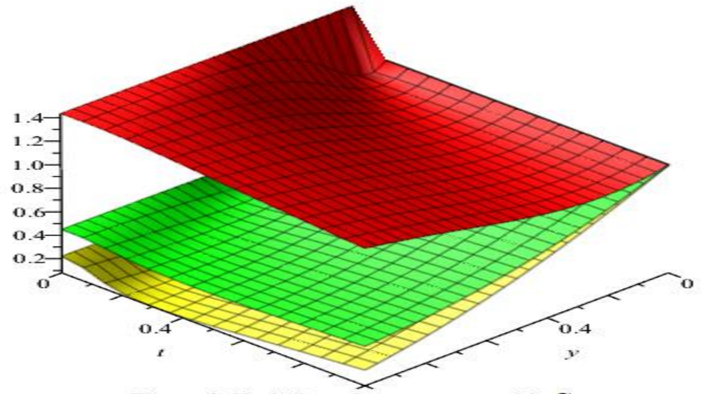

Figure 6: Variation of temperature with Sc

From figures7, 8 and 9, we can conclude that with the increase of Lewisnumber (Le), temperatureincreases.

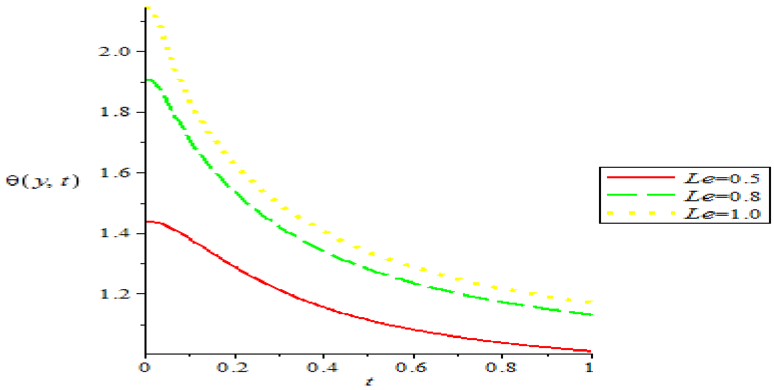

Figure 7: Variation of temperature with Le

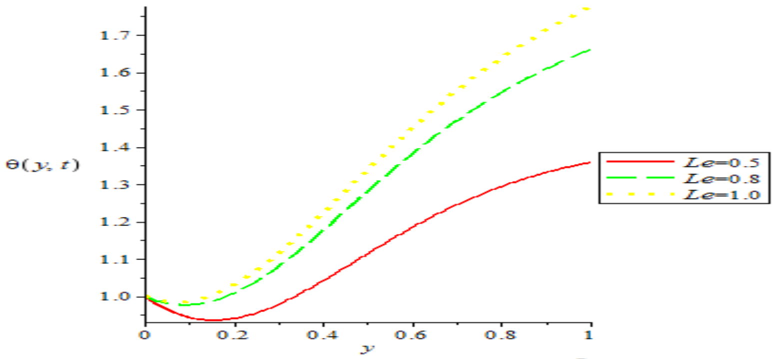

Figure 8: Variation of temperature with Le

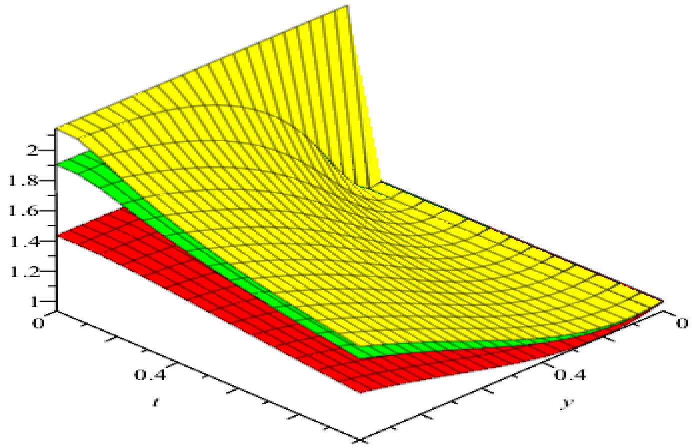

Figure 9: Variation of temperature with Le

From figures10, 11 and 12, we can conclude that with the increase of Eckertnumber (Ec), temperature increases.

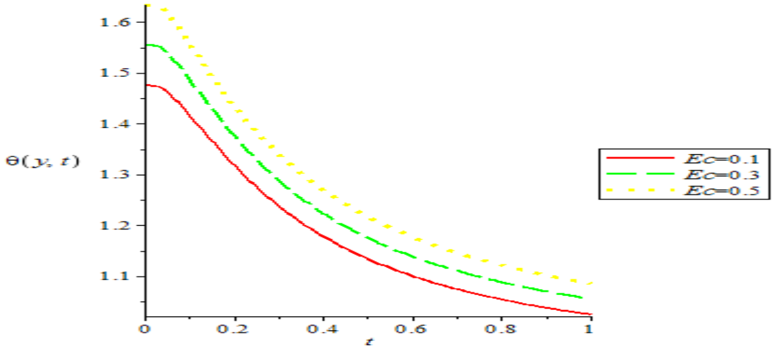

Figure 10: Variation of temperature with Ec

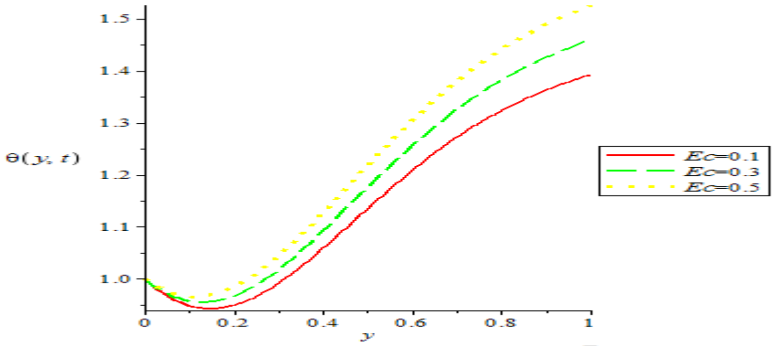

Figure 11: Variation of temperature with Ec

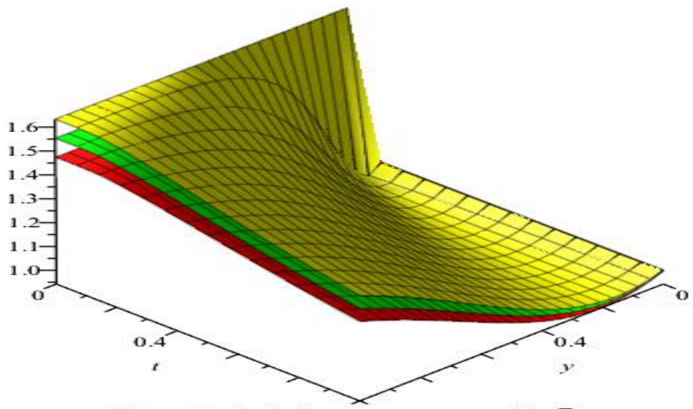

Figure 12: Variation of temperature with Ec

From figure 13, 14 and 15, we can conclude that with the increase of reaction rate (σ_1), species concentration decreases.

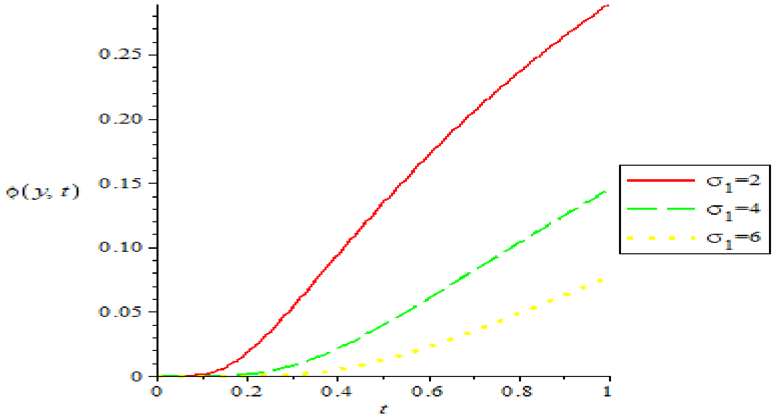

Figure 13: Variation of concentration with σ_1

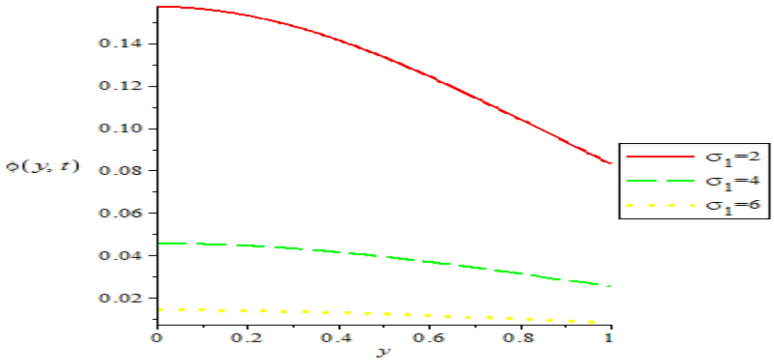

Figure 14: Variation of concentration with σ_1

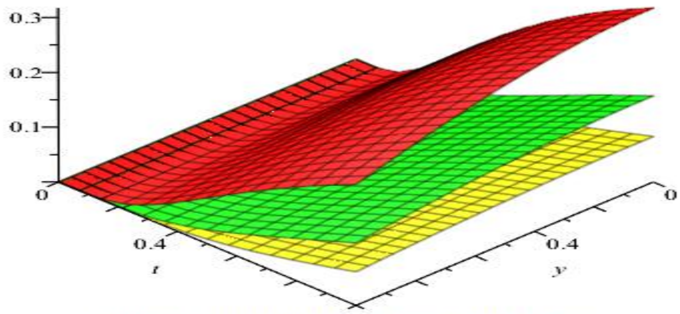

Figure 15: Variation of concentration with σ_1

V. Conclusion

From the studies made on this paper we conclude as under.

1. Schmidt number enhances the fluid velocity and decreases the fluid temperature.
2. Lewis number enhances the fluid temperature.
3. Eckert number enhances the fluid temperature.
4. Reaction rate number decreases the species concentration.

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