Process Noise Parameters of Beamforming Nodes in Wireless Sensor Networks (WSNs)

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***Abstract*— Signals from collaborative beamforming (CB) nodes are always arriving out of phase at the intended receiver due to unsynchronized clock frequencies of different oscillators of these nodes. Beamforming nodes have to synchronize their carrier frequencies in order to eliminate any phase offset of the received signal. To do this, estimation, correction and prediction of phase offsets due largely to these drifts needs be carried out on Software Defined-Radio (SDR) while employing non-linear filters like the Extended Kalman Filter (EKF) so as to obtain a near zero (0) phase offset. This paper presents a method of computing the NI USRP-2920 process noise parameters of the phase noise (dBc/Hz) data obtained with keysight N9320B spectrum analyzer (SA). The effects of phase offset measurements at various offset frequencies of the carrier are shown. Power-law noise model is applied to compute the Allan variance and later the process noise parameters that will be applied with the EKF. It will be noticed that the Allan variance curve truly depicts the standard slope characteristics of white frequency and random walk frequency noises which are special considerations when dealing with USRP's oscillators.**

Keywords—beamforming; wireless sensor networks (WSN); phase noise, usrp.

I. INTRODUCTION

When two or more antennas that assume virtual array arrangement transmits a common message signal collectively/jointly to an intended receiver, then they are said to have beam formed their signals otherwise known as CB [1]. CB has the advantages of longer transmission range coverage that is directional with total transmission energy being shared among the beamforming nodes.

Frequency and phase synchronization of sensor nodes for distributed beamforming experimentally have been in the forefront of research for the past decade. Practical implementation of CB for wireless sensor network (WSN) by synchronizing the low signal sine waves of the local oscillators of each node was carried out in wired form [2, 3] and wireless form [4-7] using closed-loop 1-bit feedback control.

Frequency stability of USRP devices is important as it is expected that they should produce the same frequency level for a given time period but, due to the difference in phase noise experienced by the local oscillators (LOs) in the equipment, this is always not the case. Phase noise from USRP transmitters limits the performance of the received signals and produces frequency drifts and phase offsets in CB nodes [8]. To improve the performance of these devices, a non-linear filter should be used to linearize the frequency drift of the beamforming nodes and the EKF is one of such. The process and measurement noise parameters of the CB nodes' LO needs to be calculated so that the filters will correctly predict and correct the frequency and phase offsets of the received signals.

The short-term frequency stability measure in the frequency domain, that is phase noise, is looked at in this paper. This phase noise is needed for the calculation of the process noise covariance matrix, Q, of the EKF so as to obtain a near zero (0) phase offset of received signal. The single side band (SSB) phase noise (dBc/Hz) measurements offset from a specific carrier frequency which is taken for various offset values is used to obtain the process noise parameters through the Allan variance of the power-law noise model.

II. PHASE NOISE MODEL

Consider an ideal frequency oscillator that produces the signal:

$$V(t) = A\sin(2\pi\nu_o t) \quad (1)$$

where A is the signal amplitude and ν_o the oscillator carrier frequency (Hz). The signal that comes out of the real oscillator is express by [9] as;

$$V(t) = A\sin\big(2\pi\nu_o t + \emptyset(t)\big) \quad (2)$$

where $\emptyset(t)$ is the phase noise of the signal. If

$$V(t) = A\sin\big(2\pi\nu_o t + x(t)\big) \quad (3)$$

where $\mathrm{x(t)} = \frac{\emptyset(t)}{2\pi\nu_o}$ is the time fluctuation due to phase noise, then the frequency deviation due to this time fluctuation is

$$\mathrm{y(t)} = \frac{1}{2\pi\nu_o}\frac{d\emptyset(t)}{dt}$$

A. Allan Variance

The equation for the SSB (the ratio of the power density at a desired offset frequency from a known carrier to the total power of the carrier signal) phase noise in logarithmical scale (dBc/Hz) is:

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$$\mathrm{L(f)} = 10^{\left(\frac{\mathrm{L_{dBc(f)}}}{10}\right)} \quad (4)$$

The spectral density of the fractional frequency fluctuation (1/Hz) of the SSB phase noise is;

$$\mathrm{S_y(f)} = \frac{2\mathrm{L(f)}f^2}{\nu_0{}^2} \quad (5)$$

where $2\mathrm{L(f)} = \mathrm{S_\emptyset(f)}$ is the spectral density of phase fluctuations (rad^2/Hz).

The Allan variance ($\sigma_y^2(\tau)$) in the time domain can now be calculated since the $\mathrm{S_y(f)}$ value at all frequency offset is known.

$$\sigma_y^2(\tau) = 2\int_0^{f_H} \mathrm{S_y(f)} \frac{\sin^4(\pi\tau f)}{(\pi\tau f)^2} \mathrm{df} \quad (6)$$

The measuring system higher cutoff frequency, $\mathrm{f_H}$, is required for the noise to be finite.

A combination of the power-law noises whose spectral density of the fractional frequency fluctuation is directly proportional to the side band offset frequency can be modeled to represent the instability of frequency sources [10]. Equation (7) gives the Allan variance in the frequency domain for the power-law noises while (8) is that of the noises considered in this paper representing a model for the instability of the NI USRP-2920 (white frequency noise and random walk frequency noise) LO. These variances will be use in computing the process noise parameters that will be used in the EKF process noise covariance matrix.

$$\sigma_y^2(\tau) = \frac{3f_H h_2}{(2\pi\tau)^2} + [1.038 + 3\ln(2\pi\tau f_H)]\frac{h_1}{(2\pi\tau)^2} + \frac{h_0}{2\tau} + 2\ln(2)h_{-1} + \frac{(2\pi)^2 h_{-2}\tau}{6} \quad (7)$$

$$\sigma_y^2(\tau) = \frac{h_0}{2\tau} + \frac{(2\pi)^2 h_{-2}\tau}{6} \quad (8)$$

where h_2, h_1, h_0, h_{-1} and h_{-2} are the white phase noise, flicker phase noise, white frequency noise, flicker frequency noise and random walk frequency noise, respectively.

III. EXTENDED KALMAN FILTER

The Kalman filter is a set of mathematical equation (that is recursive) that estimates the state of a linear system, that is, it can estimate the past, present and future states of a system at every time step t. But when the process in question is of the non-linear type as is the case with the USRP LOs' drifting over time even after being synchronized during cases like applications in collaborative beamforming, then a non-linear filter is required.

The EKF (a non-linear filter) is a filter that linearizes a non-linear system about its current mean and covariance [11]. Detail work on the EKF is given in [12, 13]. A comparison of the EKF and unscented kalman filter (UKF) for virtual reality application was analyzed by [14] based on performance and computational overhead. The two filters were found to deliver the same performance scale with the EKF having lesser computational overhead. Contrary to [14], the UKF showed better performance in the four different simulation cases in [15]. This is to say that the performance and computational overhead of these two filters depends on the application they are being used on.

Consider a non-linear system described by a set of stochastic difference and measurement equations for the CB nodes' offsets;

$$x_{k+1} = F(x_k) + w_k \quad (9)$$

$$z_k = h(x_k) + v_k \quad (10)$$

$F(x_k)$ and $h(x_k)$ are the process and observation non-linear functions with k being the previous time step in the process function. w_k is the process noise vector that is assumed to be drawn from the zero mean multivariate normal distribution with covariance Q; $w_k \sim \mathcal{N}(0, Q)$. v_k is the measurement noise vector assumed to be zero mean Gaussian white noise with covariance R; $v_k \sim \mathcal{N}(0, R)$. x_{k+1} and z_k are the State and Observation vector.

The process noise and measurement noise are uncorrelated with their initial state vector (say x_0) as well as the random noise themselves, that is;

$$E[w_k X_0^T] = 0 \quad \text{for } \forall\, k \quad (11)$$

$$E[v_k X_0^T] = 0 \quad \text{for } \forall\, k \quad (12)$$

$$E[w_k v_j^T] = 0 \quad \text{for } \forall\, (k \text{ and } j) \quad (13)$$

The phase and frequency offsets of each of the CB nodes as a model by [4] is $x_k = [\emptyset_k, \omega_k]^T$ with ω_k as $2\pi f_k$. The feedback rate, T, in the transition matrix can either be fixed or varied indicating stationary or mobile nodes and as such might change in the course of the filter performance, that is, $F = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$. Haven used Taylors series to expand the $F(x_k)$ and $h(x_k)$ functions based on the two-state model of [16] , the forecast State and forecast Error Covariance of the system called the Time update (predict equations) are;

$$x_{k+1|k} = F(x_{k|k}) \quad (14)$$

$$P_{k+1|k} = FP_{k|k}F^T + Q \quad (15)$$

The measurement updates (Kalman gain, state and error covariance) are thus;

$$K_k = P_{k|k-1}H_k^T(H_k P_{k|k-1} H_k^T + R)^{-1} \quad (16)$$

$$x_{k|k} = x_{k|k-1} + K_k\left(z_k - h(x_{k|k-1})\right) \quad (17)$$

$$P_{k|k} = (I - K_k H_k)P_{k|k-1} \quad (18)$$

with the Jacobian of the measurement function h being $H = \frac{\partial h}{\partial x}\Big|_k$, $(H_k P_{k|k-1} H_k^T + R)$ the innovation covariance and $\left(z_k - h(x_{k|k-1})\right)$ the innovation or measurement residual.

One of the major setbacks of the EKF is that prediction equations provide a very poor estimate of the true mean and Covariance matrix due to non-linearity. This paper tries to calculate the optimum process noise parameters that will be used in the process noise covariance matrix and as such direct spectrum method is used to measure the SSB phase noise (dBc/Hz) of the oscillator.

IV. EXPERIMENTAL RESULTS

Average values for the process noise of the NI USRP-2920 device was used due to differences in their aging, temperature and other manufacturers' device tolerance. Allan variance was employed to characterize the frequency stability of the USRP's oscillator [17] under the presence of different noise type. White frequency noise and random walk frequency noise parameters were calculated from Allan variance values haven measured the phase noise data for the USRP using the direct spectrum method. Figure 1 shows the experimental setup used in acquiring the phase noise data. The values of the phase noise at offset frequencies of 1kHz to 1 MHz range were taken using Keysight N9320B [18] SA.

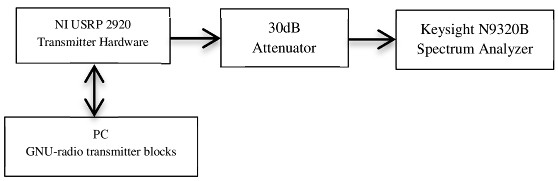

Fig. 1. Phase noise experimental setup

A pilot tone signal was generated at 100 MHz from the NI USRP-2920 to the SA using GNU-radio toolkit v. 3.7.1 [19]. The signal was sampled at 500 kHz and the gain of the USRP sink is set at 0dB to avoid damaging the SA as can be seen in Figure 2.

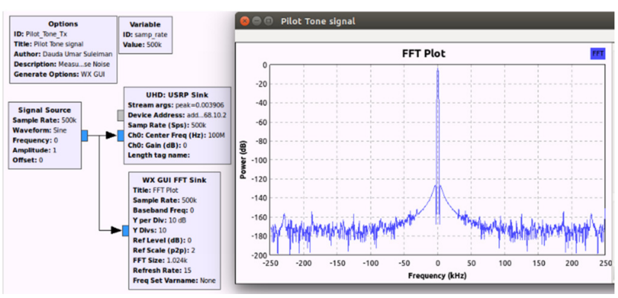

Fig. 2. Pilot tone signal using GNU-radio toolkit

The Keysight N9320N SA allows measurement of phase noise values in dBc/Hz at different frequency offset setting. The SA was allowed to warm-up for a minimum period of 30 minutes and all calibrations were done according to manufacturer's guide before readings were taken. Detail phase noise values at offsets of 1 kHz, 10 kHz, 100 kHz and 1 MHz are shown in Figure 3. Phase noise values in Figure 3 at all frequency offset except at 1 MHz is found to be lower when compared to those of N210 and B100 USRP used with WBX daughterboard [20] (see Table I for additional information).

(a)

(b)

(c)

(d)

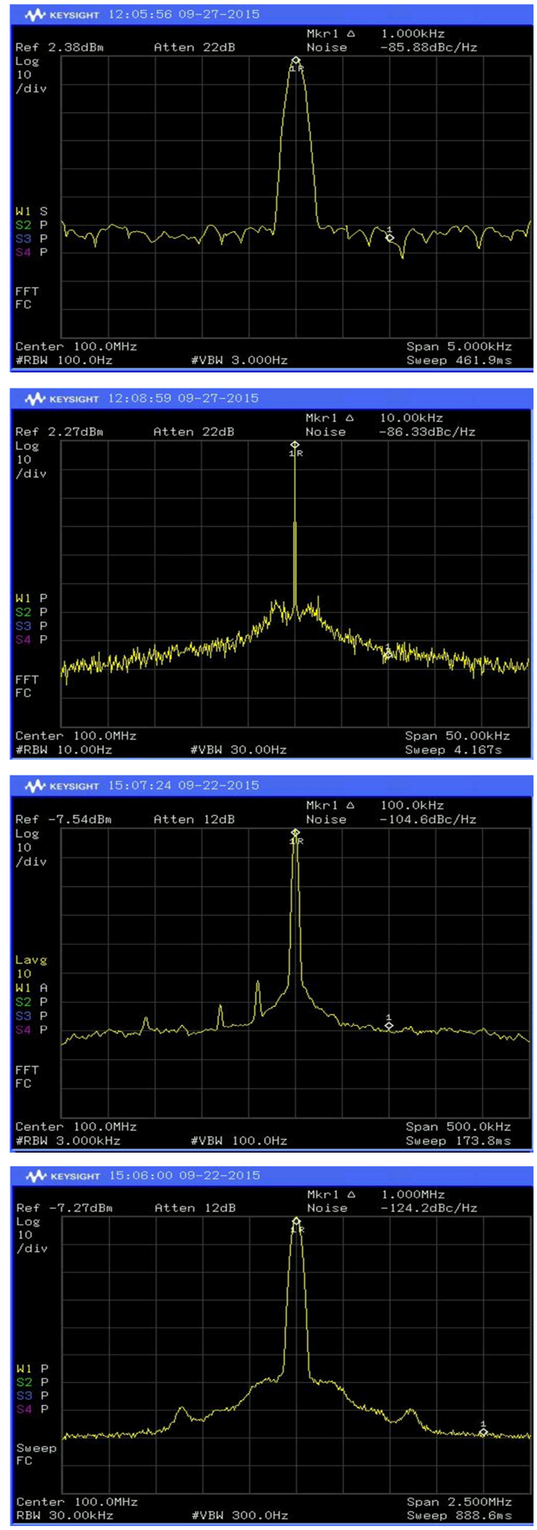

Fig. 3. Phase noise values at various frequency offset
(a) -85.88 dBc/Hz at 1 kHz (b) -86.33 dBc/Hz at 10 kHz
(c) -104.6 dBc/Hz at 100 kHz (d) -124.2 dBc/Hz at 1 MHz

Each phase noise values is the difference between the carrier power (1R) and the specific noise level (1) as can be seen in Figure 3. The four (4) selected offset frequencies were based on standard auto scale (10^a Hz; where **a** = 0,1,2, ….6) of the keysight N9320B SA where significant differences in phase noise values was not noticed for a= 0,1,2 and 3, and as such only the 1 kHz was choosen along side the 10 kHz, 100 KHz, and 1 MHz for easier comparison with [20]. The summary of the graphs in Figure 3 is shown in Figure 4 with phase noise values in the vertical axis and their corresponding offset values on the horizontal axis.

The Allan variance curve was obtained by calculating the variance at different tau (τ) values using (8). The variance was averaged over the four (4) measurements. It can be observed from the curve of Figure 5 that it truly depicts the standard slope characteristics of white frequency and random walk frequency noises as seen in [9].

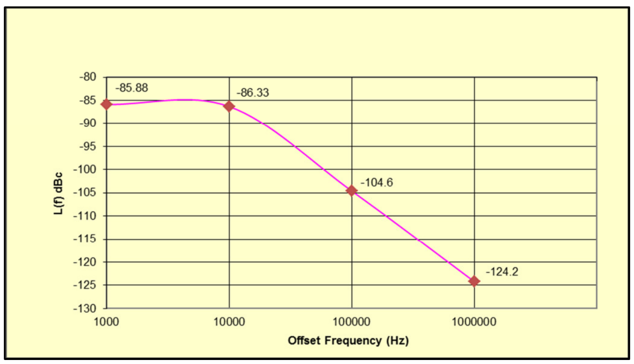

Fig. 4. Phase noise Vs frequency offset

TABLE I. NI USRP-2920 PROCESS NOISE PARAMETERS

Offset frequency	*Phase Noise (dBc/Hz)*	
	NI 2920 Experimental	*N210, B100 [20]*
1 kHz	-85.88	-
10 kHz	-86.33	-80
100 kHz	-104.60	-100
1 MHz	-124.20	-137

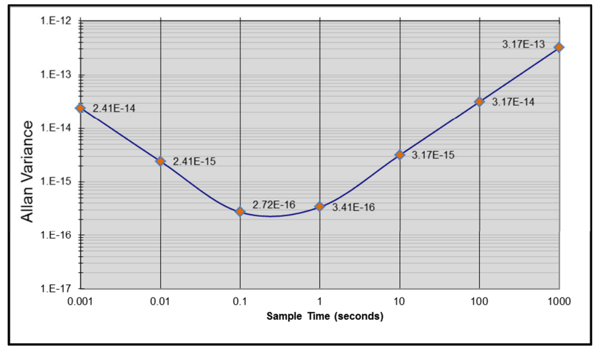

Fig. 5. Allan variance curve at various tau values

Table II shows the parameter values calculated from the averaged Allan variance, the white frequency noise $\left(q_1 = \frac{h_0}{2}\right)$ and random walk frequency noise $\left(q_2 = \frac{(2\pi)^2 h_{-2}}{2}\right)$ from (8). These values can directly be used in the Q matrix of (15) for the correction and prediction of the optimum Kalman filter state measurement matrix (17).

TABLE II. NI USRP-2920 PROCESS NOISE PARAMETERS

Units	*Parameters*	*Symbol*	*Value*
sec	White frequency noise	q_1	3.80×10^{-17}
Hz	Random walk frequency noise	q_2	1.50×10^{-15}

The equation for the Q matrix of (15) is;

$$Q(T) = \omega_c^2 T \begin{bmatrix} q_1 + q_2 \frac{T^2}{3} & q_2 \frac{T}{2} \\ q_2 \frac{T}{2} & q_2 \end{bmatrix} \quad (19)$$

where T is the sampling period and $\omega_c$the angular frequency of the carrier.

CONCLUSION

This paper has successfully obtained the process noise parameters of the NI USRP-2920 USRP device using a combination of the white frequency noise (q_1) and random walk frequency noise (q_2) of the power-law noise model. Allan variance values in the frequency domain were computed and later the process noise parameters that will be applied with the EKF. It was noticed that the Allan variance curve truly depicts the usual slope features of white frequency and random walk frequency noises which are special considerations when dealing with USRP's LOs. These process noise parameters are critical for CB in WSN as they are used in the EKF for carrier frequency and phase offset recovery.

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