Analytical Solution of Unsteady Boundary Layer Flow of a Nanofluid past a Stretching Inclined Sheet with Effects of Magnetic Field

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Abstract— Flow of a nanofluid in a boundary layer in an inclined moving sheet at angle Θ is considered analytically. The Mathematical formulation consists of the Magnetic parameter, thermophoresis, and Brownian motion. Previously published work considered convective boundary condition. The present study considered an inclined stretching sheet at angle Θ in one dimension and considered thermal conditions of non convective heating and heat flux. Solutions to momentum, temperature and concentration distribution depends on seven parameters, Magnetic parameter M, Lewis number Le, Prandtl number Pr, thermophoresis parameter Nt, the Brownian motion parameter Nb, unsteady parameter c and Grashof numbers Gr and Gc. The non linear coupled Differential equations were solved using the improved Adomian decomposition method and a good agreement was established with the numerical method (Shooting technique). Analytical result is also presented graphically to illustrate the effect of the earlier listed parameters on Momentum, temperature and nanofraction boundary layers. Momentum boundary layer increases with increase in Grashof numbers, angle of inclination and unsteady parameter.
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Keywords— Adomian Decompostion Method (ADM), Nanofluid, Inclined sheet, magnetic field effects, Numerical Method (NM)

1 Introduction

Altan *et al.* (1979), the flow of fluid on stretching sheets is useful in industries for wire drawing, hot rolling, extrusion and metal spinning,. It is useful to get the concept of flow characteristics and heat flow of process so that the refined product attained the required specifications. Several cases dealing with fluid flow and heat flow on a moving wall was studied with non-Newtonian fluids and Newtonian fluids with imposed magnetic fields and electric fields, different conditions, and power law varies for the moving velocity. Both similarities and solutions at mesh points of the convective model were studied. Representatives samples of the recent literatures on the work are listed by reference (Prasad *et al.*, 2010).

Adegun et al (2017) investigate numerically the effect of some geometric parameters and flow variables on heat transfer augmentation in annuli with equi-spaced internal longitudinal fins along the external wall. The results obtained show that for 50≤Re≤500, total Nusselt number increases with increase in Re while Re>500, there was no significant increase in Nusselt number. Idowu and Jimoh(2017) carried out the effect of kuvshinshiki fluid on Magnetohydrodynamics (MHD) heat and mass transfer flow over a vertical porous plate with chemical reaction of nth order and thermal conductivity. The results shows that the visco-elastic of kuvshinshiki fluid type is growing as concentration profile increases, while the velocity and temperature profile falls.

Beg et al. (2014) listed numerical solution of both two-phase equations and single equation for Nanofluid equation phenomena. Rashidi et al. (2014) considered the single phase and two phase of flow field and heat transfer of water-copper nanofluids in an irregular wall numerically. Abu-Nada et al. (2010) show the effects of variable properties in convective nanofluid transport. Rashidi et al. (2013) illustrated that one of the laws of thermodynamics is applicable to magneto-hydro-dynamics nanofluid flow in a porous rounding disk.

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The stagnation movement of a nanofluid over a moving wall was considered by Mustafa et al. (2011) analytically. Yusuf et al. (2016) presented analytical solution of a nanofluid in an inclined permeable wavy channel with Soret and Duffour effects using the Adomian Decomposition Method. This work is a new development in the literature in which an analytical solution of an unsteady one dimensional nanofluid in an inclined stretching sheet with magnetic field effect is proposed using the Adomian Decomposition Method.

2 Problem Formulation

Consider a One dimensional unsteady nanofluid flow in an inclined stretching sheet at angle Θ. The flow is along y=0 and the stretching velocity is assumed to be U and 0 far away from the stretching sheet. The temperature and concentration are constant at y=0, ie T_W and C_W respectively and T_∞ and C_∞ at large values of y. Following the work of Makinde and Aziz (2011) with magnetic field effect without convective heating, the one dimensional unsteady with suction parameter is govern by the following:

Continuity equation

$$\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum equation:

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\upsilon\left(\frac{\partial^2 u}{\partial y^2}\right)-\frac{\sigma B_0{}^2}{\rho}u+g\beta(t)(T-T_\infty)\operatorname{Sin}\Theta$$
$$+g\beta(t)(C-C_\infty)\operatorname{Sin}\Theta \tag{2}$$

Energy equation:

$$\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial y}=\alpha(\frac{\partial^2 T}{\partial y^2})+\frac{\upsilon}{c_p}(\frac{\partial u}{\partial y})^2$$
$$+\tau\left(D_B(\frac{\partial C}{\partial y}\frac{\partial T}{\partial y})+\frac{D_T}{T_\infty}\left(\left(\frac{\partial T}{\partial y}\right)^2\right)\right) \tag{3}$$

Nanofraction equation:

$$\frac{\partial C}{\partial t}+v\frac{\partial C}{\partial y}=D_B\left(\frac{\partial^2 C}{\partial y^2}\right)+\left(\frac{D_T}{T_\infty}\right)\left(\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

Subject to the boundary conditions:

$$\left.\begin{array}{llllll} y=0: & u=U, & v=v_0, & T=T_0, & C=C_0, & t\le 0 \\ y\to\infty: & u\to 0, & & T\to T_\infty, & C\to C_\infty, & t>0 \end{array}\right\} \quad (5)$$

Where u is the velocity, $v = v_0$ is the suction parameter, t is the time, α is the thermal diffusivity, υ is the kinematic viscosity, B_0 is external magnetic field, C_P is the specific heat capacity at constant pressure, D_B is the Brownian diffusion coefficient, D_T is the thermopheric diffusion coefficient and τ is the ratio between the effective heat capacity of the fluid with ϱ being the density, g is the acceleration due to gravity and σ is the electrical conductivity.

From the following dimensionless variables:-

$$\eta=\frac{y}{2\sqrt{\upsilon t}},\ \mathrm{u}=Uf(\eta),\ \theta(\eta)=\frac{T-T_h}{T_0-T_h}, \quad \text{and}$$

$$\phi(\eta)=\frac{C-C_h}{C_0-C_h}$$, we have that

$$\left.\begin{array}{l} \eta=\dfrac{y}{2\sqrt{\upsilon t}} \\ \eta^2=\dfrac{y^2}{4\upsilon t} \\ 2\eta\dfrac{\partial\eta}{\partial t}=-\dfrac{y^2}{4\upsilon t}=-\dfrac{\eta}{2t} \\ \dfrac{\partial\eta}{\partial t}=\dfrac{-\eta}{2t} \\ u=Uf \\ v=v_0 \\ \dfrac{\partial u}{\partial y}=\dfrac{\partial u}{\partial f}\dfrac{\partial f}{\partial\eta}\dfrac{\partial\eta}{\partial y}=\dfrac{Uf^{/}}{2\sqrt{\upsilon t}} \\ \dfrac{\partial u}{\partial t}=\dfrac{\partial u}{\partial f}\dfrac{\partial f}{\partial\eta}\dfrac{\partial\eta}{\partial t}=\dfrac{-U\eta}{2t}f^{/} \\ \dfrac{\partial^2 u}{\partial y^2}=\dfrac{\partial\eta}{\partial y}\dfrac{\partial}{\partial\eta}\left(\dfrac{Uf^{/}}{2\sqrt{\upsilon t}}\right)=\dfrac{Uf^{//}}{4\upsilon t} \end{array}\right\} \quad (6)$$

where η, f(η), Ɵ(η), ϕ(n) are the dimensionless fluid distance, velocity profile, temperature profile, and concentration.

Introducing equation (6) and (7) into equations (1) to (5), the equation reduced to the following local similarity solution:-

$$\left.\begin{array}{l} f^{//}+2(\eta+c)f^{/}-Mf+Gr_T\theta Sin\Theta+Gr_C\phi Sin\Theta=0 \\ \theta^{//}+2(\eta+c)\Pr\theta^{/}+\Pr N_b\phi^{/}\theta^{/}+\Pr N_t\theta^{/2}=0 \\ \phi^{//}+2Le(\eta+c)\phi^{/}+\dfrac{Nt}{Nb}\theta^{//}=0 \end{array}\right\} \quad (8)$$

with corresponding boundary conditions:

$$\left.\begin{array}{llll} f(0)=1, & \theta(0)=1, & \chi(0)=1, & \eta=0 \\ f(\infty)=0, & \theta(\infty)=0, & \chi(\infty)=0, & \eta\to\infty \end{array}\right\} \quad (9)$$

$$\left.\begin{array}{l} T=T_\infty+(T_0-T_\infty)\theta \\ C=C_\infty+(C_0-C_h)\phi \\ \dfrac{\partial T}{\partial t}=(T_0-T_\infty)\dfrac{\partial\eta}{\partial t}\dfrac{\partial\theta}{\partial\eta}=-(T_0-T_\infty)\left(\dfrac{\eta}{2t}\right)\theta^{/} \\ \dfrac{\partial T}{\partial y}=(T_0-T_\infty)\dfrac{\partial\eta}{\partial y}\dfrac{\partial\theta}{\partial\eta}=(T_0-T_\infty)\left(\dfrac{1}{2\sqrt{\upsilon t}}\right)\theta^{/} \\ \dfrac{\partial^2 T}{\partial y^2}=\dfrac{\partial}{\partial y}\left(\dfrac{\partial T}{\partial y}\right)=\dfrac{\partial\eta}{\partial y}\dfrac{\partial}{\partial\eta}\left((T_0-T_\infty)\left(\dfrac{1}{2\sqrt{\upsilon t}}\right)\theta^{/}\right)=\dfrac{(T_0-T_\infty)}{4\upsilon t}\theta^{//} \\ \dfrac{\partial C}{\partial t}=(C_0-C_\infty)\dfrac{\partial\eta}{\partial t}\dfrac{\partial\phi}{\partial\eta}=-(C_0-C_\infty)\left(\dfrac{\eta}{2t}\right)\phi^{/} \\ \dfrac{\partial C}{\partial y}=(C_0-C_\infty)\dfrac{\partial\eta}{\partial y}\dfrac{\partial\phi}{\partial\eta}=(C_0-C_\infty)\left(\dfrac{1}{2\sqrt{\upsilon t}}\right)\phi^{/} \\ \dfrac{\partial^2 C}{\partial y^2}=\dfrac{\partial}{\partial y}\left(\dfrac{\partial C}{\partial y}\right)=\dfrac{\partial\eta}{\partial y}\dfrac{\partial}{\partial\eta}\left((C_0-C_\infty)\left(\dfrac{1}{2\sqrt{\upsilon t}}\right)\phi^{/}\right)=\dfrac{(T_0-T_h)}{4\upsilon t}\phi^{//} \end{array}\right\} \quad (7)$$

in which : $Gr_T=\dfrac{4tg\beta(T_W-T_\infty)}{U}$, $Gr_C=\dfrac{4tg\beta_0(C_W-C_\infty)}{U}$, $M=\dfrac{4t\sigma B_0^{\,2}}{\rho}$, $\Pr=\dfrac{\upsilon}{\alpha}$,

$Le=\dfrac{\upsilon}{D_B}$, $N_b=\dfrac{(\rho c)_p D_B(C_W-C_\infty)}{(\rho c)_f\upsilon}$,

$N_t=\dfrac{(\rho c)_p D_T(T_W-T_\infty)}{(\rho c)_f T_\infty\upsilon}$, and $c=-v_0\sqrt{\dfrac{t}{\upsilon}}$

are thermal Grashof number, concentration Grashof number, Magnetic field, Prandtl number, Lewis number, Browning motion, thermopheric parameter and unsteady parameter.

Introducing the improved Adomian decomposition method, equation (8) can be written as

$$\left.\begin{array}{l} f(\eta)=\alpha e^{-\eta}+L_1^{-1}\left[-2(\eta+c)f^{/}+Mf-Gr_T\theta Sin\Theta-Gr_C\phi Sin\Theta\right] \\ \theta(\eta)=\beta e^{-\eta}+L_1^{-1}\left[-2(\eta+c)\Pr\theta^{/}-\Pr N_b\phi^{/}\theta^{/}-\Pr N_t\theta^{/2}\right] \\ \phi(\eta)=\gamma e^{-\eta}+L_1^{-1}\left[-2Le(\eta+c)\phi^{/}-\dfrac{Nt}{Nb}\theta^{//}\right] \end{array}\right\}$$

(10)

Where $L_1^{-1}=\iint[\bullet]\,d\eta d\eta$

Decomposing the dependent variables in (10) by introducing the Adomian polynomials, we have

$$\left.\begin{array}{l}\sum_{n=0}^{\infty} f_n = \alpha e^{-\eta} + L_1^{-1}\left[-2(\eta+c)\sum_{n=0}^{\infty} f_n^{/} + M\sum_{n=0}^{\infty} f_n - Gr_T\sum_{n=0}^{\infty}\theta_n Sin\Theta - Gr_C\sum_{n=0}^{\infty}\phi_n Sin\Theta\right] \\ \sum_{n=0}^{\infty}\theta_n = \beta e^{-\eta} + L_1^{-1}\left[-2(\eta+c)\Pr\sum_{n=0}^{\infty}\theta_n^{/} - \Pr N_b\sum_{n=0}^{\infty} A_n - \Pr N_t\sum_{n=0}^{\infty} B_n\right] \\ \sum_{n=0}^{\infty}\phi_n = \gamma e^{-\eta} + L_1^{-1}\left[-2Le(\eta+c)\sum_{n=0}^{\infty}\phi_n^{/} - \frac{Nt}{Nb}\sum_{n=0}^{\infty}\theta_n^{//}\right]\end{array}\right\} \quad (11)$$

Where $A_n = \theta_{n-k}^{/}\phi_k^{/}$ and $B_n = \theta_{n-k}^{/}\theta_k^{/}$, are called Adomian polynomials

Therefore,

$$\left.\begin{array}{l} f_n = L_1^{-1}\left[-2(\eta+c) f_{n-1}^{/} + Mf_{n-1} - Gr_T\theta_{n-1}Sin\Theta - Gr_C\phi_{n-1}Sin\Theta\right] \\ \theta_n = L_1^{-1}\left[-2(\eta+c)\Pr\theta_{n-1}^{/} - \Pr N_b\sum_{k=0}^{n}\phi_{n-k}^{/}\theta_k^{/} - \Pr N_t\sum_{k=0}^{n}\theta_{n-k}^{/}\theta_k^{/}\right] \\ \phi_n = L_1^{-1}\left[-2Le(\eta+c)\phi_{n-1}^{/} - \frac{Nt}{Nb}\theta_{n-1}^{//}\right]\end{array}\right\} \quad (12)$$

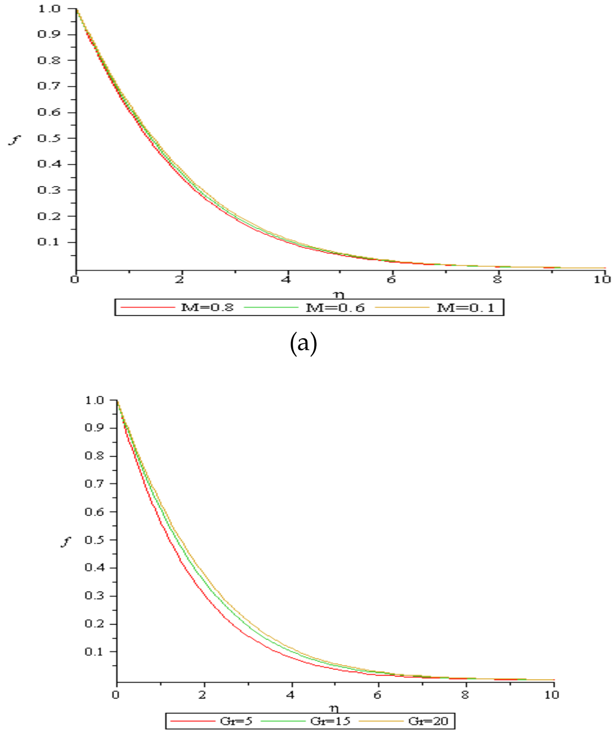
Where $f_0 = \alpha e^{-\eta}$, $\theta_0 = \beta e^{-\eta}$ and $\phi_0 = \gamma e^{-\eta}$

The solutions are

$$\left.\begin{array}{l} f(\eta) = \alpha e^{-\eta} - 8Sin\Theta Grc\left(\eta e^{-\eta} + 2e^{-\eta}\right) + \ldots \\ \theta(\eta) = \beta e^{-\eta} - \Pr\left(-2\beta\left(\eta e^{-\eta} + 2e^{-\eta} + ce^{-\eta}\right) + \right) \quad \ldots \\ \phi(\eta) = \gamma e^{-\eta} + \frac{\left(2Le\gamma N_b c - N_t\beta + 4Le\gamma N_b + 2\right)}{N_b} \quad \ldots \end{array}\right\} \quad (13)$$

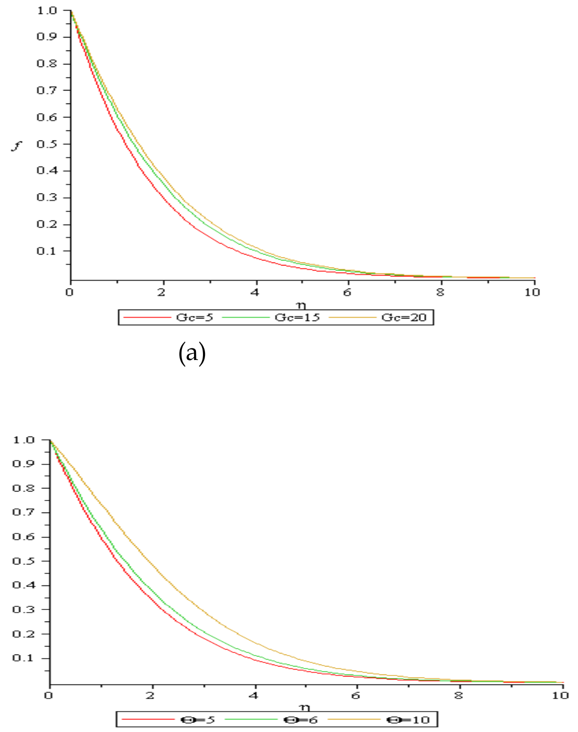
3 Results and Discussion

The nonlinear coupled ordinary differential equation in (8) with boundary condition (9) has been solved using the improved ADM as showed above and the solutions are presented in (13). Table 1 showed that there is a good agreement between ADM and NM (Numerical Method). The values of the auxiliary constants α, β, and γ are obtained by invoking the initial conditions in other for the boundary conditions to satisfy and the graphical variation of the physical properties are presented below:



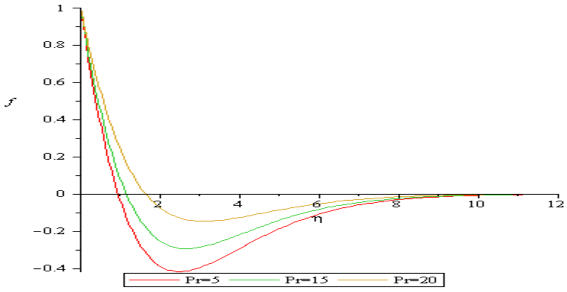
(a) (b)

Fig. 1: Variation of Magnetic parameter and thermal Grashof number on velocity profile

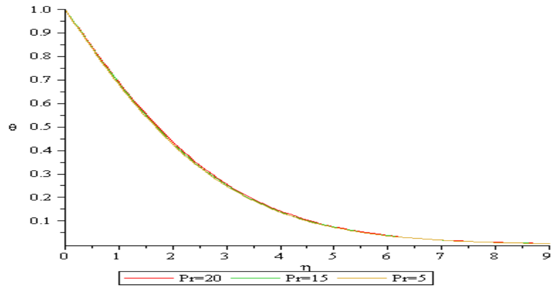


(a) (b)

Fig. 2: Variation of Concentration Grashof number and angle of inclination on velocity profile



(a)



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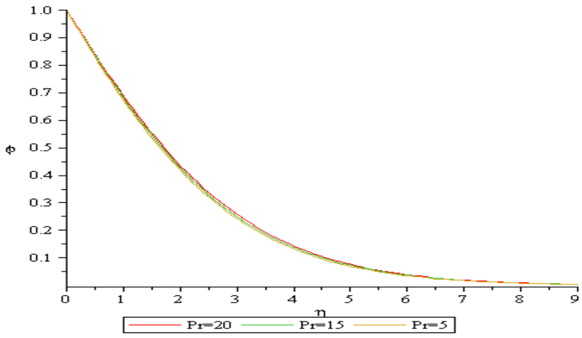
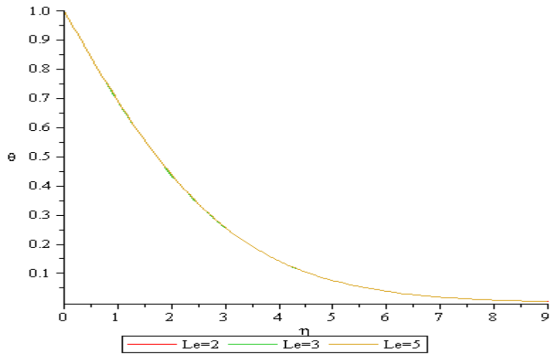
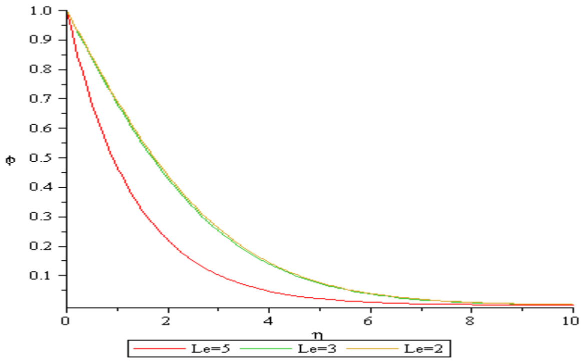


(c)

Fig. 3: variation of Prandtl number on velocity, temperature and concentration profile

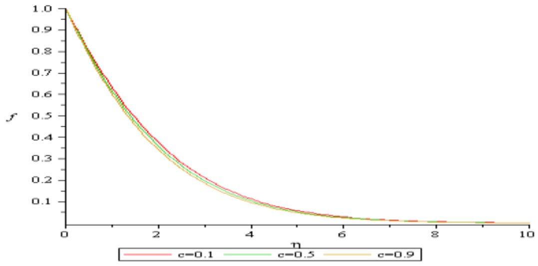


(a)

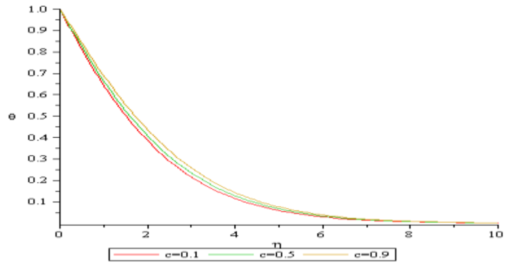


(b)

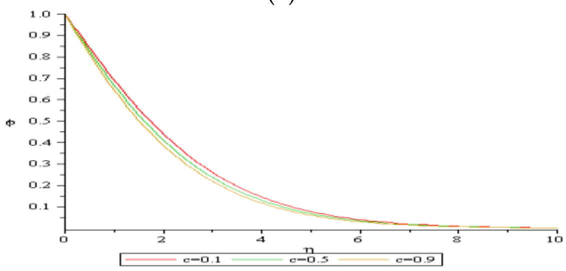
Fig. 4: Variation of Lewis number on temperature and concentration profile



(a)

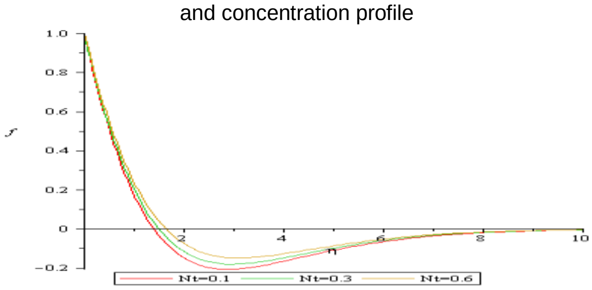


(b)

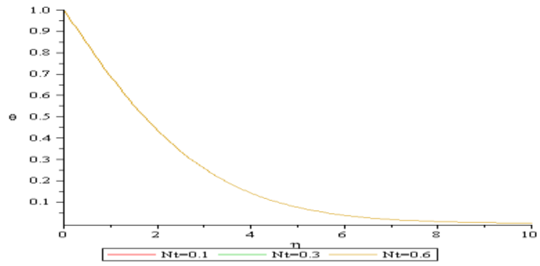


(c)

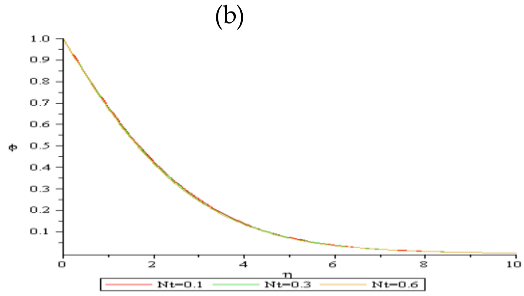
Fig. 5: variation of unsteady parameter on velocity, temperature and concentration profile



(a)



(b)



(c)

Fig. 6: Variation of thermopheris parameter on velocity, temperature and concentration profile

Figure 1 present the effects of magnetic parameter (a) and thermal Grashof number (b) on velocity profile. The fluid velocity shrink for higher values of magnetic parameter as a result of the drag like force and thickens for higher values of thermal Grashof number because of the buoyancy effect. Figure 2 show the variation of concentration Grashof number (a) and angle of inclination (b) on velocity profile. The velocity of the fluid increases with an increase in the concentration Grashof number due to the presence of buoyancy effect and also increases as the angle of inclination also increases due to the presence of gravity.

Figure 3 depict the effects of Prandtl number on velocity (a), temperature (b) and fluid concentration (c). As Prandtl number increases the velocity and concentration profile are enhanced while the fluid temperature dropped. It is also observed that the velocity profile takes a larger domain before attaining free stream compare to the temperature and concentration. Figure 4 present the variation of Lewis number on temperature (a) and concentration (b) profiles. Increase in Lewis number leads to decrease in concentration profile while the fluid temperature rises.

Figure 5 is the variation of unsteady parameter on the fluid velocity (a), temperature (b) and concentration profile (c). As the unsteady parameter gains momentum, the velocity and temperature boundary layer of the fluid thickens while the concentration dropped. Figure 6 show the effects of thermopheris parameter on the velocity (a), temperature (b) and concentration (c) boundary layer. This parameter enhances the fluid velocity boundary layer and temperature, but has an insignificant effect on the concentration boundary layer. The rate of increase in the temperature is very low.

4 Conclusion

The problem of laminar flow in an inclined stretching sheet has been considered in one dimension with magnetic field effect (M) without convective heating. The local similarity solution were obtained and solved using the improved ADM and the analytical solution were presented which depends on magnetic parameter (M), thermal and concentration Grashof number (Gr and Gc respectively), Lewis number, and Prandtl number. It was found that:-

1. All the graphs presented in this work clearly satisfied the boundary conditions.
2. A slight flow back was observed on varying the Prandtl number on the velocity profile.
3. The analytical result presented in this work gives a solution at every point unlike the numerical results presented by Makinde and Aziz (2011) which only give results at mesh points.
4. The flow velocity is in favour of gravity as depicted in Figure 2 (b).
5. This work, is recommended to industries that specialises in the development of high thermal conductivity fluid to serve as guide as to how each of the physical property influence the fluid velocity, temperature and concentration profile.

Table 1: Comparison of Skin Friction (-f”)

M	NM	ADM
0.1	1.0519	1.0488
0.2	1.0977	1.0955
0.3	1.1402	1.1418
0.4	1.1844	1.1832
0.5	1.2256	1.2248
0.6	1.2655	1.2649
0.8	1.3419	1.3416
1	1.4144	1.4142

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