The Pollutant Concentration Regime in a Flow due to Variable Time-Dependent Off-Diagonal Dispersion

Jimoh, O. R.* and Aiyesimi, Y. M.

Department of Mathematics, Federal University of Technology, Minna, Nigeria

* Corresponding author's email: razaq.jimoh@futminna.edu.ng

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ABSTRACT

In this paper, an Eigen Functions expansion technique was used to obtain an analytical solution of two-dimensional contaminant flow problem with non-zero initial concentration. The equation which describes the two-dimensional contaminant flow model is a partial differential equation characterized by advection, dispersion, adsorption, first order decay and zero-order source. It was assumed that the adsorption term was modeled by Freudlich isotherm. The off-diagonal dispersion parameter was incorporated into the two-dimensional contaminant model in order to expand the scope of the analysis. The model equation was non-dimensionalized before the parameter expanding method was applied. The resulting equations were solved successively by Eigen functions expansion technique. This research establishes that the pollutant concentration declines with increase in distances in both directions as the off-diagonal dispersion coefficient, zero-order source coefficient and vertical dispersion coefficient increases.

Keywords: Advection, dispersion, adsorption, contaminant, off-diagonal dispersion

1.0 Introduction

The occurrence of dispersion of contaminant in soil, water channels, groundwater and surface water has been an evolving research in geology and hydro-geological centers for many years. This is not unconnected with the increased awareness of the effect of significant contamination of groundwater and surface water by industrial and human activities such as agricultural chemicals, accidental spills, landfills, toxic wastes and buried hazardous materials on our health.

The flow of contaminants through surface soil, groundwater and surface water environments was modeled by transport equations (Bear,1997). In order to understand the flow of contaminant in the hydrological formation more accurately, a critical survey of contaminant flow lines are desired. The problem encompasses providing the solution of transport equation, the travel time of water via the flow lines and to predict the activities of the parameters which changes the concentration in the course of flow.

Most researchers posit that the contaminant flow is predominantly horizontal as found in (Bear,1997), but in addition, others affirmed nonetheless that appreciable vertical flow components may occur in the domain of vertically penetrating wells, bore holes and streams (Brainard and Gelhar, 1991). In an effort to provide solutions to the contaminant flow problems, a lot of successes were achieved by some researchers but mostly on one-dimensional cases with various initial and boundary conditions. This includes the study of the influence of the retardation factor on the contaminant in a nonlinear contaminant flow (Okedayo and Aiyesimi, 2005). On the dispersion of solute, the effect of horizontal dispersion of miscible fluid flow in one dimension through porous media was examined (Ramakanta and Mehta, 2010).

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In line with the desire to understand the behavior of contaminant in a flow, an analytical solution to temporally dependent mixing through semi-infinite homogeneous porous medium by Laplace transform technique (LTT) was provided (Yadav *et al*., 2011). Computational analyses on the effect of reactive and non-reactive contaminant on the flow were carried out on one-dimensional non-linear contaminant flow with an initial continuous point source and discovered that the concentration decreases with increase in time and distance from the origin for the non-reactive case by homotopy perturbation method (Yadav *et al*., 2011).

In this research, we present the pollutant concentration regime in the flow due to variable time-dependent off-diagonal dispersion. The two-dimensional contaminant flow problem incorporating flow in both horizontal directions, off-diagonal dispersion parameters in addition to first-order decay and zero order sources was solved using the parameter expanding technique and Eigen functions expansion method.

2.0 Formulation of the Problem

An incompressible fluid flow through a finite homogeneous porous media with non-zero initial concentration in the transport domain is considered. It is assumed that the flow is two-dimensional and in the direction of *x* and *y*-axis. The concentration of the source is assumed at the origin (i.e. at time *t=0*). it is assumed that the contaminant invades the groundwater level from point source in a finite homogeneous porous media.

Following Bear (1997), Yadav *et al*., (2011), Suciu (2014), Lee and Kim (2012), Mahato *et al*. (2015), Singh (2015) and Jimoh *et al*. (2018), we introduce the off-diagonal dispersion parameter into the two dimensional contaminant concentration flow equation and obtained:

$$\begin{aligned}\frac{\partial C}{\partial t}+\frac{\partial S}{\partial t}&=D_{xx}\frac{\partial^2 C}{\partial x^2}+D_{yy}\frac{\partial^2 C}{\partial y^2}+D_{xy}\frac{\partial^2 C}{\partial x\partial y}+D_{yx}\frac{\partial^2 C}{\partial y\partial x}\\&-u(t)\frac{\partial C}{\partial x}-v(t)\frac{\partial C}{\partial y}-\gamma(t)C+\mu(t),\end{aligned} \tag{1}$$

where C is the concentration of the contaminant in the flow, S is the concentration of the contaminant adsorbed to the porous media, D_{xx} is the dispersion in the longitudinal direction, D_{yy} is the dispersion in the vertical direction, D_{xy} and D_{yx} are off-diagonal dispersion coefficients. $\gamma(t)$ is the decay parameter, $\mu(t)$ is the source term, $u(t)$ and $v(t)$ are the velocities in the horizontal and vertical directions respectively,

$$\left.\begin{aligned}D_{xx}&=\frac{\alpha_L u^2}{\sqrt{u^2+v^2}}+\frac{\alpha_T v^2}{\sqrt{u^2+v^2}}\\D_{yy}&=\frac{\alpha_L v^2}{\sqrt{u^2+v^2}}+\frac{\alpha_T u^2}{\sqrt{u^2+v^2}}\end{aligned}\right\} \tag{2}$$

$$D_{xy} = D_{yx} = \frac{(\alpha_L - \alpha_T)uv}{\sqrt{u^2 + v^2}}, \tag{3}$$

as in Batu (2006).
The adsorbed contaminant is considered directly proportional to the contaminant concentration. i.e.,

$$S = K_d C \tag{4}$$

$$\frac{\partial S}{\partial t} = K_d \frac{\partial C}{\partial t}, \tag{5}$$

as found in Dawson (1993).

Following the relationship (4) and (5), equation (1) may be rewritten as

$$R\frac{\partial C}{\partial t} = D_{xx}\frac{\partial^2 C}{\partial x^2} + D_{yy}\frac{\partial^2 C}{\partial y^2} + 2D_{xy}\frac{\partial^2 C}{\partial x \partial y} - u(t)\frac{\partial C}{\partial x} - v(t)\frac{\partial C}{\partial y} - \gamma(t)C + \mu(t) \tag{6}$$

D_{xy} is the initial off-diagonal dispersion component. The initial and boundary conditions are chosen as

$$\left.\begin{aligned} &C(x,y,t) = c_i ; x \geq 0, y \geq 0, t = 0 \\ &C(x,y,t) = C_0(1 + e^{-qt}); x = 0, y = 0, t < 0 \\ &C(x,y,t) = c_p ; x \to L, y \to L, t \geq 0 \end{aligned}\right\} \tag{7}$$

We let

$$\left.\begin{aligned} &D_x = D_{x0}h(t) \\ &u(t) = u_0 h(t) \\ &v(t) = v_0 h(t) \\ &D_y = D_{y0}h(t) \\ &\mu(t) = \mu_0 h(t) \\ &\gamma(t) = \gamma_0 h(t) \\ &D_{xy} = D_{xy0}h(t) \\ &D_{xy0}h(t) = \frac{(\alpha_L - \alpha_T)u_0 v_0}{\sqrt{u_0^{\,2} + v_0^{\,2}}}, \end{aligned}\right\} \tag{8}$$

where $h(t)$ is arbitrary function of time as used (Yadav *et al*. 2011and Batu, 2006). D_{x0} is initial horizontal dispersion coefficient, D_{y0} is the initial vertical dispersion coefficient, D_{xy0} is the initial off-diagonal dispersion coefficient, v_0 is the initial vertical velocity, u_0 is the initial horizontal velocity, μ_0 is the initial zero-order source coefficient and γ_0 is the initial first order decay coefficient. We also introduced a new time variable as used in Olayiwola *et al*. (2013):

$$\left.\begin{aligned} &\tau = \frac{1}{R}\int_0^t h(t)dt \\ &h(t) = Re^{-qt} \end{aligned}\right\} \tag{9}$$

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By substituting the components of equations (8) and (9) in (6), the following equation is obtained:

$$\frac{\partial C}{\partial \tau} = D_{x0}\frac{\partial^2 C}{\partial x^2} + D_{y0}\frac{\partial^2 C}{\partial y^2} + 2D_{xy0}\frac{\partial^2 C}{\partial x \partial y} - u_0\frac{\partial C}{\partial x} - v_0\frac{\partial C}{\partial y} - \gamma_0 C + \mu_0 \tag{10}$$

$$\left.\begin{aligned} &C(x,y,\tau) = c_i;\, x \ge 0, y \ge 0, \tau = 0 \\ &C(x,y,\tau) = C_0(2 - q\tau);\, x = 0, y = 0, \tau \ge 0 \\ &C(x,y,\tau) = c_p;\, x \to L, y \to L, \tau \ge 0 \end{aligned}\right\} \tag{11}$$

where all the parameters as defined previously.
A new special variable is introduced below as used in (Batu, 2006):

$$\eta = x + y\sqrt{\frac{D_{y0}}{D_{x0}}} \tag{12}$$

Then, substituting equation (12) in equation (10), we obtain

$$\frac{\partial C}{\partial \tau} = D\frac{\partial^2 C}{\partial \eta^2} - U\frac{\partial C}{\partial \eta} - \gamma_0 C + \mu_0 \tag{13}$$

$$\left.\begin{aligned} &C(\eta,\tau) = C_i;\ \eta \ge 0, \tau = 0 \\ &C(\eta,\tau) = C_0(2 - q\tau);\ \eta = 0, \tau \ge 0 \\ &C(\eta,\tau) = c_p;\ \eta \to L, \tau \ge 0 \end{aligned}\right\} \tag{14}$$

where

$$D = D_{x0}\left(1 + \left(\frac{D_{y0}}{D_{x0}}\right)^2 + 2\frac{D_{xy0}}{D_{x0}}\sqrt{\frac{D_{y0}}{D_{x0}}}\right) \tag{15}$$

$$U = \left(u_0 + v_0\sqrt{\frac{D_{y0}}{D_{x0}}}\right) \tag{16}$$

2.1 Non-Dimensionalization

Equation (13) is non-dimensionalized by using the following dimensionless variables:

$$\left.\begin{aligned} &\tau = \frac{L}{U}\tau' \\ &\eta = \eta' L \\ &C = C_0 C^* \\ &\partial\tau = \frac{L}{U}\partial\tau' \\ &\mu_0 = \frac{\mu' C_0}{\tau} \end{aligned}\right\} \tag{17}$$

On substituting the above dimensionless variables in equation (13), the following equation is obtained.

$$\frac{\partial C^*}{\partial \tau'} = \frac{D}{LU}\frac{\partial^2 C^*}{\partial \eta'^2} - \frac{\partial C^*}{\partial \eta'} - \frac{\gamma_0 L C^*}{U} + \frac{L}{C_0 U}\mu_0 \tag{18}$$

For convenience, the primes are dropped and obtained

$$\frac{\partial C^*}{\partial \tau} = D_2 \frac{\partial^2 C^*}{\partial \eta^2} - \frac{\partial C^*}{\partial \eta} - \gamma_0 C^* + \mu_1 \tag{19}$$

where,

$$D_2 = \frac{D}{LU} \tag{20}$$

and μ_1 is the new dimensionless initial zero-order source coefficient.

The dimensionless form of equation (13) and (14) is

$$\left.\begin{aligned} &\frac{\partial C^*}{\partial \tau} = D_2 \frac{\partial^2 C^*}{\partial \eta^2} - \frac{\partial C^*}{\partial \eta} - \gamma C^* + \mu_1 \\ &C^*(\eta,0) = \frac{c_i}{c_0} \\ &C^*(0,\tau) = 2 - q\tau,\ \tau \ge 0 \\ &C^*(1,\tau) = \frac{c_p}{c_0},\ \tau \ge 0 \end{aligned}\right\} \tag{21}$$

2.2 Solution of the Model

The above problem (21) is solved by using parameter expanding method. The parameter expanding method breaks the equation (21) into simpler ones which can be easily solved successively. To achieve this, let

$$1 = a\gamma_0 \tag{22}$$

in the advection term of equation (21) and

$$C^*(\eta,\tau) = C_0(\eta,\tau) + \gamma_0 C_1(\eta,\tau) + \ldots \tag{23}$$

as in (He, 2006; Sweilam and Khader, 2010; Olayiwola *et al*., 2013). When equation (23) is substituted in equation (21), the following was obtained.

$$\begin{aligned} &\frac{\partial}{\partial \tau}(C_0(\eta,\tau) + \gamma_0 C_1(\eta,\tau) + \gamma_0^{\,2} C_2(\eta,\tau) + \ldots) \\ &= D_2 \frac{\partial^2}{\partial \eta^2}(C_0(\eta,\tau) + \gamma_0 C_1(\eta,\tau) + \gamma_0^{\,2} C_2(\eta,\tau) + \ldots) \\ &- a\gamma_0 \frac{\partial}{\partial \eta}(C_0(\eta,\tau) + \gamma_0 C_1(\eta,\tau) + \gamma_0^{\,2} C_2(\eta,\tau) + \ldots) \\ &- \gamma_0(C_0(\eta,\tau) + \gamma_0 C_1(\eta,\tau) + \gamma_0^{\,2} C_2(\eta,\tau) + \ldots) + \mu_1 \end{aligned} \tag{24}$$

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Equating coefficients of corresponding terms on both sides of equation (24), the resulting equations are as given below:

$$\left.\begin{aligned} &\frac{\partial C_0(\eta,\tau)}{\partial\tau} = D_2\frac{\partial^2 C_0}{\partial\eta^2} + \mu_1 \\ &C_0(\eta,0) = \frac{c_i}{c_0} \\ &C_0(0,\tau) = 2 - q\tau \\ &C_0(1,\tau) = \frac{c_p}{c_0} \end{aligned}\right\} \tag{25}$$

$$\left.\begin{aligned} &\frac{\partial C_1(\eta,\tau)}{\partial\tau} = D_2\frac{\partial^2 C_1}{\partial\eta^2} - a\frac{\partial C_0}{\partial\eta} - C_0 \\ &C_1(\eta,0) = 0 \\ &C_1(0,\tau) = 0 \\ &C_1(1,\tau) = 0 \end{aligned}\right\} \tag{26}$$

Equations (25) and (26) are transformed to satisfy the homogeneous boundary conditions and solved successively using the Eigen functions expansion method. To accomplish this, a function is chosen which satisfies the given boundary conditions. i.e.,

$$w(\eta,\tau) = \alpha(\tau) + \eta(\beta(\tau) - \alpha(\tau)) \tag{27}$$

where the boundary conditions

$$\left.\begin{aligned} &\alpha(\tau) = 2 - q\tau \\ &\beta(\tau) = \frac{c_p}{c_0} \end{aligned}\right\} \tag{28}$$

Let,

$$C_0(\eta,\tau) = v_0(\eta,\tau) + w(\eta,\tau) \tag{29}$$

That is

$$C_0(\eta,\tau) = v_0(\eta,\tau) + (2 - q\tau) + \eta(c_p - (2 - q\tau)) \tag{30}$$

By application of change of variables,

$$\frac{\partial C_0}{\partial\tau} = \frac{\partial C_0}{\partial v_0}\times\frac{\partial v_0}{\partial\tau} + \frac{\partial C_0}{\partial w_0}\times\frac{\partial w_0}{\partial\tau} \tag{31}$$

$$\frac{\partial C_0}{\partial\tau} = \frac{\partial v_0}{\partial\tau} + (\eta - 1)q \tag{32}$$

i.e.
$$\frac{\partial C_0}{\partial \eta} = \frac{\partial v_0}{\partial \eta} + \left(\frac{c_p}{c_0} - (2 - q\tau) \right) \tag{33}$$

$$\frac{\partial^2 C_0}{\partial \eta^2} = \frac{\partial}{\partial \eta}\left(\frac{\partial C_0}{\partial \eta} \right) \tag{34}$$

Therefore,

$$\frac{\partial^2 C_0}{\partial \eta^2} = \frac{\partial^2 v_0}{\partial \eta^2} \tag{35}$$

We substitute (32) and (35) in equation (25) to obtain

$$\frac{\partial v_0}{\partial \tau} = D_2 \frac{\partial^2 v_0}{\partial \eta^2} + \mu_0 - (\eta - 1)q \tag{36}$$

Then,

$$C_0(0,\tau) = v_0(0,\tau) + w(0,\tau) \tag{37}$$

$$\Rightarrow v_0(0,\tau) = 0 \tag{38}$$

Similarly,

$$C_0(1,\tau) = v_0(1,\tau) + w_0(1,\tau) \tag{39}$$

$$v_0(1,\tau) = 0 \tag{40}$$

For the initial condition, $C_0(\eta,0) = 0$

$$C_0(\eta,0) = v_0(\eta,0) + 2 + \eta\left(\frac{c_p}{c_0} - 2 \right) = \frac{c_i}{c_0} \tag{41}$$

$$\Rightarrow v_0(\eta,0) = \frac{c_i}{c_0} + 2(\eta - 1) - \eta \frac{c_p}{c_0} \tag{42}$$

The partial differential equation (25) with the homogeneous boundary conditions (38) and (40) and the initial condition (42) is solved using the Eigen Function expansion method and obtain the following result:

$$v_0(\eta,\tau) = \sum_{m=1}^{\infty} C_m(\tau) \sin(m\pi\eta) \tag{43}$$

$$C_0(\eta,\tau) = v_0(\eta,\tau) + w(\tau) \tag{44}$$

i.e.,
$$C_0(\eta,\tau) = w(\tau) + \sum_{m=1}^{\infty} C_m(\tau) \sin(m\pi\eta) \tag{45}$$

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$$C_0(\eta,\tau)=(2-q\tau)(1-\eta)+\eta\frac{c_p}{c_0}+\sum_{m=1}^{\infty}\left(\begin{array}{l}\frac{2}{D_2(m\pi)^3}(q-\mu_1(\cos(m\pi)-1))\left(1-e^{\left(-D_2(m\pi)^2\tau\right)}\right)\\+\left(\begin{array}{l}-\frac{2c_i}{m\pi c_0}(\cos(m\pi)-1)\\-\frac{4}{m\pi}+\frac{2c_p}{m\pi c_0}\cos m\pi\end{array}\right)e^{\left(-D_2(m\pi)^2\tau\right)}\end{array}\right)\sin(m\pi\eta) \tag{46}$$

Similarly, when equation (26) was solved by Eigen Functions expansion method and the following results was obtained:

$$C_1(\eta,\tau)=-\sum_{n=1}^{\infty}\left(\sum_{n=1}^{\infty}\frac{2}{D_2(n\pi)^3}(q-\mu_1(\cos(n\pi)-1))\left(\begin{array}{l}\frac{1}{D_2(n\pi)^2}\\-\tau e^{\left(-D_2(n\pi)^2\tau\right)}\\-\frac{e^{\left(-D_2(n\pi)^2\tau\right)}}{D_2(n\pi)^2}\end{array}\right)\right)\sin(n\pi\eta)-\sum_{n=1}^{\infty}\left(\sum_{n=1}^{\infty}\left(\begin{array}{l}-\frac{2c_i}{n\pi c_0}(\cos(n\pi)-1)\\-\frac{4}{n\pi}+\frac{2c_p}{n\pi c_0}\cos n\pi\end{array}\right)e^{\left(-D_2(n\pi)^2\tau\right)}\tau\right)\sin(n\pi\eta) \tag{47}$$

The solution of the contaminant flow equation (21) where $c_i \neq 0$ is therefore,

$$C^*(\eta,\tau)=C_0(\eta,\tau)+\gamma C_1(\eta,\tau)+\ldots \tag{48}$$

where $C_0(\eta,\tau)$ and $C_1(\eta,\tau)$ are as given in (47) and (48) respectively.

3.0 Results and Discussion

The semi-analytical result which was obtained in equation (48) is plotted into graphs with the help of input data and Maple software (Maple 16) package as presented in the following figures.

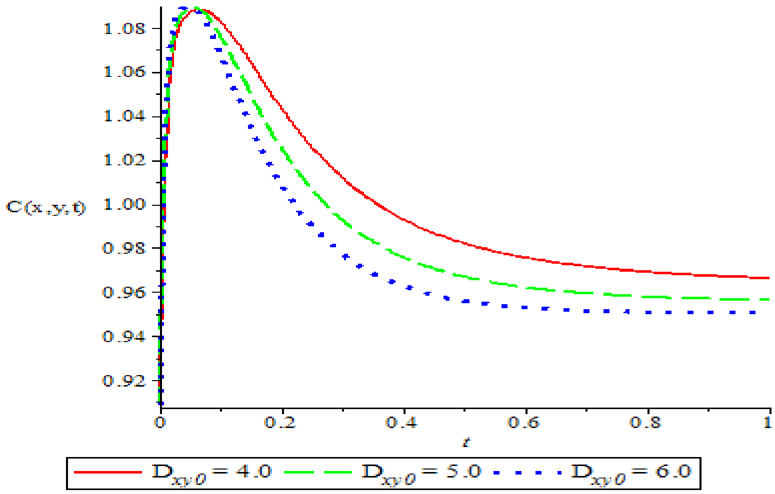

Figure 1: Contaminant Concentration profile for $D_{xy0} = 4, D_{xy0} = 5, D_{xy0} = 6$ when $c_0 = 1, \mu_1 = 0.3$, $c_p = 1, \gamma_0 = 0.1, q = 3, D_{xo} = 1, D_{y0} = 1.5, u_0 = 0.1, v_0 = 0.1$ with y and x fixed as 0.5.

Figure1 is the graph of contaminant concentration with respect to time for varying off-diagonal dispersion coefficient from 4.0 to 6.0. The graph reveals that as the off-diagonal dispersion coefficient intensifies, there is decline in concentration as time increases. Figures 2 and 5 are the concentration profile of contaminant for varying vertical dispersion coefficient. From the graphs, increase in the dispersion in upward direction decline the concentration of the pollutant in x and y directions.

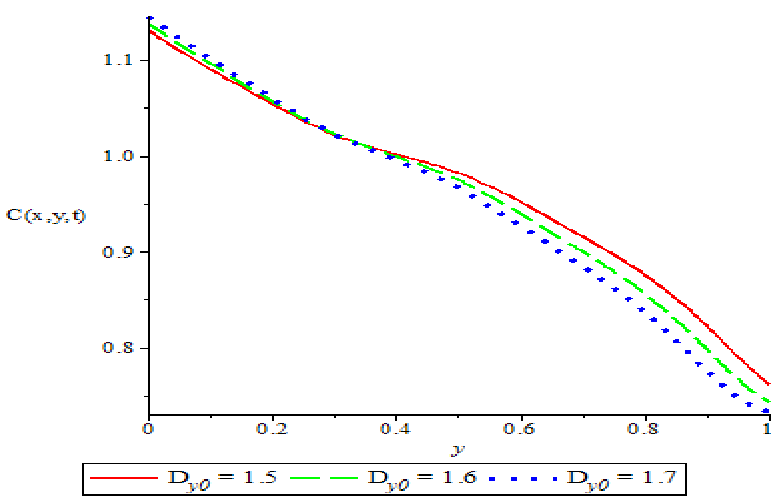

Figure 2: Contaminant Concentration profile for $D_{y0} = 1.5, D_{y0} = 1.6, D_{y0} = 1.7$ when $c_0 = 1$, $\mu_1 = 0.3, c_p = 1, \gamma_0 = 0.1, q = 3, D_{xo} = 1, D_{xy0} = 4, u_0 = 0.1, v_0 = 0.1$ with x and t fixed as 0.5.

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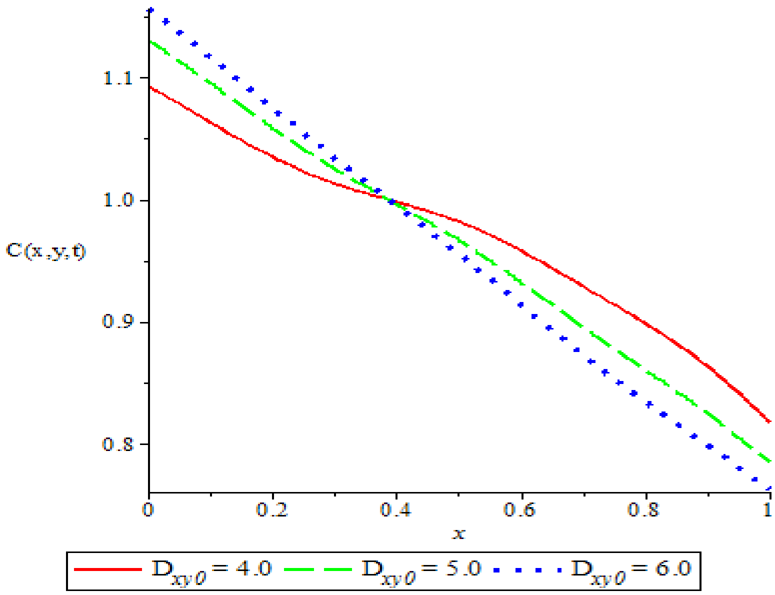

Figure 3: Contaminant Concentration profile for $D_{xy0} = 4, D_{xy0} = 5, D_{xy0} = 6$ when $c_0 = 1, \mu_1 = 0.3,$ $c_p = 1,\ \gamma_0 = 0.1,\ q = 3,\ D_{xo} = 1,\ D_{y0} = 1.5,\ u_0 = 0.1, v_0 = 0.1$ with y and t fixed as 0.5.

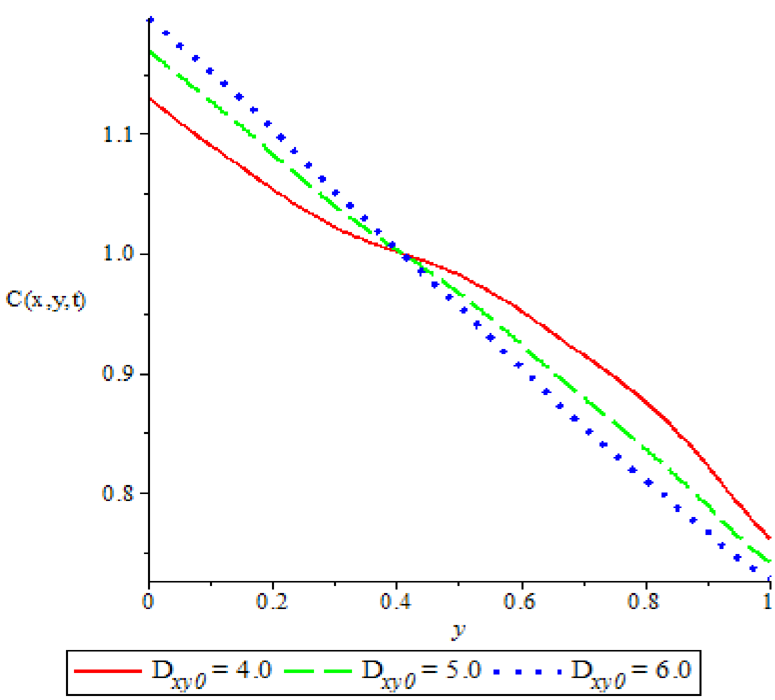

Figure 4: Contaminant Concentration profile for $D_{xy0} = 4, D_{xy0} = 5, D_{xy0} = 6$ when $c_0 = 1, \mu_1 = 0.3,$ $c_p = 1,\ \gamma_0 = 0.1,\ q = 3,\ D_{xo} = 1,\ D_{y0} = 1.5,\ u_0 = 0.1, v_0 = 0.1$ with x and t fixed as 0.5.

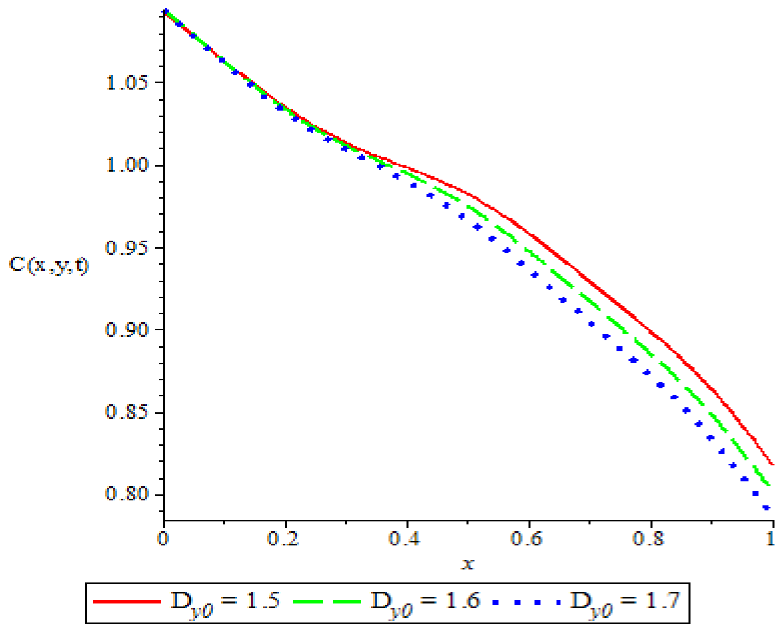

Figure 5: Contaminant Concentration profile for $D_{y0} = 1.5, D_{y0} = 1.6,\ D_{y0} = 1.7$ when $c_0 = 1$, $\mu_1 = 0.3, c_p = 1,\ \gamma_0 = 0.1,\ q = 3,\ D_{yo} = 1.5,\ D_{xy0} = 4,\ u_0 = 0.1, v_0 = 0.1$ with y and t fixed as 0.5.

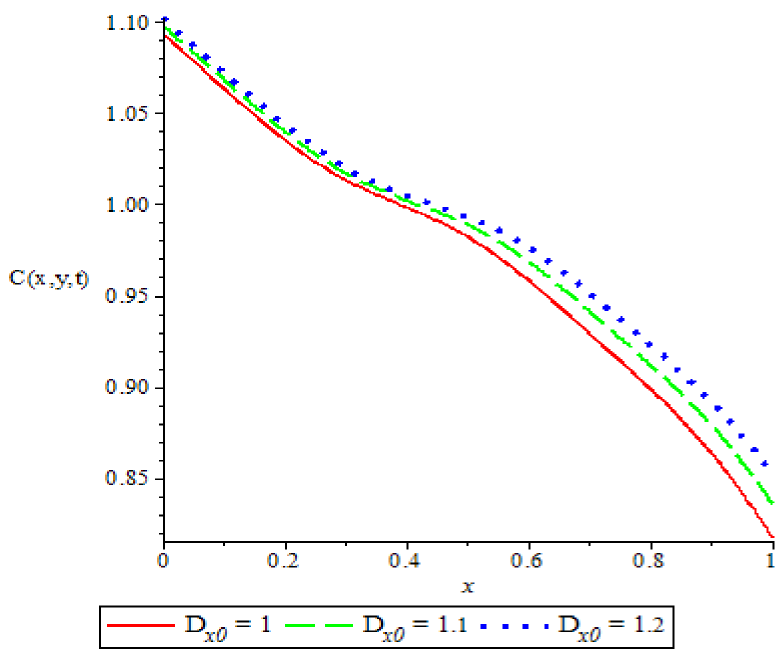

Figure 6: Contaminant Concentration profile for $D_{x0} = 1, D_{x0} = 1.1,\ D_{x0} = 1.2$ when $c_0 = 1$, $\mu_1 = 0.3, c_p = 1,\ \gamma_0 = 0.1,\ q = 3,\ D_{yo} = 1.5,\ D_{xy0} = 4,\ u_0 = 0.1, v_0 = 0.1$ with y and t fixed as 0.5.

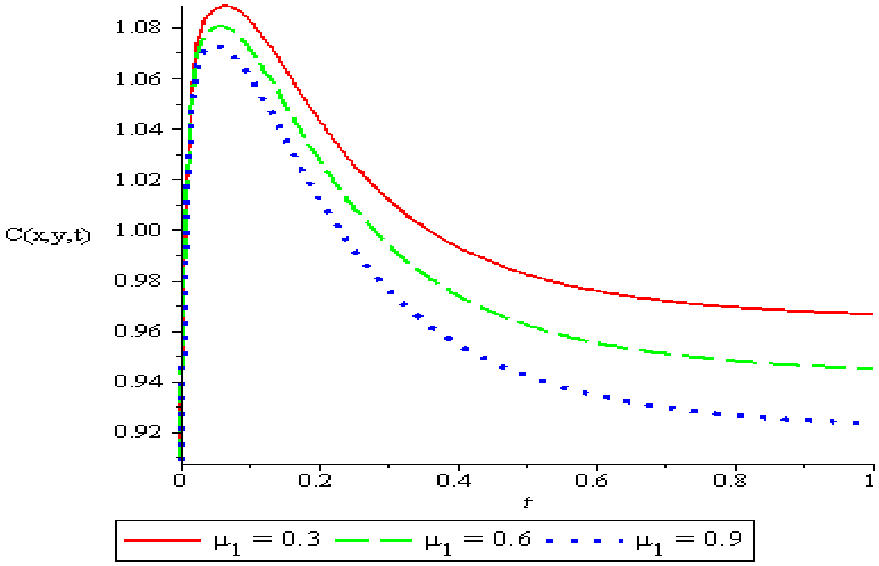

Figure 7: Contaminant Concentration profile for $\mu_1 = 0.3,\ \mu_1 = 0.6,\ \mu_1 = 0.9$, when $c_0 = 1, c_p = 1$, $\gamma_0 = 0.1,\ q = 3,\ D_{xo} = 1,\ D_{y0} = 1.5,\ u_0 = 0.1,\ D_{xy0} = 0.4\ v_0 = 0.1$ with y and x fixed as 0.5.

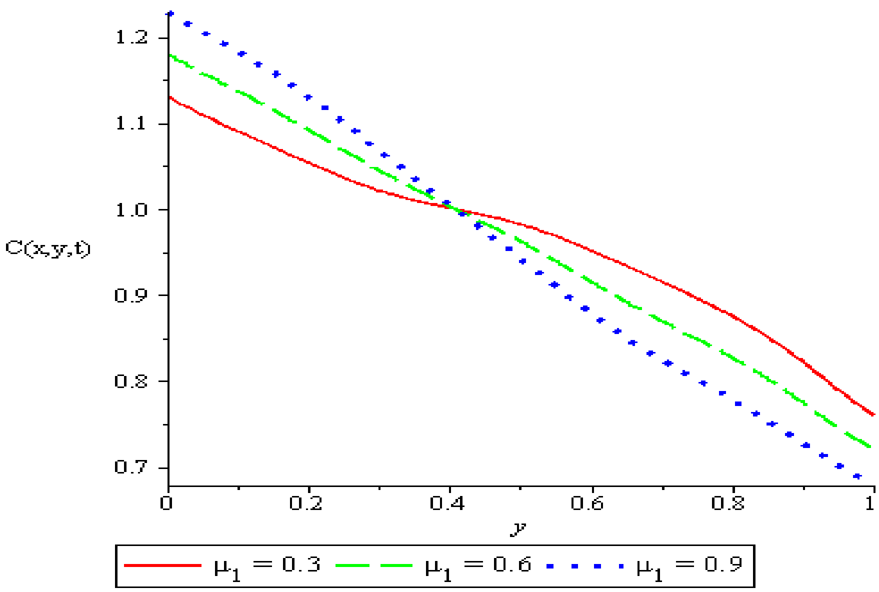

Figure 8: Contaminant Concentration profile for $\mu_1 = 0.3,\ \mu_1 = 0.6,\ \mu_1 = 0.9$, when $c_0 = 1, c_p = 1$, $\gamma_0 = 0.1,\ q = 3,\ D_{xo} = 1,\ D_{y0} = 1.5,\ u_0 = 0.1, v_0 = 0.1$ with x and t fixed as 0.5.

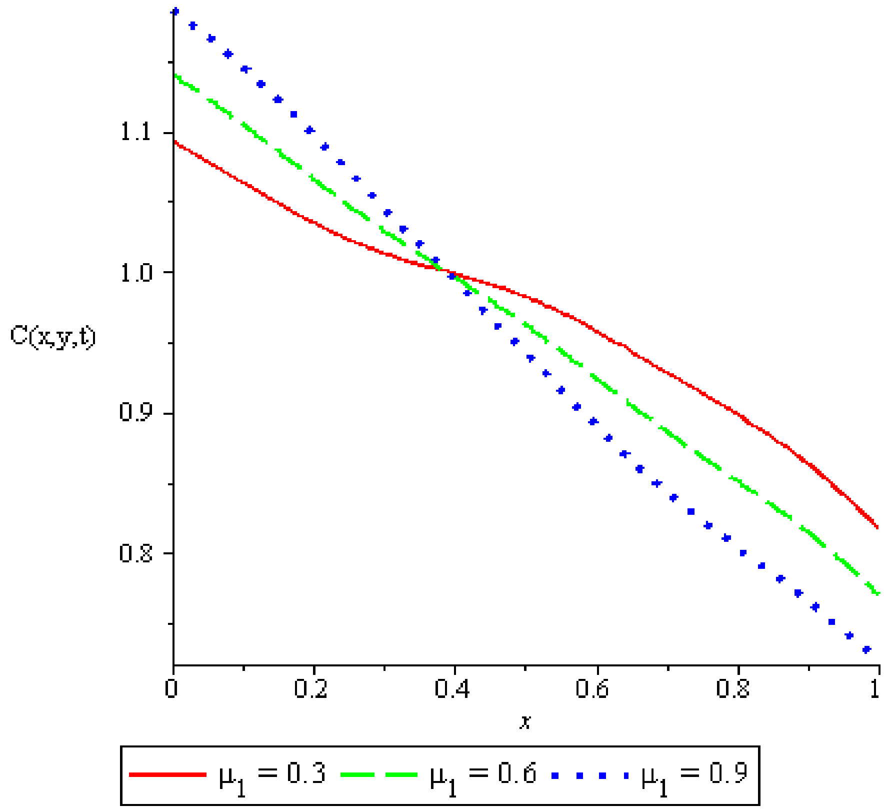

Figure 9: Contaminant Concentration profile for $\mu_1 = 0.3$, $\mu_1 = 0.6$, $\mu_1 = 0.9$, when $c_0 = 1, c_p = 1$, $\gamma_0 = 0.1$, $q = 3$, $D_{xo} = 1$, $D_{y0} = 1.5$, $u_0 = 0.1, v_0 = 0.1$ with y and t fixed as 0.5.

The impact of off-diagonal dispersion on the contaminant concentration is illustrated by figures 3 and 4. These figures show that the concentration of pollutant decreases with increase in distances x and y as a result of rise in off-diagonal dispersion. Lastly, in figure 6, as the horizontal dispersion coefficient increases, the concentration of pollutant declines as the horizontal distance x increases. The results obtained in this study may assist the geologist in knowing the distance from the source of contaminant that is good for location of wells. These are the distances at which the contaminant concentration is zero. Figures 7, 8 and 9 show that there is a drop in contaminant concentration as the zero-order source coefficient increases.

The intersection of the lines in Figures 8 and 9 only show the faster rate of concentration decline the zero-order source coefficient increases. It has no physical implication with respect to this research article.

4.0 Conclusion

In this work, the effect of off-diagonal dispersion and zero-order source of contaminant on the concentration are studied. The two-dimensional contaminant flow problem incorporating the off-diagonal dispersion coefficient and zero-order source has been solved by the method of Eigen functions expansion. Findings from the study show that the contaminant concentration declines with increase in time and distances as the off-diagonal dispersion, horizontal dispersion and vertical dispersion coefficients increase. The study also revealed that as the zero-order source coefficient increases, there is concentration drop as distance and time increases.

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