Recent Developments in the Use of Time-Frequency Analysis for Radar-based Applications

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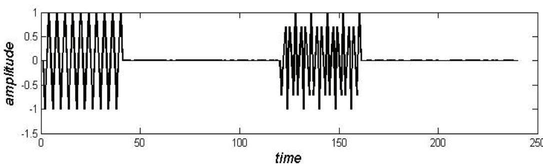
***Abstract*— Time-frequency analysis is an evolution of mathematical ideas and concepts used in the analysis of time-varying spectra of signals in order to cater for various problems in different fields. One of such fields of interest is radar related problems. This paper reviews the recent advancements made in this area with the aid of time-frequency (t-f) tools within the last year till date, it presents a good starting point for researchers interested in radar applications of t-f analysis. It investigates the nature of tools used, the problem(s) solved, advantages and limitations of method used where applicable. Illustrations of some basic concepts are also presented when necessary. The short time Fourier transform (STFT) and Wigner-Ville distribution (WVD) still remain the two main key time-frequency distributions that have undergone various modifications to mitigate radar related problems. It is also seen that considerable amount of most recent works deals with micro-Doppler identification and radar signals analysis.**

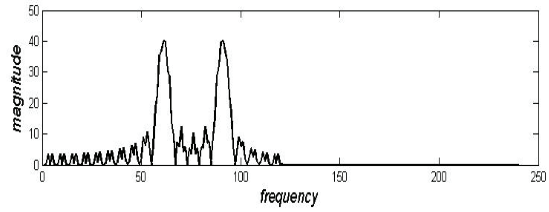
Keywords*— time-frequency, time-varying spectra, radar, micro-Doppler, short time Fourier transform (STFT), Wigner-Ville distribution (WVD).

I. Introduction

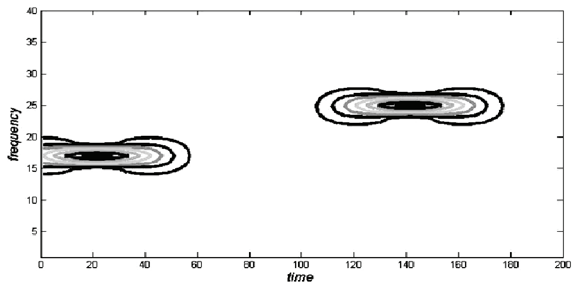
The conventional time domain and its associated frequency domain representation obtained with the aid of Fourier transform are insufficient to obtain the characteristics of a non-stationary signal. This is because the former can only present when the characteristics of a signal change, and the latter on how it changes but not both at the same time. The time-frequency (t-f) tools were developed to provide both characteristics at the same instance [1]. This concept is illustrated in fig. 1 using a radar signal.



a) Time representation



b) Frequency representation



c) Time-frequency representation

Fig. 1. Concept of time-frequency analysis (All units in samples)

The figures above show a generated frequency hopping radar signal on a pulse-to-pulse basis, with frequencies of 10MHz and 15 MHz, sampling frequency of 40 MHz, pulse width (PW) and pulse repetition interval (PRI) of 1 and 2 microseconds respectively. In fig. 1a, the time domain shows the time characteristics of the signal, but cannot clearly show that frequency of the first pulse is different from that of the other pulse. Fig. 1b shows that there are two frequencies present in the radar signal but doesn’t show when these frequencies occur. Fig. 1c shows both the time and frequency characteristics of the signal in t-f plane with the aid of time-frequency distribution (TFD). This concept is even clearer when the practical non-stationary signals are considered due to interferences from various sources. This forms the basis of why time-frequency tools are crucial in the field of biomedicine, sonar, communications, manufacturing, multimedia and radar as considered in this paper [2] [3].

There are two key distributions that other TFDs spur out from when t-f analysis is considered. The short time Fourier transform (STFT) used in figure 1 is the oldest and linear TFD, a direct extension of the classical Fourier transform. It uses

978-1-4799-7498-6/15/$31.00 ©2015 IEEE

sliding window to localize the content of the signal in a specific domain. The localization in one domain causes smear in the other domain due to the uncertainty principle accounting for the major problem associated with this TFD [4]. The Wigner-Ville distribution (WVD), an extension of the Wigner distribution is the other key TFD that uses a form of quadratic function to concentrate the signal energy along the signal's instantaneous frequencies (IFs). Unfortunately, the WVD comes with the undesired cross terms as a result of its quadratic nature. All other TFDs that have been enacted since inception of t-f analysis have been done so through the actual use, modification or at the very least based on concepts of these key two TFDs [1-3].

The paper investigates usage and modifications of the TFD that has been carried out within the past year in order to solve radar related problems such as micro-Doppler (m-D) signature detection and removal, production of high resolution radar images, analysis and classification of radar waveforms among others. Sections II and III present t-f analysis associated with radar-based interference and mitigation techniques, while sections IV and V discuss the analysis of radar signals and images respectively. A conclusive remark is present at the end.

II. Micro-Doppler Analysis

Various t-f analyses were used to analyse various sources of radar interference such as m-D, narrow band interference (NBI) and ionospheric decontamination. A reasonable amount of researches within the previous year dealt with m-D analysis due to its importance to this field. m-D can be defined as small vibration or signatures caused by micro-motion dynamics causing interference to radar echoes which can be due to motions in a vehicle, sea clutter or environmental sources [3].

In [5], the novel short time fractional Fourier transform (STFrFT) was used for m-D signal removal in a spiky seas clutter environment. The method achieved a high m-D detection probability in a low signal-to-clutter ratio and demonstrates high superiority over the classical STFT. The STfrFT is a hybrid TFD made up of fractional Fourier transform (FrFT) and STFT, attaining the merits of both distributions while minimizing their demerits. The STFT and STFrFT are given in (1) and (2) respectively.

$$STFT(t,f) = F_{t\to f}\{s(\tau)w(\tau - t)\} \quad (1)$$

$$STFrFT_{\alpha}(t,u) = F_{t\to f}\{s(\tau)w(\tau - t)K_{\alpha}(\tau,u)\} \quad (2)$$

Where $F_{t\to f}$ denotes taking a Fourier transform with respect to τ, $s(\tau)$ is the signal, $w(\tau)$is the window. In the case of STFrFT, a Gaussian window is used due its number of advantages. $K_{\alpha}(\tau,u)$ is the transformation function due to FrFT given in [5], capable of transforming each component of the m-D signal into peak in the TFD, thereby concentrating the signal into few coefficients. Alternatively [6], the Fourier Bessel transform (FBT) was used in conjunction with FrFT to extract m-D features of radar echo before performing STFT to estimate the motion parameters. Mathematically, the FBT $F(p)$ of a function $f(r)$ is given as;

$$F(p) = 2\pi \int_{-\infty}^{\infty} f(r)J_0(2\pi pr)rdr \quad (3)$$

$$f(r) = 2\pi \int_{-\infty}^{\infty} F(p)J_0(2\pi pr)pdp \quad (4)$$

Where $J_0(2\pi pr)$ are the zeroth-order Bessel functions and p is the transform variable. Two additional test environments; rotating reflectors and rotating synthetic aperture radar (SAR) antenna in addition to strong sea clutter environment were used to test this method.

Furthermore superiority of using interpolators to estimate missing or incomplete sampled data for micro-Doppler radar signature estimation using WVD and Choi-Williams distribution (CWD) was demonstrated in [7]. The equation for WVD is given in (5);

$$WVD(n,k) = \sum_{m=-\frac{N}{2}}^{\frac{N}{2}-1} x(n+m)x^{*}(n-m)e^{-\frac{j2\pi mk}{N}} \quad (5)$$

Where $x(n)$ is the signal and N is the signal length. CWD is a form of WVD distribution that uses an alternative kernel function to cater for the cross term associated with WVD. Interpolation is a classical signal processing procedure used to change a signal from one sampling rate to the other, acting like a low pass filtering process. Under uniform sampling environment, interpolation is given as;

$$y(n) = \sum_{m} x(n-m)h(m) \quad (6)$$

Where *x(n)*, *h(n)* and *y(n)* is original signal, interpolation kernel in time domain, and the interpolated signal respectively. The sinusoidal multi-component frequency modulated (FM) signal was used as the test m-D signal in this work. Another work that models the m-D signal using a multi-component SFM signal was presented in [8]. Their novel t-f technique, Sinusoidal Frequency Modulation Fourier Transform (SFMFT), offers a reduced complexity and better computational efficiency over others, and hence the SFMFT was used for parameter estimation based on global data. In its discrete form the resulting SFMFT is given as

$$X(k) = \sum_{n=0}^{N-1} \ln[x(n)]\left(e^{-\frac{j2\pi nk}{N}}\right) \quad (7)$$

This method used has high spectrum estimation accuracy when compared to the existing method.

The linear classical STFT was used to estimate m-D signals with the aid of IF and direction pattern algorithms in [9]. The STFT is used to obtain the time-frequency representation (TFR) of the target echo, then the IF algorithm based on peaks of the TFR along frequency axis is used to obtain the IF. Thereafter, the direction pattern algorithm is used to separate the m-D signal. The mask used for the direction pattern in order to obtain the basic feature of the m-D signal is given in table 1.

TABLE I. THE DIRECTION MASK

1			2
1		2	
1	2		
4	3	3	3
5	4		
5		4	
5			4

Table 1 is used to calculate the gray mean of the frequency pixel in order to determine the pixel's direction vector from the maximum mean. It is important to point out that no parameter estimation or target recognition was carried out in this research.

III. OTHER RADAR-BASED INTERFERENCE MITIGATION TECHNIQUES

Even though the m-D signal dominated recent researches on radar based interference using t-f analysis, there were still other sources of interference associated with some specific types of radar. The STFT and the independent subspace analysis (ISA) were used to mitigate NBI of SAR [10]. The NBI results from interference due to man-made and radio frequencies leading to the bandwidth of jamming signals being relatively narrower (less than 1%) than that of the useful target echoes. This poses a great hindrance on SAR image formation and accurate Doppler parameter estimation. The minimum description length criterion of the ISA given in (8) is used to correctly divide the signal subspace into NBI subspace and useful signal subspace;

$$L = argmin\{-N(q-L)log\rho(L) + 0.5L(2q-L)logN\} \quad (8)$$

Where L is the number of NBI components, N is the number of data samples in a pulse, q is the number of single values, $\rho(L)$ is the ratio of the geometric mean of the $(q-L)$ smallest singular value to their arithmetic mean. The method used has a good peak-to-sidelobe ratio (PSLR) and the best integrated sidelobe ratio (ISLR) when compared to other methods.

Recently, detecting ship target was marred by two sources of interference based on the radar in consideration. A hybrid STFT-smoothed pseudo WVD (SPWVD) joint TFD was used to suppress large amplitude, slow-varying type ionospheric phase decontamination to improve ship target detection for over-the-horizon radar (OTHR) [11]. The hybrid system is used to improve the uncertainty principle of the STFT while at the same time suppressing the cross terms associated with WVD. The method together with other presented methods effectively produced an improved clutter spectrum for better ship recognition.

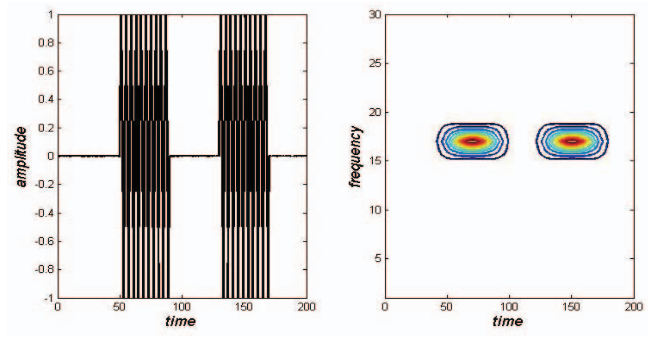
In the other related work based on improving ship detection [12], a t-f coherence indicator was used for azimuth ambiguities removal for ship detection in SAR. Simply put, a t-f coherence indicator performs cross-correlation between the obtained t-f representations (TFR) around different spectral locations; its matrix is shown in (9);

$$C_{TF} = E(S_{TF}S^{*}{}_{TF}) = \begin{matrix} C_{11} & \cdots & C_{1R} \\ \vdots & \ddots & \vdots \\ C_{R1} & \cdots & C_{RR} \end{matrix} \quad (9)$$

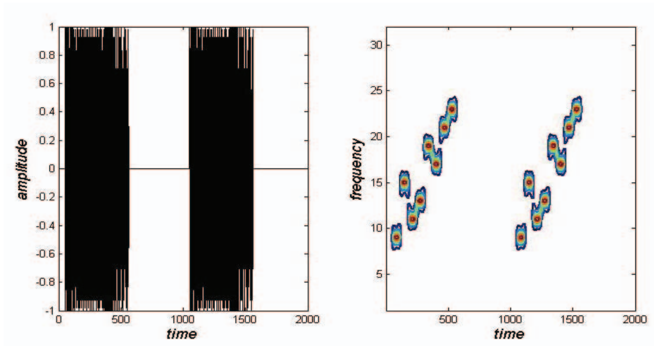
Where C_{TF} is the coherency matrix, R represents a spectral location. The maximum likelihood (ML) estimate of C_{TF} is used to obtain the normalised version used in this work. The t-f coherence indicator is used to remove ghost echoes generated by the various t-f coherence characteristics between real ship target echoes and ambiguous echoes. Results obtained show that the method developed was able to effectively mitigate ambiguities originating from ships due to their highly coherent characteristics

IV. RADAR SIGNAL ANALYSIS

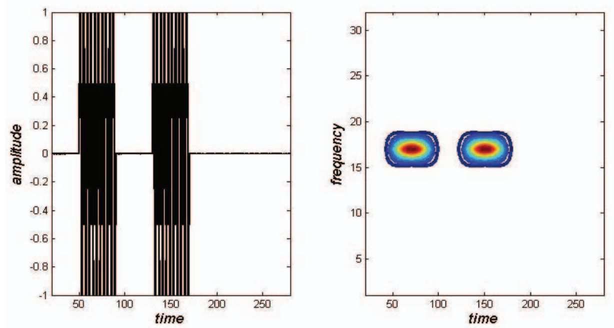
Currently, radar signals can be roughly classified into three groups; the common classical radar signal of simply modulated pulse and constant PRI, the pulse modified radar signals such as the pulse compression and frequency hopping radar signals used mostly by military to achieve low probability of intercept (LPI) and the PRI modified radar signals used by different radar types to achieve specific functions. An example of each class and its corresponding TFR is shown in fig. 2.



a) Simple pulsed



b) LPI-FSK



c) Staggered PRI

Fig. 2. Examples of radar signal groups (All units in samples)

In fig. 2a, the classical radar signal of small pulse width, modulated sinusoidally within a pulse and of constant PRI is shown. A Costas coded frequency shift keying (FSK) modulation LPI signal is shown in fig. 2b, whereby eight frequencies are now within a longer pulse width in order to meet LPI requirements. In fig. 2c., staggered PRI signal is shown of different first and second PRI, usually used by moving target indication (MTI) radar to eliminate blind speed. Further details of these types of radar signals can be found in [13].

Recent developments dealt majorly with LPI signals due to their interesting characteristics and unique applications. T-f rate distribution was used to classify different types of poly-phase coded LPI signals (frank, P1, P2, P3 and P4 codes) with the aid of LFM characteristics [14]. The t-f rate distribution is given in (10).

$$TFR(t,\Omega) = \int x(t+\sqrt{\tau})x(t-\sqrt{\tau})e^{-j\Omega\tau}d\tau \quad (10)$$

Where *x(t)* is the signal in consideration. The result obtained was good (low SNR) and justified the use of this method. In [15], the low-complexity spectrogram TFD and other related basic signal processing tools were used to analyse and classify correctly there LPI signals (Linear FM, FSK, and phase coded) together with simple pulsed and staggered radar signals. The spectrogram is the square magnitude of the STFT shown in (11);

$$SP(t,f) = |STFT|^2 = \left|F_{t\to f}\{s(\tau)w(\tau-t)\}\right|^2 \quad (11)$$

Where $s(\tau)$ and $w(\tau)$ is the signal and window function (hamming in this case) respectively. Results obtained demonstrated and validated the efficiency of the analysis and classification model used.

Also on LPI signals analysis, a robust algorithm based on the Markov random field (MRF) model was used in conjunction with the B-distribution in order to estimate overlapping instantaneous frequencies (IFs) of multi-component radar signals in low SNR environments [16]. The B-distribution has significant cross-term suppressions and the MRF model works on graph obtained from TFR of the B-distribution. Good normalized mean absolute error (NMAEs) for various LPI test signals that were considered was obtained.

It was also shown in [17] that accurate detections of LFM continuous waveform (LFMCW) – one of the most common LPI signals – can be achieved faster by decreasing the window length of PWVD and Wigner Hough transform (WHT). A rectangular window was used for the PWVD and the test signal is detected from the WHT peak detection after the WHT transform is carried out. Results obtained demonstrated the quality of the method as accurate classification was achieved at low SNR.

STFT is used in conjunction with the wave entropy in order to identify phase-coded modulation (PCM) for the pulse Doppler (PD) radar processing in [18]. The wave-entropy is used to track the phase changes of the PCM signals in t-f domain and its derivation is shown in (12) and (13).

$$di(t,w) = \frac{STFT(t_i,w_i)}{sum(STFT(t_i,w_i))} \quad (12)$$

$$E_{wave} = -\sum_{i=1}^{N} d_i \log\left[d_i\right] \quad (13)$$

The method developed works in a reasonable SNR environment for both single and poly-phase coded modulation.

V. RADAR IMAGING ANALYSIS

Analysis of radar images is very important in the field of radar signal processing, especially in SAR related applications. In some cases, radar signals are transformed into images in order to easily obtain some specific characteristics of the signal. The radar cross-section (RCS) also comes into consideration in the section. RCS can be roughly referred to as the area obtained from the reflected power of a target from a specific point of view (receiver's) to that obtained through the power of the target scatted in all directions [19]. The larger the target size, the larger the RCS is going to be, hence commercial airlines usually have a larger RCS, while military aircrafts usually have a smaller RCS due to the importance of stealth.

Two recent researches delved into RCS analysis using t-f techniques for SAR applications [20][21]. A vector network analyser (VNA) experimental setup, with a form of STFT component, is used to analyse RCS value of cylinder, cone, sphere and rectangular plane surface [20]. The relationship between the RCS value and incident wave was verified. Alternatively in a more sophisticated setup [21], the multi-dimensional continuous wavelet transform (MCWT) was used together with other statistical signal processing tools to analyse the RCS of canonical targets (sphere, trihedral, dihedral, flat plates) in high resolution laboratory radar imagery. The wavelet transform is similar to STFT, with the difference of using a Gaussian sliding window in the frequency domain due to unique characteristics of images. Entropy, kurtosis and skewness are used to evaluate the performance of method used.

The STFT is combined with spherically invariant random vector (SIRV) product model for high resolution polarimetric SAR (PolSAR) image feature extraction in an urban area [22]. The complex measurement of the SIRV production is given (14);

$$k = \sqrt{\tau}.\mathrm{z} \quad (14)$$

Where τ is a random variable and z is an independent complex circular Gaussian vector. An almost automatic man-made target extraction was achieved using the method developed.

VI. CONCLUSION

This paper has shown that despite being the oldest TFD since the inception of t-f analysis, the STFT is still being used for solving radar related problems. Most times, some other signal processing tool is used before using STFT, such as FBT for m-D analysis, or after using it such as the use of direction pattern for m-D detection or wave entropy to track phase changes in PD processing. Even more common is a hybrid TFD formed from STFT and another TFD such as itself (spectrogram) for radar signal analysis and classification and FrFT (STFrFT) for micro-Doppler removal. The PWVD, as the alternative to WVD in order to suppress the cross-terms disadvantage of the WVD, is also used for m-D analysis and also in conjunction with other common effective tools such as WHT for better IF estimation and even STFT for ionospheric phase decontamination removal for better ship recognition.

Recent developments associated with ground penetrating radar (GPR) were not considered in order to maintain a clear and concise review and to meet the type of publication scope. All the figures shown in this work relating to radar signals despite being based on current technology are meant for illustration as the practical radar signal usually has a much longer PRI and at least four PRIs of a captured signal to be analysed must be shown.

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