Inter-Pulse Analysis of Airborne Radar Signals Using Smoothed Instantaneous Energy

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***Abstract*—This paper investigates the effect of five window functions on the smoothing of the less computationally complex instantaneous energy for the estimation of time parameters of three radar signals in the presence of additive white Gaussian noise (AWGN). The windows considered, all of one variable parameter (N) are rectangular, Hamming, Hanning, Bartlett and Blackman. Simulation results show that the rectangular window that possesses widest main lobe width performs best while Blackman window of narrowest main lobe width performs poorest. In addition, 100% percent probability of time parameters estimation is achieved at signal-to-noise ratio (SNR) of 4dB for simple modulated pulse signal, -1dB for staggered pulsed signal and 5dB for linear frequency modulated signal in the worst case window scenario.**

***Index Terms*—instantaneous energy, main lobe width, signal-to-noise ratio**

I. Introduction

Electronic intelligence (ELINT) system, a component of electronic support (ES) is basically associated with the intercepting and analysis of radar signals. Its main feature is to determine the pulse (time) characteristics such as pulse width (PW) and pulse repetition period (PRP), i.e. inter-pulse analysis and also to determine the frequency characteristics of received radar signal. These characteristics are then used to classify it in order to determine its electronic order of battle (EOB) [1]. Apart from the classical use of PW and PRP to determine range resolution and unambiguous range respectively [2], they can also be used to estimate angle of target and can undergo various variations to achieve specific functions [1], [3]. Recent and advanced systems uses radar signals of staggered PRP and pulse compression modulation to eliminate blind speed in moving target indicating (MTI) systems, protection against electronic counter measures (ECM), achieve low probability of intercept (LPI) and modern waveform design [4]-[6].

Due to low peak transmit power and the use of spread spectrum waveform of pulse compression radar, it is necessary to adopt signal processing techniques that can cater for this. Time-frequency analysis is identified as key and robust solution technique especially in recent times [7]-[9], but unfortunately of high computational complexity. The objective of this work is to use the instantaneous energy (IE) of less computational complexity to estimate time parameters of radar signal together with the investigation of effect of various window functions on the estimation. Energy analysis has recently been used in various fields such as radar [10], seismology [11] and wireless sensor networks [12] to achieve specific functions.

Section II, III and IV of this paper describes the signal models, window functions characteristics used, and the IE analysis method respectively. Results obtained are presented and discussed in Section V.

II. Received Signal Model

The received signal is modeled as follows:

$$y(t) = x(t) + n(t) \tag{1}$$

where $x(t)$ is the received radar signal sampled at 40MHz and its corresponding noise, $n(t)$ is modeled as additive white Gaussian noise (AWGN) with zero mean and unity variance. The three signals used in this paper are:

1) Linear FM - a type of pulse compression radar signals,
2) Staggered pulse - a type of PRI variations radar signals,
3) Simple modulated signal - the classical radar signal.

Their signal parameters are shown in Table I.

The chirp ratio, α of the LFM is obtained from the bandwidth (BW) and signal duration (T). SiP and StP have the same modulation with the difference being the latter having two PRPs, one short and the other long.

TABLE I. Airborne Radar Signal Parameters

Signal Type	Frequency Parameters	Time Parameters	Equations
Linear FM Pulse (LFM)	f_0=2MHz, $f1$=17MHz	PW=2µs, PRP=100µs	$x(t) = A\cos\left(2\pi\left(f_0 + \frac{\alpha t}{2}\right)t\right)$, $\alpha = \frac{BW}{T}, BW = f_1 - f_o$
Staggered Modulated Pulse (StP)	fc=10MHz	PW=1µs, PRP=50µs, 200µs	$x(t) = A\cos(2\pi f_c t)$,
Simple Modulated Pulse (SiP)	fc=10MHz	PW=1µs, PRP=50µs	$x(t) = A\cos(2\pi f_c t)$,

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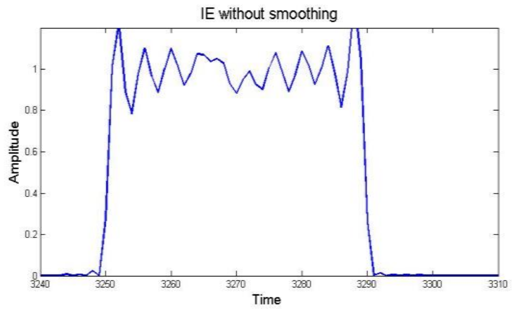
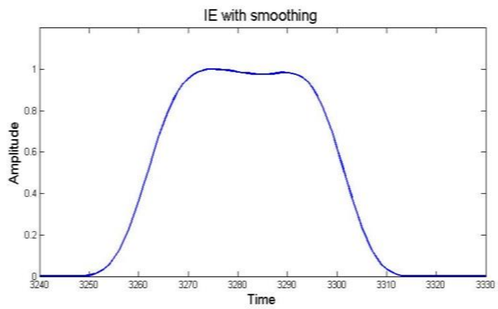
III. Smoothing Windows

Window functions are considered in this paper for the purpose of smoothing the IE. They are used to eliminate the out-of-points in the IE envelope for better estimation of time parameters. Recently, this concept of smoothing for better estimation was used in the field of radar for direction finding [13] and time delay estimation [14]. Typical effect of smoothing at SNR of -2dB is illustrated in Fig. 1. It can be seen that estimation will be better with smoothing.



a) Without smoothing



b) With smoothing

Figure 1. Instantaneous energy with and without smoothing using Hamming windows of 24 sample points for simple pulsed radar at SNR of -2dB

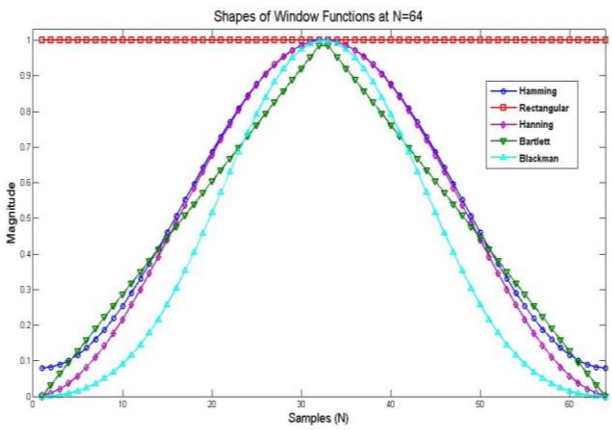


Figure 2. Shapes of window functions.

Apart from smoothing, window functions are used in the field of radar for the design of non-linear frequency modulation (NLFM) signals [15] and pulse compression radar processing [16]. The classical window functions considered in this paper are rectangular, Hamming, Hanning, Bartlett and Blackman. All of them have one variable parameter, N and are of linear phase response, making them suitable for smoothing. Their shapes are shown in Fig. 2.

Rectangular window has a higher side lobe than the others but at the same time has a smaller main lobe width [17]. Table II is a collection of the mathematical representation for the window functions considered in this work.

TABLE II. Window Functions Mathematical Representations

Window Function Name	Time Domain Sequence $h(n), 0 \le n \le N-1$
Rectangular	1
Hamming	$0.54 - 0.46\cos(2\pi n / N-1)$
Hanning	$0.5 - 0.5\cos(2\pi n / N-1)$
Blackman	$0.42 - 0.5\cos(2\pi n / N-1) + 0.08\cos(4\pi n / N-1)$
Bartlett	$1-2\left\|(n-(N-1)/2)/N-1\right\|$

The Hamming and the Hanning (also known as Von hann) are basically the same window with the general formula as;

$$\alpha-(1-\alpha)\cos(2\pi n / N-1) \qquad (2)$$

where α is a parameter having the value of 0.5 for Hanning and 0.54 for Hamming. The slight increase reduces the ripple ratio by 50% [18]. The additional cosine term in the Blackman window function reduces the ripple ratio, but at a cost of increase in the main lobe width. Lastly the triangular window is implemented by the Bartlett function.

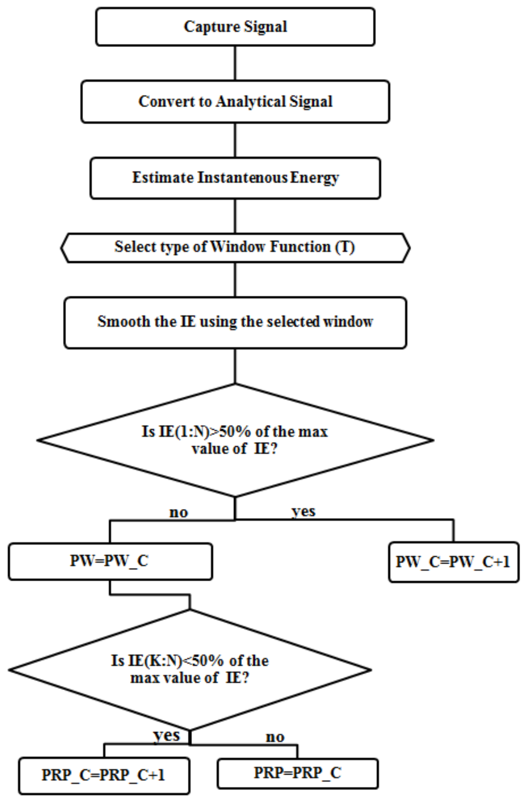


Figure 3. Inter-Pulse analysis by instantaneous energy flowchart

IV. Inter-Pulse Analysis by IE Methodology

Recent works [19]-[21] have reported different methods for estimating time parameters of radar signals.

The PRI transform based on complex-valued autocorrelation was used to detect up to 2 or 3 single pulse trains with a 30% PRI jitter from 100 pulses [19] and a new estimator of time of arrival (TOA) and PW based on solutions of linear equations obtained from three convolution peaks was presented [20]. Most recently, a combination of fast Fourier transform (FFT) and filters were used for achieving 100% probability of PW and TOA estimation at SNR of 4dB [21]. This work extends the use of instantaneous energy (IE) to analyse and estimate the PW and PRP of airborne radar signals. The flowchart of Fig. 3 describes the methodology for implementing the inter-pulse analysis. It presents different steps undertook to estimate the time parameters of the radar signals; PW and PRP. These steps are further explained in the upcoming sub-sections.

A. Hilbert Transform

This transform is used to introduce a phase lag of 90°. For a signal *x(n)*, its Hilbert transform is given by [22];

$$H\{x(n)\} = \underset{t \leftarrow f}{F^{-1}}\{(-j \operatorname{sgn} f)\underset{t \rightarrow f}{F}\{x(n)\}\} \tag{3}$$

As such;

$$\begin{aligned} H\{\cos(2\pi f_o n)\} &= \sin(2\pi f_o n) \\ H\{\sin(2\pi f_o n)\} &= -\cos(2\pi f_o n) \end{aligned} \tag{4}$$

The Hilbert transform is used in this work to produce the analytic form or the complex form of the signal in order to eliminate the non-required negative frequencies generated by Fourier transform. It provides two main advantages [22];

1) Halves the total bandwidth, hence allowing sampling at half the Nyquist rate without aliasing
2) Making sure that an approximate value of unity is obtained when the instantaneous energy is calculated irrespective of the amplitude value.

The mathematical representation for analytic signal is given by:

$$x_a(n) = x(n) + j.H\{x(n)\} \tag{5}$$

B. Instantaneous Energy

The instantaneous energy is used in this work to determine the time characteristics of the incoming radar signal so as to cut down processing burden as compared to other methods such as time-frequency analysis. Instantaneous energy (after Hilbert transform) at instantaneous of time is given by the convolution of $x_a(n)$ and $x_a^*(n)$. Thus,

$$E_i(n) = x_a(n) * x_a^*(n) \tag{6}$$

where $x_a^*(n)$ is the conjugate of $x_a(n)$. This instantaneous energy is now smoothed through the use of the finite impulse response (FIR) filter function as given in (7) [23]:

$$E_{i,s}(n) = \sum_{k=0}^{N} b(k) E_i(n-N) \tag{7}$$

where $E_{i,s}$ denotes the smoothed instantaneous energy and b denotes the 'averaged' filter coefficients obtained from the type of window function selected.

V. Results and Discussions

A question arises on selecting appropriate window length (N) for the smoothing so as results from different window can converge appropriately at a specific value. As such a sub-analysis was carried out to examine the effect of different windows on the PW estimation of simple pulsed radar. Results obtained are present in Table III.

TABLE III. Effect of Window Length on PW Estimation of a Simple Pulsed Radar

Length N / Window	1	2	3	4	5	6	7	8	9	10
Rectangular	39	40	41	42	41	42	43	44	45	46
Hamming	39	40	39	40	41	40	41	42	43	42
Hanning	39	40	41	40	41	42	41	42	43	42
Blackman	39	40	39	40	41	40	41	42	41	42
Bartlett	39	0	39	40	41	40	41	42	43	42

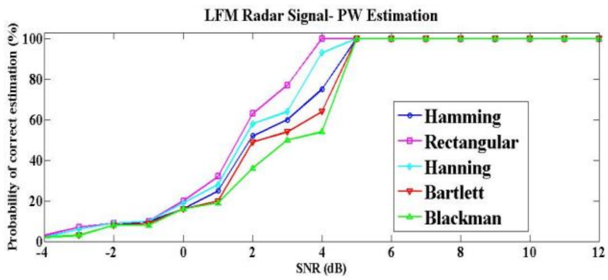
The simple radar signal was generated at a PW of 1 µs (40 samples). It can be seen in Table III that at N=5, PW estimation is 41 (1.025 µs) and is constant irrespective of the type of window selected. As such, window length of 5 (0.125 µs) was used for the whole setup. It is also important to point out that a reasonable threshold of 50% was chosen for the time characteristics estimation to cater for the presence of LPI signals.

To verify the performance of this inter-pulse analysis, Monte Carlo simulation is performed for each test signal and set of windows by running the system for 100 loops for SNR range from -4dB to 12dB and probability of correct estimation (P_{ce}) were determined at the end of 100 loops. The SNR and P_{ce} are given by;

$$SNR(dB) = 10\log\left(\frac{P_x}{P_v}\right) \tag{8}$$

$$P_{ce}(\%) = \left(\frac{E_C}{E_T}\right) * 100\% \tag{9}$$

where P_x and P_v is the signal power (gain inclusive) and noise power respectively, E_C is no. of PW or PRP correct estimates and E_T is no. of total estimates (in our case E_T =100). Fig. 4 shows the PW and PRP estimation performance for the LFM.



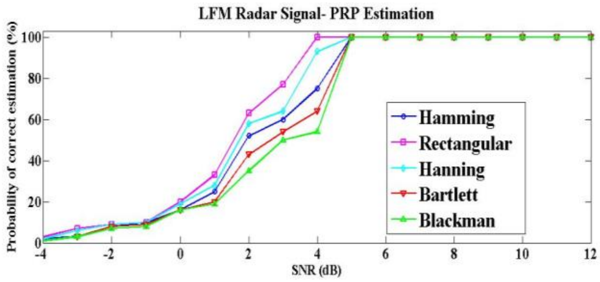


Figure 4. Effect of various window functions on inter-pulse analysis of LFM signal

It is seen that irrespective of the time parameter being estimated, the rectangular window performs best having a 100 percent correct estimation at 3dB, followed by Hanning, Hamming and Bartlett. Blackman has the worst performance, requiring an additional SNR of 2dB to achieve perfect estimation. This is the same case with StP and SiP signals with the only difference being the SNR required to obtain 100 percent correct estimation as shown in Fig. 5 and Fig. 6.

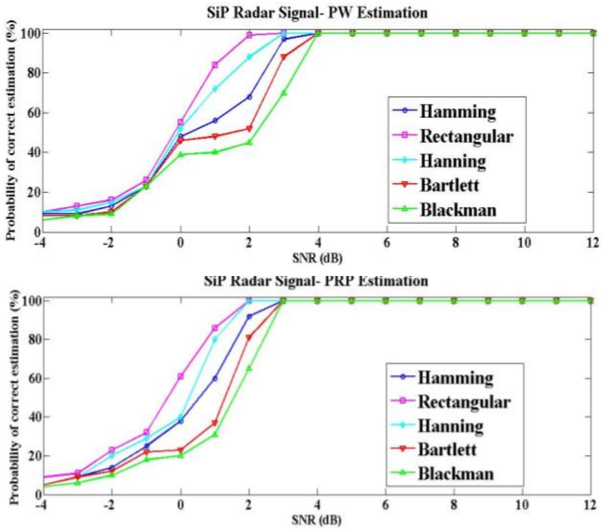


Figure 5. Effect of various window functions on inter-pulse analysis on SiP signal

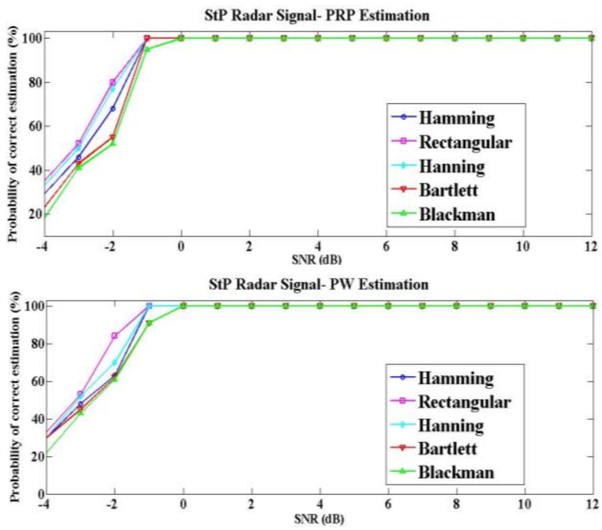


Figure 6. Effect of various window functions on inter-pulse analysis on StP signal

It has been established in [17] that as the main lobe width becomes narrower, the smoothing provided by the window is reduced. This is the reason why rectangular having the widest main lobe width performs best and Bartlett having the narrowest main lobe width is the poorest when smoothing is considered.

The StP signal performs best (100% P_{ce} at SNR=-1dB) as shown in Fig. 6 due to its higher number of samples for better processing followed by its counterpart signal of the same modulation, the SiP signal (100% P_{ce} at SNR=4dB) as shown in Fig 5. The LFM performs poorest (100% P_{ce} at SNR=5dB) due to its more sophisticated modulation and hence complexity compared to others.

VI. Conclusion

The paper has examined the effect of window functions on smoothing instantaneous energy to analyse and estimate time parameters of airborne radar signals. Results obtained shows 100 percent probability of correct estimation for all the test signals considered at SNR of 5dB. It was confirmed that smoothing is directly proportional to the main lobe width at constant window size. Therefore the windows functions based on this criteria in descending order of performance are rectangular, Hanning, Hamming, Bartlett and Blackman. Hence rectangular window is recommended for smoothing instantaneous energy when inter-pulse analysis is carried out. Modulation and number of samples plays important role in estimation, accounting for the different performance of estimation obtained from the test signals considered. Thus, estimated parameters can be used as input to a classifier network.

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