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Semi-Markov Modeling in Discrete Time and State for the Study of Desertification in Nigeria

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ABSTRACT

The area of land lost to desert encroachment in a year in Nigeria has been considered as a stochastic process. A semi-Markov and gap size model has been utilized to study the process. The model defines a discrete-time, discrete-state process in which successive state occupancies are governed by the transition probabilities of the Markov process. The model was tested by examining the historical data for desertification in Nigeria. The result indicates a slow and continuous loss of Nigeria cropland of about.1.4 % to desert in 200years. The result could be useful to the government and Non-governmental organizations for the control of desert in Nigeria.

Keywords: *encroachment, stochastic, discrete-time, discrete-state, probabilities, transition*

INTRODUCTION

Vegetation means the plant cover of the earth which includes trees and grasses of different kinds. Vegetation of an area is about considering the existence of plants cover in that area. There is hardly any vegetation that has not been affected by human activities in the country. Farming, logging, grazing, hunting, urbanization, road construction and other development activities by the rapidly

expanding population have together reduced the nation's natural plant cover to isolated remnants.

Following Iloege (1981), there are two broad belts of plant group that can be found in Nigeria (forest and savannah) and within each group it is possible to distinguish three sub-types ; salt-water swamp, fresh water swamp, high forest . The savannah comprises of guinea savannah, Sudan savannah and Sahel savannah. The transition between the last two grass regions is the subject of this paper. This transition is as a result of Sahara Desert moving southwards at a rate of 0.6 kilometers yearly and that is referred to as Desertification. Desertification according to the experts is the rapid depletion of plant life and loss of top soil at desert boundaries and in semi-arid regions usually caused by a combination of drought and overexploitation of grasses and other vegetation by people.

According to experts, desertified areas around the globe expand by about 50,000 to 70,000 square kilometers every year, and such areas now total 38 million square kilometers or one fourth of the earth's total land area.

The area in desert is expanding, largely at the expense of grassland and crop Acevedo, M.F., La Point T.W. and Elrod D.A. (1995).

Drought and desertification have been occurring persistently in the arid and semi-arid zone of northern Nigeria with devastating social and economic impact for decades. The primary causes of desertification in Nigeria have been identified as overgrazing, overexploitation , deforestation and poor irrigation practices leading to negative impacts like; resource use conflicts, problem of food security etc.
An official federal government assessment placed the impact of desert encroachment as causing the visible sign of the shift in vegetation from grass and bushes and in the final stages, expansive desert like sand. This same danger assessment estimated that between 50% and 75% of Bauchi, Borno, Gombe, Kastina, Kano, Jigawa, Kebbi, Sokoto, Yobe, Adawama, and Zamfara States in Nigeria are being affected by desertification (Copyright 2003)
Desertification and the associated persistent drought constitute most serious environmental problem facing the northern part of Nigeria. Desertification as an environmental hazard is posing a lot of problems to many nations in the world and as a result of that many people have studies desertification so as to device a solution to desertification problems. For instance, The Federal ministry of Environment of Nigeria awarded a contract for collation and characterization of hydro meteorological data. As advocated by the United nation (U N) convention to combat Desertification in Nigeria signed in 1997. The ministry finalized a National Action Programme (N A P) to combat desertification in collaboration to relevant international organizations and local communities, sector and donor agencies. This is in line with articles 10 of the Nations convention to combat desertification. (Federal Ministry of Environment, .National Action Programme to combat Desertification (2005)). Moritmore (1989) recorded the impact of desertification and drought in Dambatta district of Kano. He observed that drought affected both agriculture and human. The desertification is viewed as an adverse environmental process and degradation as a persistent decrease in the capacity of an ecosystem to deliver ecosystem services.

The natural causes of desertification include physical condition of soil, vegetation, topography as well as the inherent extreme climatic variability as evidenced in periodic drought and other causes of desertification include overgrazing, over exploitation, deforestation and poor irrigation practices Nasiru (2010). The issue of desertification and its associated dangers, especially as it affects the northern part of Nigeria, is being discussed without concrete evidence at tackling the crisis. Despite the fact that Nigeria is the country with the largest population in Africa, yet its people are badly affected by drought and a fast rising population, especially in the arid northern part of the country. The sandy soil is usually low in organic matter, nitrogen and phosphorous and may degrade rapidly under conditions of intensive rainfall when over-used occurs in this general sandy environment, denuded patches may appear when the wind blown sand becomes mobile Moritmore (1989). However, the deserts advance can only be stopped if Nigeria's authorities start taking the threat more seriously.

AIM AND OBJECTIVE OF THIS PAPER

The aim of this paper is to x-ray the semi-Markov model as well as apply a semi-Markov model to study desert encroachment in Nigeria, estimate interval transition probabilities of desertification use a semi-Markov model to estimate the future occurrence of desert encroachment in Nigeria. However, scope of this study is limited to the study of the transition of Sahel savannah to the Sudan savannah using semi-Markov model.

MATERIALS AND METHODS

Semi-Markov Model

A semi-Markov model to desert encroachment in the Northern part of Nigeria with emphasis on the land measured in square kilometres lost to desert in a year. This is where the transition from one state of the land status to another may not necessarily occur at discrete time instants. We therefore look at a situation where the time between transitions may be in several units of time interval, and where the transition time can depend on the transition being made. This leads us to a generalization of a Markov process called the semi-Markov process Howard (1971). In other words, we shall consider the desert encroachment as a semi-Markov process running in discrete time and State.

Since desert encroachment is considered in this research work as a form of Markov process, it is worth defining what this process entails. The basic concept of a Markov process is that of "state" of a system and state "transition". It is a process that runs in time. At any given point in time, the process is in a given state and could possibly make a transition to another state after a period of time. A Markov process in discrete time and discrete state is called a Markov chain.

We may give a mathematical definition of a Markov chain as a sequence X_0, X_1, - - - - of discrete random variables with the property that the conditional probability distribution of X_{n+1} given X_0, X_1, - - - - X_n depend only on the value of X_n but not further on X_0, X_1, - - - - X_{n-1}. That is for any set of values, $h, i,$ - - - j in the discrete state space,

$$P(X_{n+1} = j / X_0 = h, \text{- - - - -}, X_n = i) = P(X_{n+1} = j / X_n = i) = P_{ij} \quad i, j = 1, 2, 3, 4,5,6$$

The matrix P whose entries are the P_{ij} 's is called the transition probability matrix for the process. The above chain is a first order Markov chain. In this process, the probability of making transition to a future state does not depend on the previous state but only depend on the present state. In other words, the probability of making a transition to a future state does not depend on the past history. The matrix P and the initial state transition probabilities

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completely specify the process. If the transition probabilities depend on time, then the Markov chain is non-homogeneous, otherwise, it is homogeneous. In this project we shall only consider the Markov chain that does not depend on time. Thus we have stationary transition probabilities.

The Markov process discussed above has the property that state changes can only occur at the appropriate time instants. However, given the nature of the process (Desertification), transition may not actually occur at these time instants. We therefore consider a 'situation' where the time between transition may be several of units of time and where the transition time can depend on the transition that is being made. This leads to a general form of Markov process called a semi-Markov process Howard (1971).

In precise terms, let P_{ij} be the probability that the vegetation cover that is in state 'i' on its last transition will enter state 'j' on its next transition, i, j =1, 2, 3, 4,5,6. The transition probabilities must satisfy the following

$$P_{ij} \geq 0, \; i, j = 1, 2, 3, 4, 5, 6.$$

$$\text{and} \quad \sum_{j=1}^{6} P_{ij} = 1, \quad i = 1, \; 2, \; 3, \; 4, 5, 6 \qquad (1)$$

Whenever the desertification enters state 'i' it remains there for a time T_{ij} in state i before making a transition to state 'j'. T_{ij} is called the 'holding time' in state i. The holding times are positive integer valued random variables each governed by a probability distribution function f_{ij} (m) called the holding time distribution function for a transition from state i to state j.

Thus $P(T_{ij} = \text{m}) = f_{ij}$ (m). m = 1, 2, 3, . . .

$$i, j = 1, 2, 3, 4, 5, 6.$$

We assume that the means μ of all holding time distribution are finite and that all holding times are at least one year in length. That is,

$$f_{ij}(0) = 0$$

To completely describe the semi-Markov process we must specify holding time distribution functions in addition to the transition probabilities. For a fixed value of i T_{ij} is the same for each value of j, (i, j = 1, 2, 3, 4, 5, 6).

Let F_{ij} (n) be the cumulative probability distribution of T_{ij}.

$$F_{ij}(\text{n}) = P(T_{ij} \leq \text{n}) = \sum_{m=\phi}^{n} f_{ij}(m) \qquad (2)$$

and F_{ij} (n) be the complementary cumulative probability distribution of T_{ij}.

$$F_{ij}(\text{n}) = 1 - F_{ij}(\text{n}) = P(T_{ij} > \text{n}) = \sum_{m=n+1}^{\infty} f_{ij}(m)$$

Suppose the desert enters state i. Let Y_i be the time it spent in state i before moving out of the state i. Then Y_i is called the waiting time in state i.

We let w_i (m) be the probability distribution function of Y_i. Then

$$w_i(\text{m}) = P(Y_i = \text{m}) = \sum_{j=1}^{n} P_{ij} f_{ij}(m) \qquad (3)$$

The cumulative probability distribution $\bar{W}_i$(n) and the complimentary cumulative probability distribution $\bar{W}_i$ (n) for the waiting times are given as follows

$$w_i(m) = P(Y_i \leq n)$$

$$= \sum_{m=1}^{n} W_i(m) \quad (4)$$

$$= \sum_{m=1}^{n} \sum_{j=1}^{6} P_{ij}\, f_{ij}(m)$$

$$= \sum_{j=1}^{6} P_{ij}\, f_{ij}(n)$$

and

$$w_i(n) = P(Y_i > n)$$

$$= 1 - w_i(ns)$$

$$= \sum_{m=n+1}^{\infty} w_i(m)$$

$$\sum_{m=n+1}^{\infty} \sum_{j=1}^{6} P_{ij}\, f_{ij}(m)$$

$$= \sum_{j=1}^{6} P_{ij}\, F_{ij}(m) \quad (5)$$

Development of the Model and Model Assumptions

Desert encroachment is considered as a process that runs in time. Some suitable states of the desert encroachment are specified and the description of the manner in which the process moves from one state to another is given. The states of the desert encroachment are finite. It should be readily observed that there is no unique set of states and the progress through the states can be described in a variety of ways. The choice of states should therefore be governed by the intended use of the model and the availability of data.

The basic assumption in developing the model is that the transition from one state to a different state should not occur at time t = 0 (year 0) and that the basic unit of time is one.

The cumulative distribution function is given by $e^{-ti\theta}$ i.e

$$f(t') = 1 - exp^{-\left(\frac{t'}{\alpha}\right)\beta} \quad (6)$$

The land mass of Sudan Savannah lost to Sahel Savannah in a year is been considered as a random variable.

The holding time in state 1 before a transition is made into state 2 has been described by Weibull distribution function given below:

Following Dubey (1967), the probability density function of a random variable T having the three parameter Weibull distributions is given by

$$f(t) = \left\{\frac{\beta}{\alpha}\left(\frac{t-c}{\alpha}\right) exp^{-\left(\frac{t-c}{\alpha}\right)\beta} \quad : \alpha, \beta, c > 0\right.$$

And the percentile parameters α and β of the Weibull are estimated as

$$\beta = \frac{log - (\log(1-p1)) - \log(-\log(1-p2))}{logtp\ 1 - logtp\ 2}$$

$$\therefore \quad \beta* = \frac{log - (\log(1-p1)) - \log(-\log(1-p2))}{logyp\ 1 - logyp\ 2}$$

$$\alpha* = \exp/w \log y_1 + (1 - w) \log y_2/$$

$$\text{Where } W = 1 - \frac{Log\ k1}{k}$$

$$K_1 = -\log(1 - p_1)$$

Where $\beta*$ is the percentile estimator of β based on two ordered sample observations from a Weibull population.

RESULTS AND DISCUSSION

The results of the computation gotten from the percentile point estimates of the Weibull distribution with parameters α and β with their corresponding times which is the assumed model for the research work are presented below.

Table I

P_1=0.004	**t_1=10**	**α=39.70792745**
P_2=0.996	**t_2=60**	**β=4.03407951**

When substituting α and β data in table I into equations 5 and 6 which is the required model for the interval transition probabilities from state 1 to state 2.

However, following the classification of the vegetation of Nigeria by Iloege (1981) gap size model proposed by Acevedo, M.F., La Point T.W. and Elrod D.A. (1995) and the data obtained from the Federal Ministry of Environment of Nigeria. National Action programme to combat Desertification, the surface area of Nigeria is approximately 91.07 million hectares. We inferred from this research work that the country is presently losing about 351,000 hectares of landmass to desert condition every year, which is advancing southwards.

Table 2: The interval transition probabilities from state 1 to state 2

t	$\emptyset_{12(t)}$
1	0.001974581
5	0.003274626
7	0.003663456
18	0.005098882
30	0.00616781
50	0.007533841
150	0.012074232
170	0.012795341
192	0.013584282

Table 2 shows the interval transition probabilities and we discovered that desert is encroaching gradually year after year and that the probabilities converge slowly. Looking at the results of the interval transition probabilities, it is observed that there is increment of approximately 0.0002 in the first 5years.In the next 18years, there is increment of 0.00 1. Converting the results from the interval transition probabilities into percentage, the results showed that desertification increased by about 0.02% in the first year, about 0.4% in the 7th year, about 0.6% in the 30th year, 0.8 % in the 50th year, about 1.2% in the 150th year, about 1.3% in the 170th year, about 1.4% in the 192nd year and about 1.4% for the two hundredth year period. Representing the results in square kilometres, we have 182 sq km in the first year, 3,643 sq km in the 7th year, about 7,286 sq km in 50th year, 10,928 sq km in the 150th year, 11,891 sq km in the 170th year and about 12,750 sq km in the 192nd year. These represent the Sudan savannah land that may be lost to Sahel savannah in the corresponding time period. Looking at the scores, we observed that the existing value of 0.6km per year is got in the 30th year of our prediction. Comparing the results with that of Abubakar (2010), Abubakar used semi-Markov model to study desert in Nigeria in discrete state and continuous time. In his work, we discovered that continuous time cannot be easily read, while we considered discrete state and discrete time.

In Abubakar (2010) result, 92 sq km and 1,847 sq km of the northern parts of Nigeria cropland was lost to desert in the 7 year and 192 year respectively while in our work, 3,643 sq km and 12,750 sq km of Sudan savannah was lost to desert in 7 year and 192 year respectively. These scores are lower than our scores. The size of his scores could be as a result of the difficulty in using continuous time in calculating the interval transition probabilities and so we discovered that using discrete time is better and easier than continuous time. From the transition probabilities, we discovered that the probabilities converge slowly just as in our graph. It is discovered that the desert encroachment takes a longer period for its impact to be seen in any area. The desert encroachment, if not controlled could be deadly. Desert encroachment can threaten the livelihood of communities and the survival of a nation, if not controlled. This environmental menace can devastate lands and destroy many homes. Desertification can lead to conflicts amongst communities competing for farmlands. These conflicts can sometimes lead to clashes and eventual loss of lives and properties. The problem also can lead to migration from the rural areas to the urban centre. The result therefore confirms the rate of encroachment cited in the problem for illustration.

CONCLUSION

Desertification and land degradation are the major causes of poverty, hunger, social ills, and loss of bio-diversity as well as natural resources in the affected regions in Nigeria. Nasiru,(2010). This study discusses the semi-Markov in discrete time and discrete state and the rate at which Sahel savannah (desert)encroaches into the Sudan savannah. It was observed that there is a gradual and persistent lost of Sudan savannah to Sahel and desert like vegetation. The model could be used to predict the encroachment of desert in Nigeria and the prediction could be used to manage human and material resources for the control of desert in Nigeria.

ACKNOWLEDGEMENT AND REFERENCES

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