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Prey – Predator Model on the Interaction between the Drawdown Level of an Aquifer and Maize Yield

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ABSTRACT: Groundwater is a major source of water for irrigation purposes and for sustainable growth of Agricultural development. In this paper we formulated a mathematical model to analyse the interaction between the Draw-down levels in an unconfined aquifer with maize yield, using the parameters; aquifer recharged rate α, rate of interaction between the draw down level of the aquifer and the maize yield β, draw down level of the aquifer h, and the maize yield y. The aim of this paper is to analyse the interaction between crop yield and water table and to determine the effect of draw down level on maize yield. It was observed that the maize yield depends on the recharge rate of the aquifer α and the water table level h and also as the drawdown level increases, the maize yield increases. Agriculture is of paramount importance to the development of any country, it was established in this paper that a relatively small increase of water table depth beyond the optimum increase the surface irrigation requirement for maximum crop production, water table depth shallower than optimum decreases yield. ©JASEM

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Key words: Drawdown, recharged rate, maize yield, interaction

It is observed that in many developing countries, groundwater is a major source of water for domestic needs and irrigation purposes (Reichman, 2014).

The steadily growing population in Nigeria in the last four decades has put tremendous pressure on the available water resources. When the rate of groundwater abstraction exceeds the rate of natural replenishment, this results to the falling water tables (Olasehinde, 2010). According to Olasehinde (2010), groundwater exists in three major rocks in Nigeria- Igneous, sedimentary and metamorphic rocks.
Fresh water availability is considered to be a basic prerequisite for development of human activities. Groundwater offers the most abundant source of water to man (Thangarajan, 2013). Because groundwater is not visible on the surface and probably as a result of misinformation, many people undervalue the importance of groundwater in sustaining water supply needs (UN Water Report, 2013) as contained in Raghunath (2014).

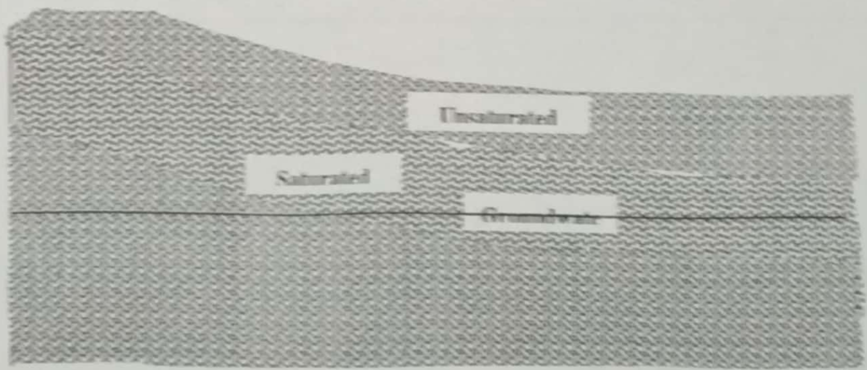

Fig 1: Unconfined Aquifer (Jirka, 2014).

Maurya (2015), worked on both low and high water table fields in Kadawa , Nigeria , In His study, He concluded that on high water table [50-80cm] site, the best yield was achieved at 15 days' irrigation interval during dry season for wheat crop. Reichman (2014), determine that relatively small increases of water table depth beyond the optimum increased the surface irrigation requirement for maximum crop

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production, and water table depth shallower than optimum decreases yields

MATERIALS AND METHODS

Water Table Aquifer is as shown in figure 2

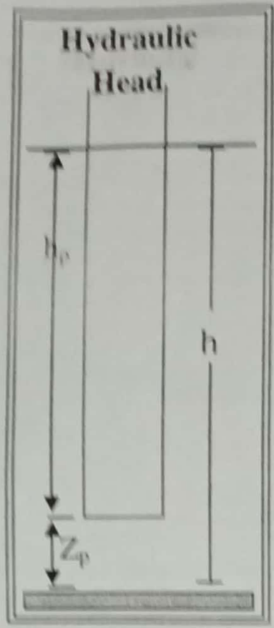

Fig 2: Water Table Aquifer (Jirka, 2014).

Legend: h_p = Pressure Head, Z_p = Elevation Head, h = Hydraulic Head

Table 1 shows the data used in this work (the initial value taken at draw-dawn level of 30m).

Table 1: Draw-down level and crop yield

Draw-down level(m)	Yield (kg / m^2)
5	0
10	0
15	0
20	0.0001
25	0.0005
30	0.1217
35	0.4127
40	0.6587
45	0.8597

Source: Mudiare (2014)

The data used (the initial value) in this work are presented in Table 1. The data consist of nine values of draw-down level (DDL) and crop yield (CY).

Prey – Predator Model: Let $h(t)$, $y(t)$ represent the prey and predator population at time t, respectively. Therefore,

$$\frac{dh}{dt} = \alpha h \qquad (1)$$

(in the absence of predator)
and,

$$\frac{dy}{dt} = -\theta y \qquad (2)$$
(in the absence of prey)

The rate of change of the draw down level of the aquifer with time rises by the rate of the recharge of the aquifer and drops by the rate of interaction between the draw down level of the aquifer and the maize yield.

$$\frac{dh}{dt} = \alpha h - \beta hy \qquad (1)$$

$h(0) = 30m = h_0$, data extracted from Egbarevba and Mudiare (2014)

where,

h	=	draw down level of the aquifer.
y	=	maize yield
α	=	recharge rate of aquifer
β	=	interaction parameter of the maize yield and draw down level of the aquifer.

The rate of change (interaction) of maize yield and the draw down level of the aquifer is given as,

$$\frac{dy}{dt} = -\theta y + \beta hy \qquad (2)$$

$y(0) = 0.1217 kg/m^2 = y_0$ Data extracted from Mudiare (2014)

Evaluating (1) and (2) asymptotically,
Such that,

$$h(t) = h_0(t) + \beta h_1(t) + \ldots \qquad (3)$$

$$y(t) = y_0(t) + \beta y_1(t) + \ldots \qquad (4)$$

$$0 < \beta < 1$$

at $t = 0$

$$h(0) = h_0(0) + \beta h_1(0) = h_0 + \beta * 0 \qquad (5)$$

$$y(0) = y_0(0) + \beta y_1(0) = y_0 + \beta * 0 \qquad (6)$$

That is,

$$h_0(0) = h_0,\ h_1(0) = 0 \qquad (7)$$

$$y_0(0) = y_0,\ y_1(0) = 0 \qquad (8)$$

Substituting equation (3) and (4) into (1) and (2) respectively, gives.

$$\frac{d}{dt}(h_0 + \beta h_1 + \ldots) = \alpha(h_0 + \beta h_1 + \ldots) - \beta(h_0 + \beta h_1 + \ldots)(y_0 + \beta y_1 + \ldots) \qquad (9)$$

$$\frac{d}{dt}(y_0 + \beta y_1 + \ldots) = -\theta(y_0 + \beta y_1 + \ldots) - \beta(h_0 + \beta h_1 + \ldots)(y_0 + \beta y_1 + \ldots) \qquad (10)$$

$$\frac{dh_0}{dt} = \alpha h_0,\ h_0(0) = h_0 \qquad (11)$$

$$\frac{dy_0}{dt} = -\theta y_0,\ y_0(0) = y_0 \qquad (12)$$

$$\frac{dh_1}{dt} = \alpha h_1 - h_0 y_0,\quad h_1(0) = 0 \qquad (13)$$

$$\frac{dy_1}{dt} = -\theta y_1 + h_0 y_0,\quad y_1(0) = 0 \qquad (14)$$

Therefore,

$$h_0(t) = h_0 e^{\alpha t} \quad (15)$$

But $h_0 = 30m$

$$h_0(t) = 30e^{\alpha t} \quad (16)$$

from (12) we have,

$$\frac{dy_0}{dt} = -\theta y_0, \quad (17)$$

$$\frac{dy_0}{y_0} = -\theta dt, \quad (18)$$

Therefore,

$$y_0(t) = y_0 e^{-\theta t} \quad (19)$$

But $y_0 = 0.1217$

$$y_0(t) = 0.1217e^{-\theta t} \quad (20)$$

From equation (13) and (14),

$$\frac{dh_1}{dt} = \alpha h_1 - h_0 y_0, \quad (21)$$

$$\frac{dh_1}{dt} - \alpha h_1 = -h_0 y_0 \quad (22)$$

$$\frac{dh_1}{dt} - \alpha h_1 = -h_0 e^{\alpha t} \cdot y_0 e^{-\theta t} \quad (23)$$

$$\frac{dh_1}{dt} - \alpha h_1 = -h_0 y_0 e^{(\alpha-\theta)t} \quad (24)$$

$$h_1(t) = \frac{h_0 y_0}{\theta}\left(e^{(\alpha-\theta)t} + e^{\alpha t}\right) \quad (25)$$

From equation (14) where,

$$\frac{dy_1}{dt} = -\theta y_1 + h_0 y_0, \quad (26)$$

and

$$\frac{dy_1}{dt} + \theta y_1 = h_0 y_0 \quad (27)$$

$$\frac{dy_1}{dt} + \theta y_1 = h_0 e^{\alpha t} \cdot y_0 e^{-\theta t} \quad (28)$$

$$\frac{dy_1}{dt} + \theta y_1 = h_0 y_0 e^{(\alpha-\theta)t} \quad (29)$$

such that,

$$y_1(t) = \frac{h_0 y_0}{\alpha} e^{(\alpha+\theta)t} + K_2 e^{\theta t} \quad (30)$$

but, $K_2 = \dfrac{h_0 y_0}{\alpha}$

$$y_1(t) = \frac{h_0 y_0}{\alpha}\left(e^{(\alpha+\theta)t} + e^{\theta t}\right) \quad (31)$$

Results and Discussion

Figure 3 shows the variation of water table level with time at different interaction rates β, α and θ using equation (25)

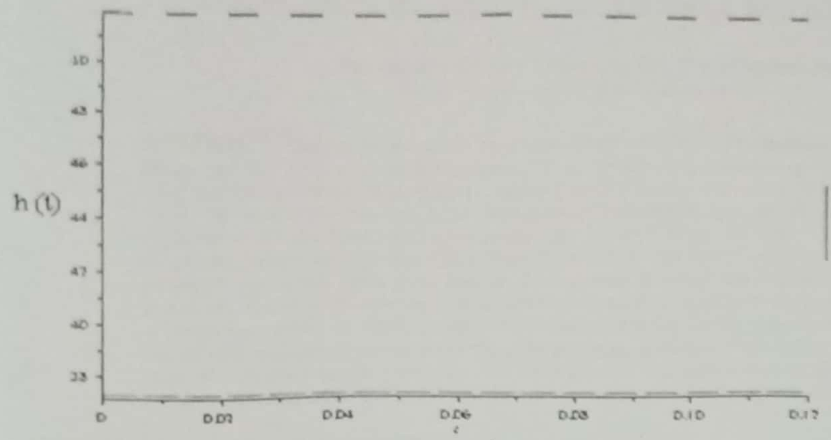

Fig 3: Variation of water table level $h(t)$ with time for different values of β, α and θ

Figure 4 shows the variation of maize yield (*y*) with time for different values of β using equation (31).

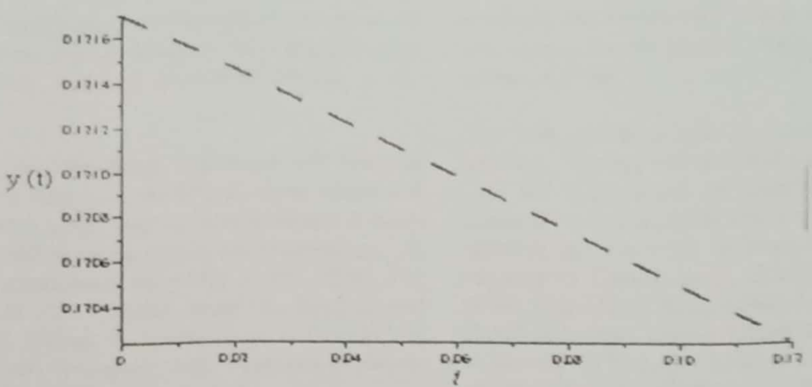

Fig 4: variation of maize yield (*y*) with time for different values of β

Conclusion: The prey-predator model has been used to mathematically model the interaction between the drawdown level and the maize yield. From figures 3 and 4 it was observed that increase in the rate of recharge of the aquifer increases the water table level above the initial level, which may lead to washing off of the top soil and nutrient, thereby decreasing the maize yield. It is therefore concluded that maize yield depends on the recharge rate of the aquifer α and the level of the water table.

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