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AN APPLICATION OF A DYNAMIC MODEL OF MALARIA SPREAD

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ABSTRACT

The use of Mathematics to solve real-world problems has become widespread. This paper describes the modeling of spread of malaria disease. Data from Pull and Grab (1974) was used to verify the model and the goodness-of-fit was used to validate the model and the observed data. The χ^2 test statistic shows that as time increases ($t \to \infty$), the proportion of affected human population by malaria at a time [$X(t)$] remains constant at significant levels of 10 % and 5 % ($\alpha = 0.1$ and $\alpha = 0.05$).

1. Introduction

Malaria is a tropical infection disease which menaces more people in the world than any other disease (Ross, 1998). WHO (1978) reported that, in Africa alone, one million children die annually of malaria before age of 5 years.

Various schemes have been advocated for control of malaria but success has been limited. For the control strategies to be more efficient, improved epidemiological knowledge is required, including a better understanding of the population dynamics leading to the spread of malaria (Ouchili, 1997).

2. Model Formulation

Let $X(t)$ represent the proportion of human population affected by malaria at time t. The proportion unaffected is $1 - X(t)$. Changes to $X(t)$ over time occur due to unaffected people and affected people recovering. Also, let r represent the recovery rate, that is, the proportion of affected people who recover in a unit time and h represent the proportion of unaffected people receiving infection bites per unit time t.

Change to $X(t)$ in a small interval, Δt, is given in equation (1) as,

$$X(t + \Delta t) - X(t) = \{h(1 - X(t)) - rX(t)\}\Delta t \tag{1}$$

As $\Delta t \to 0$;

$$\frac{dX(t)}{d(t)} = h - (r + h)X(t) \tag{2}$$

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In line with Dietz (1972), equation (2) is solved given the initial value of $X(0) = X_0$ and that $0 \le X(t) \le 1$.

The complimentary part of the solution is;

$$X_c(t) = Ae^{-(r+h)t} \tag{3}$$

And the integral part is;

$$X_p(t) = \frac{h}{r+h} \tag{4}$$

Therefore, the general solution is;

$$X(t) = Ae^{-(r+h)t} + \frac{h}{r+h} \tag{5}$$

Using $X(0) = X_0$ on equation (5), we have;

$$X(t) = -\left[\frac{h}{r+h} - X_0\right]e^{-(r+h)t} + \frac{h}{r+h} \tag{6}$$

Equation (6) gives the model for the spread of malaria disease over a period of time t

3. Parameter Estimation

There are four input parameters in the model given in equation (6). These are time (t), proportion affected by malaria at time $t = 0$ (X_0), proportion unaffected by malaria (h) and recovery rate (r). If we concentrate on newborn children in the model given in equation (6), that is, $X(0) = X_0 = 0$, then we have:

$$X(t) = \frac{h}{r+h}\left[1 - e^{-(r+h)t}\right] \tag{7}$$

In equation (7), $X(t)$ represent the proportion of newborn population affected by malaria at time (t). Let $P(t) = X(t)$ and an infected person detected with probability $k \ \forall \ 0 \le k \le 1$. Then;

$$P(t) = \frac{kh}{r+h}\left[1 - e^{-(r+h)t}\right] \tag{8}$$

Equation (8) was obtained by replacing h with kh in equation (6) as supported by Ross (1998) and it plays an important role in parameter estimation with k being treated as an additional parameter. By observing the spread of malaria in a cohort of newborn children, we use equation (8) to obtain estimates for the parameters of the model.

4. Application

The data used in this paper is extracted from Pull and Grab (1974). The values of $t_i \ \forall \ 1 \le i \le n$ used represent the age interval in months and the data captures the observed proportion of infection over a period of 24 months (2 years) and the sampling was done in line with Okafor (2002). The estimates obtained are presented in table 1:

Table 1 : The Observed And Modeled Proportion Of Population Affected

Time [t]	Proportion of Population Affected [$X(t)$]	
	$X_{observed}(t)$	$X_{modeled}(t)$
1	0.0003	0.0002
2	0.0089	0.0086
3	0.0442	0.0438
4	0.0968	0.0954
5	0.1186	0.1197
6	0.1491	0.1489
7	0.1860	0.1854
8	0.2242	0.2235
9	0.2580	0.2575
10	0.2993	0.2985
11	0.3520	0.3425
12	0.4142	0.4005
13	0.4834	0.4809
14	0.5686	0.5679
15	0.6742	0.6730
16	0.7351	0.7342
17	0.7464	0.7453
18	0.7599	0.7534
19	0.7685	0.7547
20	0.7689	0.7663
21	0.7692	0.7664
22	0.7692	0.7667
23	0.7692	0.7668
24	0.7692	0.7668

5. Validation Of The Model

One way of validating the model is through the goodness-of-fit between model and the observed data. This requires a statistical test (χ^2 test statistic). Here, we consider the dependence of proportion of population affected [$X_{modeled}(t)$] on time to remain constant.

We formulate the hypothesis as follows:

$H_0 : X_{modeled}(t)$ is independent on time t to remain constant

The alternative is guided by the assumption that the increase in the rate of infection is more pronounced in the beginning but as time t increases continuously, this rate remains constant. Therefore;

$H_1 : X_{modeled}(t)$ is dependent on time t to remain constant

Frank and Althoen (1994) suggested a measure of discrepancy between observed frequencies (O_{ij}) and expected frequencies (E_{ij}) as supplied by the chi-square (χ^2) statistic of the form;

$$\chi^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{(O_{ij} - E_{ij})^2}{E_{ij}} \tag{9}$$

and is approximately distributed as χ^2 with $(r-1)(c-1)$ degrees of freedom.

If $\chi^2 = 0$, observed and expected frequencies agree exactly, while $\chi^2 > 0$ implying that they do not agree and the larger the value of χ^2, the greater the discrepancy between them.

We calculated test statistic (χ^2) to be 36.0739, and in table 5 of appendix VIII of Frank and Althoen (1994), we found that the cumulative probability of 36.0739 falls between 0.950 and 0.975 in $\chi_{\alpha,23}^2$. The hypothesis of independence is therefore rejected for any $\alpha \geq 0.05$, which supports the argument that as time increases ($t \to \infty$), the proportion of affected human population by malaria at a time [$X(t)$] remains constant.

6. Discussion Of Results

From the results above, we observed that the proportion of affected individuals [$X(t)$] increases as time increases ($t \to \infty$). The increase is more pronounced in the beginning but as time (t) increases continuously, $X(t)$ becomes constant. Also, the chi-square testing revealed that there is an association between the model prediction and the observed data.

Conclusion And Recommendations

In conclusion, Mathematics plays a vital role in dynamics of malaria spread. To really comprehend how the disease spreads over a period of time, we need to model the system. Given the necessary parameters like time (t), proportion affected by malaria at time $t = 0$ (X_0), proportion unaffected by malaria (h) and recovery rate (r); the model arrived at shows the proportion of individuals affected by malaria at time (t).

The result obtained in this paper shows a qualitative modeling of the malaria disease and can be used to predict the health status of individual examined over a period of time. These are useful to doctors, hospitals administrators and policy makers. Hence, this work is recommended to professionals in medical field and all health bodies to guide them in preventing the spread of infectious disease.

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