ON THE APPLICATION OF SHORTEST PATH ALGORITHM IN GRAPH THEORY TO ROAD NETWORK ANALYSIS

L. A. Nafiu, A. M. Salihu and M. D. Shehu

Department of Mathematics and Computer Science
Federal University of Technology
Minna, Niger State, Nigeria
(e-mail: lanconserv@yahoo.com)

ABSTRACT

In this paper, graph theory is being applied to road network analysis of Federal University of Technology, Bosso Campus, Minna. The shortest path algorithm was described and ten data points (locations) were used. The result was generated using software developed in Visual Basic Programming Language for easy computation. It was found that the shortest and most economical route from the main gate to the exit gate is to pass through Geography Laboratory, Professor's Block and Science Complex. Hence, this route is recommended for any visitor moving from the main gate to the exit gate

Introduction

The intention of this paper is to find the shortest route from one place (main gate) to another (exit gate) in the structure of Federal University of Technology, Bosso Campus, Minna and verify a foolproof method for finding way out when stuck in the middle of the structure described.

In finding shortest route from one place to another, the utilization of shortest path algorithm in graph theory to road network analysis has often been used and always proved useful (Olayiwola, 2008).

Graph and network theory can be regarded as belonging to finite geometry. Geometric intuition is very essential both for predicting and obtaining results. Bondy and Murty (1977) opined that the basic idea of graph theory was introduced by Euler (1736) with many applications and has proven to be an extremely useful tool in analyzing various practical problems. For example, graph can be used to represent electrical or telecommunication networks, traffic systems, road path analysis, pipelines, archaeology, developmental psychology, classical studies, genetics, ecology, music and so on.

Shimon (1979) said that a graph may be undirected, meaning that there is no distinction between the two vertices associated with each edge, or its edges may be directed from one vertex to another. Graphs are represented graphically by drawing a dot for every vertex and drawing an arc between

Keywords: Graph, Path, Algorithm, Route And Network.

two vertices if they are connected by an edge. If the graph is directed, the direction is indicated by drawing an arrow. Each vertex is indicated by a point and each edge by a line joining the points which represent its ends. In Mathematics and Computer Science, graph theory is the study of graphs, mathematical structures used to model pair wise relations between objects from a certain collection. A graph in this context refers to a collection of vertices or nodes and a collection of edges that connect pairs of vertices (Bollobas, 1979).

Graph theory is the branch of Mathematics that deals with the arrangement of certain diverse objects and the relationships between these objects. It involves formulating the model of a problem in such a way that it can be approached by the techniques of graph theory. However, the way in which the modeling is carried out, and the degree to which the mathematical model accurately represents the original problem to be solved, varies considerably from problem to problem (Bronson, 1983).

A network is a set of points called nodes, and a set of curves, called branches (or arcs or links) that connect certain pairs of nodes. Only those networks considered are in which a given pair of nodes is joined by at most one branch. Nodes are denoted by uppercase letters and branches denoted by the nodes they connect. A branch is oriented if it has a direction associated with it. Schematically, directions are indicated by arrows. Two branches are connected if they have a common node. A path is a sequence of connected branches such that in the alternation of nodes and branches no node is repeated. A network is connected if for each pair of nodes in the network there exists at least one path joining the pair. If the path is unique for each pair of nodes, the connected network is called a tree (Gibbons, 1985).

Path Algorithm

If a traveler wishes to travel from place A to another place B in a shortest possible time, he may be interested in which route to take. In this case, it may not be difficult to find the solution by intelligent guesswork, but such an approach is less likely to succeed as the road network becomes more and more complicated. Columbic (1980) described an algorithm which can be used to find the shortest path between any two vertices of a given network.

Shortest Path Algorithm

Whitting and Hillier (1960) wrote on the idea of this algorithm to find the shortest path from one vertex to another in a given network. To do this, we move across the network from left to right, calculating the shortest distance from the first vertex to each of the intermediate vertices as we go. At each stage of the algorithm, we look at all vertices reached by an arc from the current vertex and assign to each such vertex a temporary label representing the shortest distance from the first vertex to that vertex by all paths considered until then. Eventually each vertex acquires a permanent label (called its potential, and denoted by a square around the label) which represents the shortest distance from the first vertex to that vertex. Once the second vertex has been assigned a potential, then we have determined the shortest distance from first vertex to the second vertex.

Chartrand and Oellemann (1993) presented the procedure to find the shortest path from one vertex to another in a road network as follows:

STEP 1: Assign to first vertex potential 0, label each vertex (say *V*) reached directly from first vertex with the distance from it to *V*. Choose the smallest of these labels and make it the potential of the corresponding vertex or vertices.

STEP 2: Consider the vertex or vertices just assigned a potential. For each such vertex *V*, look at each vertex (say *W*) reached directly from *V* and assign *W* the label.

(Potential of *V*) + (distance from *V* to *W*)

Unless *W* already has a smaller label, when all such vertices *W* have been labeled, choose the smallest label in the network which is not already a potential and make it a potential at each vertex where it occurs.

Repeat step2 with the new potential(s).

STOP : When the second vertex has been assigned a potential, this is the shortest distance from the first vertex to the second vertex.

SHORTEST PATH: Work backward from the last vertex to the first vertex and include an arc *VW* whenever

(Potential of *W*) – (potential of *V*) = (distance from *V* to *W*)

Illustration Of Shortest Path Algorithm Using Graphical Design Of Road Network Of Federal University Of Technology, Bosso Campus, Minna

We illustrate the use of this algorithm by finding the shortest distance from the main gate (MG) to the exit gate (EG) in the following road network of Federal University of Technology, Minna (Bosso Campus):

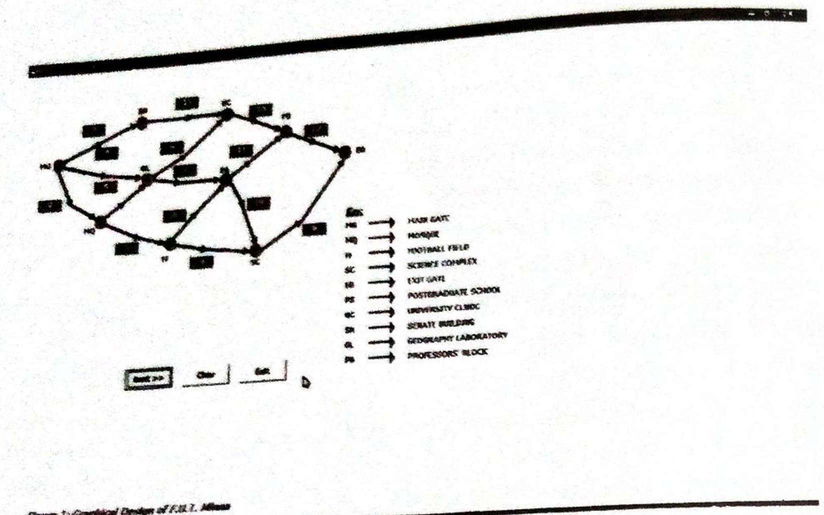

Figure 1 : Graphical Design Of F.U.T. Minna

We start by assigning to MG potential 0, since the shortest distance from MG to MG is 0 as given in figure 2.

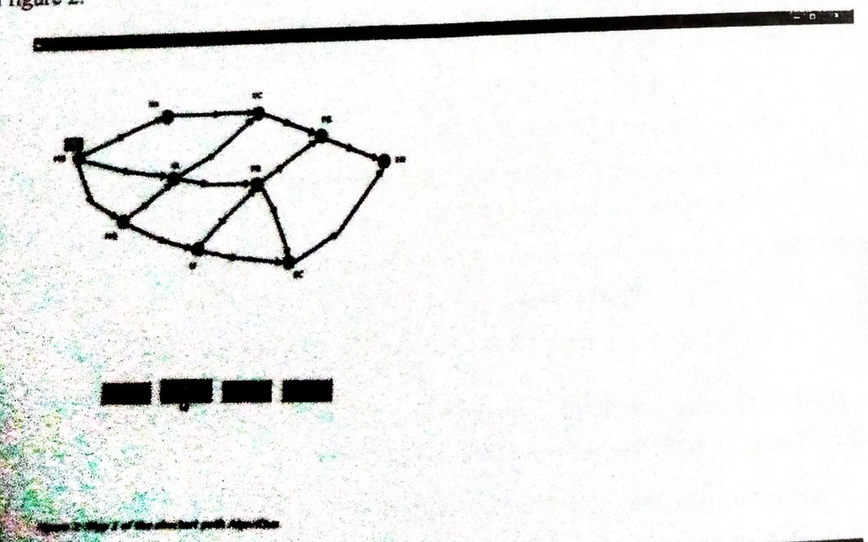

Figure 2 : Step 1 Of The Shortest Path Algorithm

We then look at those vertices reached by an arc from MG (that is SN, GL and MQ) and assign to each such vertex a temporary label equal to the potential at MG plus the distance from MG to that vertex. This gives the vertices SN, GL and MQ temporary labels of 9, 4 and 2 respectively as given in figure 3.

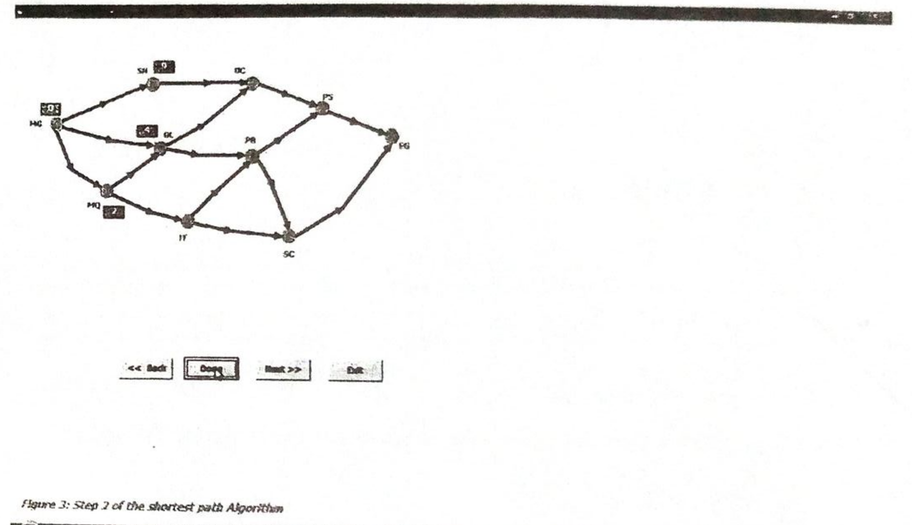

Figure 3 : Step 2 Of The Shortest Path Algorithm

We now take the smallest label that is not a potential and mark it as a potential. In this case, the relevant label is 2, at vertex MQ, so we assign to MQ a potential 2. Note that this is the shortest distance from MG to MQ, since any other path from MG exceeds 2.

Figures 4, 5, 6 and 7 give the other steps to follow in order to reach the shortest route in the structure.

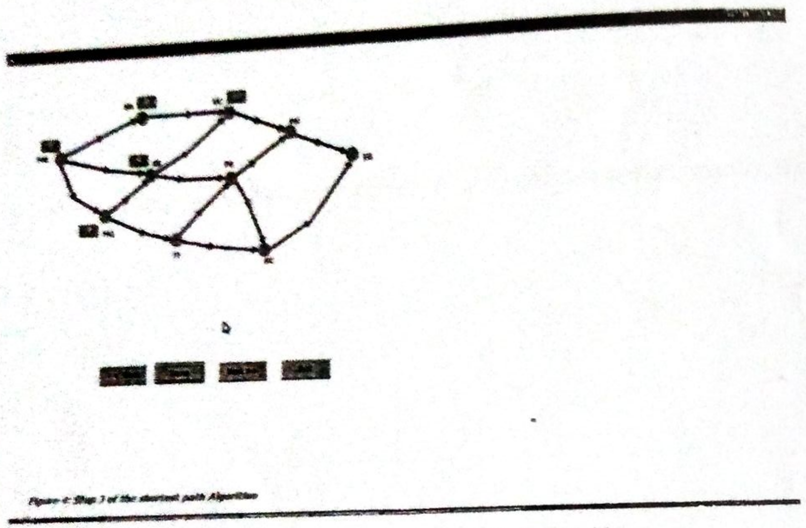

Figure 4 : Step 3 Of The Shortest Path Algorithm

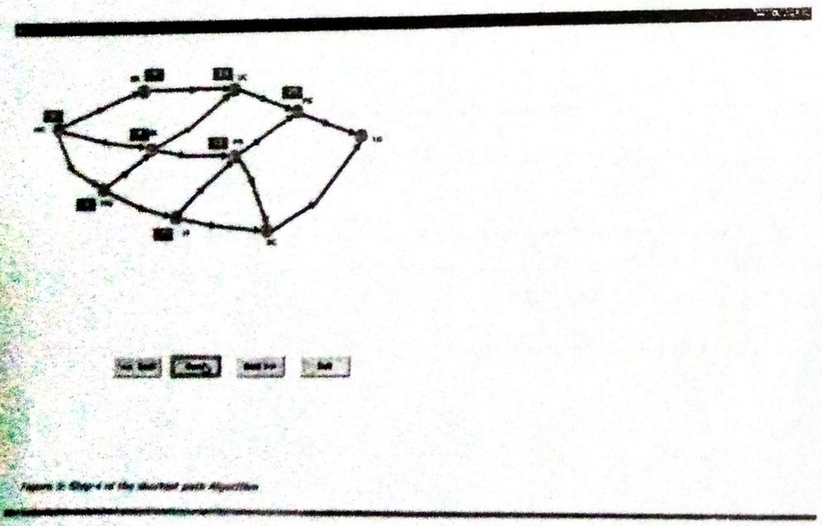

Figure 5 : Step 4 Of The Shortest Path Algorithm

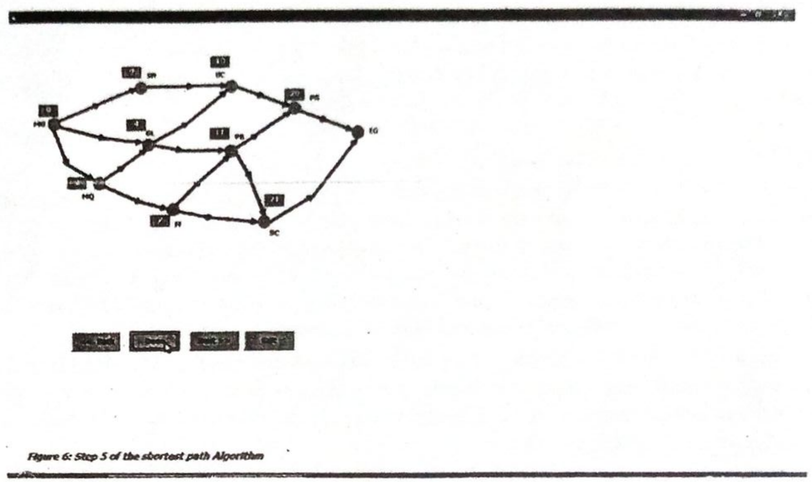

Figure 6 : Step 5 Of The Shortest Path Algorithm

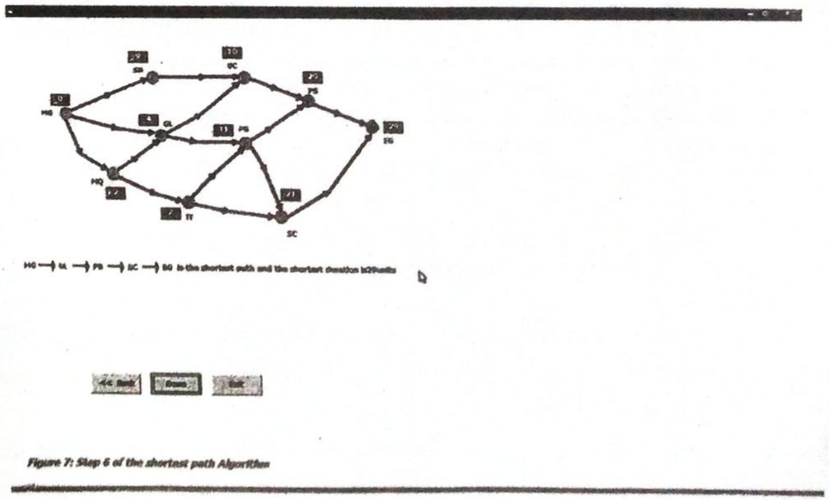

Figure 7 : Step 6 Of The Shortest Path Algorithm

Discussion

From the figures above, we observe that MG → GL → PB → SC → EG is the shortest path and the duration is 29units. Hence, the route to follow from the main gate to the exit gate is to pass through Geography Laboratory, Professors' Block and Science Complex.

Conclusion

In conclusion, from the above illustration, it shows that application of graph theory in road network analysis can be useful to study not only the road network of a particular area but also to find the shortest path or route between given starting and destination points. Road network analysis enables us to find and follow shortest route from one place to another, In this study, it is found that the shortest and most economical route to follow from the main gate to the exit gate is to pass through Geography Laboratory, Professors' Block and Science Complex.

This analysis makes it possible and simple for people within the environment of Federal University of Technology, Minna, Bosso campus to locate any building or place on campus. It also shows distance between any given two points and this makes it easy for visitors to locate any building on campus by studying the graphical analysis.

Recommendations

Road networks are essential components of map providing important contextual information, necessary for the interpretation of human presence and activities in the application of graph theory. Studying the network would actually guide the visitors' movement and identifying the shortest path provides the visitor or individuals the opportunity to use the most economical path or route in Bosso campus of Federal University of Technology, Minna. Hence, this work is recommended for anybody who wishes to know the road network of this campus with a view to knowing the main points of Bosso campus of Federal University of Technology, Minna and finding the shortest route between any two places or points. This will help to save the user's time.

REFERENCES

1. Bollobas, B. : Graph Theory: An Introductory Course. Heidelberg: Springer-Verlag, (1979).
2. Bondy, J. A. and Murty, U. S. : Graph Theory With Applications. New York: Elsevier Publishing Company, Inc. (1977).
3. Bronson, R. : Schaum's Outline Of Theory And Problems Of Operations Research. Singapore: McGraw-Hill, Inc. (1983).
4. Chartrand, G. and Oellemann, O. R. : Applied And Algorithmic Graph Theory. New York: McGraw-Hill, (1993).

5. Columbic M. C. : Algorithmic Graph Theory And Perfect Graphs. New York: Academic Press, (1980).
6. Gibbons, A. : Algorithmic Graph Theory. Cambridge: Cambridge University Press. (1985).
7. Shimon, E. : Graph Algorithms. Rockville, Maryland: Computer Science Press, (1979).
8. Olayiwola, M. E. : "An Application Of Graph Theory To Road Network Analysis". Unpublished Undergraduate Project, Federal University Of Technology, Minna, Nigeria. (2008).
9. Whitting P. D. and Hillier, J. A. : "A Method For Finding The Shortest Route Through A Road Network". Operation Res., Vol. 11: (1960), 37-40.