⁎ Hybrid Block Method for Direct Solution of General Fourth Order Ordinary Differential Equations using Power Series Function

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Abstract. This work focuses on a new block method of hybrid linear multistep method, derived by collocation and interpolation techniques for direct solution of fourth order initial value problems of ordinary differential equations using power series function as basis function. The basic properties of the block method including zero stability, error constants, consistency, order and convergence were analyzed. From the analysis, the block method derived was found to be zero stable, consistent, and convergent. Results from the new derived schemes compared with reviewed works considered in this paper show that the new method gives better accuracy in terms of error.

Keywords: Collocation, Consistency, Convergence, General FourthOrder Ordinary Differential Equations, Hybrid, Interpolation, Power Series Function, Zero Stability.

1 Introduction

The general fourth order initial value problems (IVPs) of ordinary differential equations (ODEs) of the form;

$$\left.\begin{aligned} &y^{iv}(x)=f\left(x,y,y',y'',y'''\right) \\ &y(x_0)=\phi_0, y'(x_0)=\phi_1, y''(x_0)=\phi_2, y'''(x_0)=\phi_3 \end{aligned}\right\} \quad (1)$$

where$f$ is a continuous $f:\mathbb{R}^{n+1}\rightarrow\mathbb{R}^n$are often used to describe physical systems in science, engineering such as in ship dynamics, deflection of beams, control theory and mechanics among others.

However, most of these physical problems are complex systems which solutions are extremely difficult, if not impossible to obtain an analytic solution. It is for this reason that numerical methods, which find solutions to (1) are inevitable tools.Therefore, fourth order ODEs have attracted significant interest of researchers, thereby theoretical and numerical studies dealing with (1) have appeared lately in literatures. The well-known established method of solving (1) is to first reduce it to a system of first order differential equation. The approach of reducing to a system of first order has very serious drawbacks including wastage of human and computer time due to complicated computational work and lengthy execution time (Awoyemi*et al.*, 2015). Therefore, multi-derivative linear multistep methods and implementations in a predictor-corrector mode using Taylor series algorithm to supply starting values were introduced to tackle the aforementioned drawbacks in solving differential equations. Although the implementation yielded good accuracy but they are costlier to implement in terms of the number of subroutines involved. In order to cater for this aforementioned problems, tons of researchers among whom are Areo and Omole (2015), Awoyemi*et al.* (2015), Familua and Omole (2017) and Kayode, Durodola and Bolaji (2014) have developed numerical methods of solving (1) directly.

2. Development of the Method

Following Familua and Omole (2017) power series is considered as the basis function and also as the approximate solution to (1) in the form;

$$y(x)=\sum_{j=0}^{t+c-1}\alpha_j x^j \qquad (2)$$

where $\alpha_j \varepsilon \Re$ are the real unknown parameters to be determined, $j=0(1)t+c-1, y\varepsilon C^m$, t is the interpolation points and $c$is the collocation points.The first, second, third and fourth derivatives of (2) are given by

$$\left.\begin{aligned} y'(x)&=\sum_{j=0}^{t+c-1} j\alpha_j x^{j-1}\\ y''(x)&=\sum_{j=0}^{t+c-1} j(j-1)\alpha_j x^{j-2}\\ y'''(x)&=\sum_{j=0}^{t+c-1} j(j-1)(j-2)\alpha_j x^{j-3}\end{aligned}\right\} \qquad (3)$$

and

$$y^{iv}(x)=\sum_{j=0}^{t+c-1} j(j-1)(j-2)(j-3)\alpha_j x^{j-4} \qquad (4)$$

From (1) and (4), we have that

$$f(x,y,y',y'',y''')=\sum_{j=0}^{t+c-1} j(j-1)(j-2)(j-3)\alpha_j x^{j-4} \qquad (5)$$

Collocating (5) at $x=x_{n+i}$, $i=0,\frac{1}{2},1,\frac{3}{2},2,3$ and interpolating (2) at

$x=x_{n+i}$, $i=0,\frac{1}{2},1,\frac{3}{2}$ gives a system of nonlinear equation;

$$AX=B \qquad (6)$$

where

$$\left.\begin{aligned} A &= \left[a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9\right]^T \\ U &= \left[y_n, y_{n+\frac{1}{2}}, y_{n+1}, y_{n+\frac{3}{2}}, f_n, f_{n+\frac{1}{2}}, f_{n+1}, f_{n+\frac{3}{2}}, f_{n+2}, f_{n+3}\right] \end{aligned}\right\} \quad (7)$$

$$X = \begin{pmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7 & x_n^8 & x_n^9 \\ 1 & x_{n+\frac{1}{2}h} & x^2_{n+\frac{1}{2}h} & x^3_{n+\frac{1}{2}h} & x^4_{n+\frac{1}{2}h} & x^5_{n+\frac{1}{2}h} & x^6_{n+\frac{1}{2}h} & x^7_{n+\frac{1}{2}h} & x^8_{n+\frac{1}{2}h} & x^9_{n+\frac{1}{2}h} \\ 1 & x_{n+h} & x^2_{n+h} & x^3_{n+h} & x^4_{n+h} & x^5_{n+h} & x^6_{n+h} & x^7_{n+h} & x^8_{n+h} & x^9_{n+h} \\ 1 & x_{n+\frac{3}{2}h} & x^2_{n+\frac{3}{2}h} & x^3_{n+\frac{3}{2}h} & x^4_{n+\frac{3}{2}h} & x^5_{n+\frac{3}{2}h} & x^6_{n+\frac{3}{2}h} & x^7_{n+\frac{3}{2}h} & x^8_{n+\frac{3}{2}h} & x^9_{n+\frac{3}{2}h} \\ 0 & 0 & 0 & 0 & 24 & 120x_n & 360x_n^2 & 840x_n^3 & 1680x_n^4 & 3024x_n^5 \\ 0 & 0 & 0 & 0 & 24 & 120x_{n+\frac{1}{2}h} & 360x^2_{n+\frac{1}{2}h} & 840x^3_{n+\frac{1}{2}h} & 1680x^4_{n+\frac{1}{2}h} & 3024x^5_{n+\frac{1}{2}h} \\ 0 & 0 & 0 & 0 & 24 & 120x_{n+h} & 360x^2_{n+h} & 840x^3_{n+h} & 1680x^4_{n+h} & 3024x^5_{n+h} \\ 0 & 0 & 0 & 0 & 24 & 120x_{n+\frac{3}{2}h} & 360x^2_{n+\frac{3}{2}h} & 840x^3_{n+\frac{3}{2}h} & 1680x^4_{n+\frac{3}{2}h} & 3024x^5_{n+\frac{3}{2}h} \\ 0 & 0 & 0 & 0 & 24 & 120x_{n+2} & 360x^2_{n+2} & 840x^3_{n+2} & 1680x^4_{n+2} & 3024x^5_{n+2} \\ 0 & 0 & 0 & 0 & 24 & 120x_{n+3} & 360x^2_{n+3} & 840x^3_{n+3} & 1680x^4_{n+3} & 3024x^5_{n+3} \end{pmatrix} \quad (8)$$

Solving (6) for $a_j s; j = 0(1)9$ using Maple software package and substituting it into (2) yields a continuous linear multistep method of the form

$$\left.\begin{aligned} y(x) &= \alpha_0 y_n + \alpha_{\frac{1}{2}} y_{n+\frac{1}{2}} + \alpha_1 y_{n+1} + \alpha_{\frac{3}{2}} y_{n+\frac{3}{2}} + \beta_0 f_0 + \beta_{\frac{1}{2}} f_{n+\frac{1}{2}} \\ &+ \beta_1 f_{n+1} + \beta_{\frac{3}{2}} f_{n+\frac{3}{2}} + \beta_2 f_{n+2} + \beta_3 f_{n+3} \end{aligned}\right\} \quad (9)$$

Evaluating (9) at $x = x_{n+2}$ and $x = x_{n+3}$ gives the following discrete direct schemes

$$\left.\begin{aligned} 6y_{n+1} + y_{n+2} - 4y_{n+\frac{1}{2}} - 4y_{n+\frac{3}{2}} + y_n &= -\frac{h^4}{11520}\left(f_n - 124f_{n+\frac{1}{2}} - 474f_{n+1} - 124f_{n+\frac{3}{2}} + f_{n+2}\right) \\ 45y_{n+1} + y_{n+3} - 36y_{n+\frac{1}{2}} - 20y_{n+\frac{3}{2}} + 10y_n &= \frac{h^4}{11520}\left(10f_n + 1116f_{n+\frac{1}{2}} + 5535f_{n+1} + 2860f_{n+\frac{3}{2}} + 1260f_{n+2} + 19f_{n+3}\right) \end{aligned}\right\} \quad (10)$$

The additional methods to be coupled with (10) are obtained by finding the first, second and the third derivatives of (9) and evaluating at $x = x_n, x_{n+\frac{1}{2}}, x_{n+1}, x_{n+\frac{3}{2}}, x_{n+2}, x_{n+3}$ *which are the collocation points. This gives the following discrete schemes*

$$y_{n+\frac{1}{2}} = \left. \begin{matrix} y_n + \frac{1}{2}hz_n + \frac{1}{8}h^2 w_n + \frac{1}{48}h^3 q_n + \\ h^4 \left(\begin{matrix} \frac{11849}{6967296} f_n + \frac{5947}{3628800} f_{n+\frac{1}{2}} - \frac{139}{110592} f_{n+1} \\ + \frac{11}{15552} f_{n+\frac{3}{2}} - \frac{451}{2322432} f_{n+2} + \frac{1469}{174182400} f_{n+3} \end{matrix} \right) \end{matrix} \right\} \quad (11)$$

$$y_{n+1} = \left. \begin{matrix} y_n + hz_n + \frac{1}{2}h^2 w_n + \frac{1}{6}h^3 q_n + \\ h^4 \left(\begin{matrix} \frac{23}{1215} f_n + \frac{139}{4050} f_{n+\frac{1}{2}} - \frac{149}{7560} f_{n+1} + \frac{187}{17010} f_{n+\frac{3}{2}} \\ - \frac{17}{5670} f_{n+2} + \frac{11}{85050} f_{n+3} \end{matrix} \right) \end{matrix} \right\} \quad (12)$$

$$y_{n+\frac{3}{2}} = \left. \begin{matrix} y_n + \frac{3}{2}hz_n + \frac{9}{8}h^2 w_n + \frac{9}{16}h^3 q_n + \\ h^4 \left(\begin{matrix} \frac{1467}{20480} f_n + \frac{243}{1400} f_{n+\frac{1}{2}} - \frac{9477}{143360} f_{n+1} + \frac{387}{8960} f_{n+\frac{3}{2}} \\ - \frac{243}{20480} f_{n+2} + \frac{369}{716800} f_{n+3} \end{matrix} \right) \end{matrix} \right\} \quad (13)$$

$$y_{n+2} = \left. \begin{matrix} y_n + 2hz_n + 2h^2 w_n + \frac{4}{3}h^3 q_n + \\ h^4 \left(\begin{matrix} \frac{1528}{8505} f_n + \frac{1024}{2025} f_{n+\frac{1}{2}} - \frac{104}{945} f_{n+1} + \frac{1024}{8505} f_{n+\frac{3}{2}} \\ - \frac{86}{2835} f_{n+2} + \frac{8}{6075} f_{n+3} \end{matrix} \right) \end{matrix} \right\} \quad (14)$$

$$y_{n+3} = \left. \begin{array}{l} y_n + 3hz_n + \frac{9}{2}h^2 w_n + \frac{9}{2}h^3 q_n + \\ h^4\left(\frac{9}{14}f_n + \frac{729}{350}f_{n+\frac{1}{2}} + \frac{9}{14}f_{n+\frac{3}{2}} + \frac{9}{1400}f_{n+3}\right) \end{array} \right\} \quad (15)$$

$$z_{n+\frac{1}{2}} = \left. \begin{array}{l} z_n + \frac{1}{2}hw_n + \frac{1}{8}h^2 q_n + \\ h^3\left(\begin{array}{l} \frac{23713}{1935360}f_n + \frac{403}{26880}f_{n+\frac{1}{2}} - \frac{781}{71680}f_{n+1} + \frac{1469}{241920}f_{n+\frac{3}{2}} \\ -\frac{17}{10240}f_{n+2} + \frac{139}{1935360}f_{n+3} \end{array}\right) \end{array} \right\} \quad (16)$$

$$z_{n+1} = \left. \begin{array}{l} z_n + hw_n + \frac{1}{2}h^2 q_n + \\ h^3\left(\begin{array}{l} \frac{479}{7560}f_n + \frac{1}{7}f_{n+\frac{1}{2}} - \frac{23}{336}f_{n+1} + \frac{37}{945}f_{n+\frac{3}{2}} \\ -\frac{3}{280}f_{n+2} + \frac{1}{2160}f_{n+3} \end{array}\right) \end{array} \right\} \quad (17)$$

$$z_{n+\frac{3}{2}} = \left. \begin{array}{l} z_n + \frac{3}{2}hw_n + \frac{9}{8}h^2 q_n + \\ h^3\left(\begin{array}{l} \frac{11043}{71680}f_n + \frac{567}{1280}f_{n+\frac{1}{2}} - \frac{7533}{71680}f_{n+1} + \frac{171}{1792}f_{n+\frac{3}{2}} \\ -\frac{1863}{71680}f_{n+2} + \frac{81}{71680}f_{n+3} \end{array}\right) \end{array} \right\} \quad (18)$$

$$z_{n+2} = \left. \begin{array}{l} z_n + 2hw_n + 2h^2 q_n + \\ h^3\left(\begin{array}{l} \frac{269}{945}f_n + \frac{32}{35}f_{n+\frac{1}{2}} - \frac{2}{35}f_{n+1} + \frac{32}{135}f_{n+\frac{3}{2}} \\ -\frac{1}{21}f_{n+2} + \frac{2}{945}f_{n+3} \end{array}\right) \end{array} \right\} \quad (19)$$

$$z_{n+3} = \left. \begin{array}{l} z_n + 3hw_n + \frac{9}{2}h^2 q_n + \\ h^3 \left(\begin{array}{l} \frac{27}{40} f_n + \frac{81}{35} f_{n+\frac{1}{2}} + \frac{243}{560} f_{n+1} + \frac{27}{35} f_{n+\frac{3}{2}} \\ + \frac{81}{280} f_{n+2} + \frac{81}{35} f_{n+3} \end{array} \right) \end{array} \right\} \quad (20)$$

$$w_{n+\frac{1}{2}} = \left. \begin{array}{l} w_n + \frac{1}{2}hq_n + \\ h^2 \left(\begin{array}{l} \frac{14879}{241920} f_n + \frac{263}{2520} f_{n+\frac{1}{2}} - \frac{617}{8960} f_{n+1} + \frac{143}{3780} f_{n+\frac{3}{2}} \\ - \frac{829}{80640} f_{n+2} + \frac{107}{241920} f_{n+3} \end{array} \right) \end{array} \right\} \quad (21)$$

$$w_{n+1} = \left. \begin{array}{l} w_n + hq_n + \\ h^2 \left(\begin{array}{l} \frac{1073}{7560} f_n + \frac{134}{315} f_{n+\frac{1}{2}} - \frac{11}{84} f_{n+1} + \frac{82}{945} f_{n+\frac{3}{2}} \\ - \frac{61}{2520} f_{n+2} + \frac{1}{945} f_{n+3} \end{array} \right) \end{array} \right\} \quad (22)$$

$$w_{n+\frac{3}{2}} = \left. \begin{array}{l} w_n + \frac{3}{2}hq_n + \\ h^2 \left(\begin{array}{l} \frac{1983}{8960} f_n + \frac{27}{35} f_{n+\frac{1}{2}} + \frac{81}{8960} f_{n+1} + \frac{9}{56} f_{n+\frac{3}{2}} \\ - \frac{351}{8960} f_{n+2} + \frac{3}{1792} f_{n+3} \end{array} \right) \end{array} \right\} \quad (23)$$

$$w_{n+2} = \left.\begin{matrix} w_n + 2hq_n + \\ h^2\left(\begin{matrix} \frac{284}{945}f_n + \frac{352}{315}f_{n+\frac{1}{2}} + \frac{6}{35}f_{n+1} + \frac{416}{945}f_{n+\frac{3}{2}} \\ -\frac{2}{63}f_{n+2} + \frac{2}{945}f_{n+3} \end{matrix}\right) \end{matrix}\right\} \quad (24)$$

$$w_{n+3} = \left.\begin{matrix} w_n + 3hq_n + \\ h^2\left(\begin{matrix} \frac{141}{280}f_n + \frac{54}{35}f_{n+\frac{1}{2}} + \frac{81}{70}f_{n+1} + \frac{6}{35}f_{n+\frac{3}{2}} \\ + \frac{297}{280}f_{n+2} + \frac{9}{140}f_{n+3} \end{matrix}\right) \end{matrix}\right\} \quad (25)$$

$$q_{n+\frac{1}{2}} = q_n + h\left.\left(\begin{matrix} \frac{959}{5760}f_n + \frac{35}{72}f_{n+\frac{1}{2}} - \frac{487}{1920}f_{n+1} \\ + \frac{49}{360}f_{n+\frac{3}{2}} - \frac{211}{5760}f_{n+2} + \frac{1}{640}f_{n+3} \end{matrix}\right)\right\} \quad (26)$$

$$q_{n+1} = q_n + h\left.\left(\begin{matrix} \frac{169}{1080}f_n + \frac{33}{45}f_{n+\frac{1}{2}} + \frac{11}{120}f_{n+1} \\ + \frac{8}{135}f_{n+\frac{3}{2}} - \frac{7}{360}f_{n+2} + \frac{1}{1080}f_{n+3} \end{matrix}\right)\right\} \quad (27)$$

$$q_{n+\frac{3}{2}} = q_n + h\left.\left(\begin{matrix} \frac{103}{640}f_n + \frac{27}{40}f_{n+\frac{1}{2}} + \frac{243}{640}f_{n+1} \\ + \frac{13}{40}f_{n+\frac{3}{2}} - \frac{27}{640}f_{n+2} + \frac{1}{640}f_{n+3} \end{matrix}\right)\right\} \quad (28)$$

$$q_{n+2} = q_n + h\left(\begin{array}{l} \frac{7}{45}f_n + \frac{32}{45}f_{n+\frac{1}{2}} + \frac{4}{15}f_{n+1} \\ +\frac{32}{45}f_{n+\frac{3}{2}} + \frac{7}{45}f_{n+2} \end{array}\right) \quad (29)$$

$$q_{n+3} = q_n + h\left(\frac{11}{40}f_n + \frac{81}{40}f_{n+1} - \frac{8}{5}f_{n+\frac{3}{2}} + \frac{81}{40}f_{n+2} + \frac{11}{40}f_{n+3}\right) \quad (30)$$

where $z_{n+j} = y'_{n+j}, w_{n+j} = y''_{n+j}$ and $q_{n+j} = y'''_{n+j}, j = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, 3$

Equations (11)-(30) give the hybrid block method for the direct solution of general fourth order ordinary differential equations.

3 Analysis of the Properties of the Hybrid Block Method

3.1 Order and Error Constant

Following Fatunla (1988) and Lambert (1973), the local truncation error associated with the method is defined bythe difference operator

$$L[y(x),h] = \sum_{j=0}^{k}\left[\alpha_j y(x+jh) - h^4\beta_j f(x_n+jh)\right] \quad (31)$$

where $y(x)$ is a continuous derivatives of sufficiently high order (Lambert, 1973). Expanding (31) in Taylor series about the point x, we obtainthe expression

$$L[y(x),h] = C_0 y(x) + hC_1 y'(x) + h^2 C_2 y''(x) + \ldots + h^{p+4} C_{p+4} y^{p}(x) \quad (32)$$

Where $C_0, C_1, C_2, \ldots, C_{p+4}$ are obtained as

$$\left.\begin{aligned} C_0 &= \sum_{j=0}^{k} \alpha_j \\ C_1 &= \sum_{j=0}^{k} j\alpha_j \\ C_2 &= \frac{1}{2!}\sum_{j=0}^{k} j^2\alpha_j \\ &\vdots \\ C_q &= \frac{1}{q!}\sum_{j=0}^{k} j^q q(q-1)(q-2)(q-3)\sum_{j-1}^{k}\beta_j j^{q-4}; q = 0,1,2,\ldots,p+4 \end{aligned}\right\} \quad (33)$$

In the sense of Lambert (1973), a method is of order p and error constant C_{p+4} if

$$C_0 = C_1 = C_2 = \ldots = C_p = C_{p+1} = C_{p+2} = C_{p+3} = 0, C_{p+4} \neq 0 \quad (34)$$

The $C_{p+4} \neq 0$ is called the error constant and $C_{p+4}h^{p+4}y^{p+4}(x_n)$ is the principal local truncation error at the point x_n.

Using the concept above, the first scheme in (10) has order 6 and the error constant is given by $C_{p+4} = \frac{1}{3096576}$.

3.2 Consistency of the Method

A linear multistep method is said to be consistent if it has order $p \geq 1$ and the first and second characteristics polynomials which are defined as $\rho(r) = \sum_{j=0}^{k} \alpha_j r^j$ and $\sigma(r) = \sum_{j=0}^{k} \beta_j r^j$, where r, the principal root satisfies the following conditions;

$$\left.\begin{aligned} &(i) \quad \sum_{j=0}^{k} \alpha_j = 0 \\ &(ii) \quad \rho(1) = \rho'(1) = 0 \\ &(iii) \quad \rho^{iv}(1) = 4!\sigma(1) \end{aligned}\right\} \quad (35)$$

Considering the first scheme in (10) the order $p = 6 > 1$ and it has been investigated to satisfy conditions (36). Hence the scheme is consistent.

3.3 Zero Stability of the Method

A numerical method is said to be zero-stable if the roots $z_s = 1,2,3,\ldots,N$ of the characteristics polynomial $p(z) = \det\left(zA^0 - A^1\right)$ satisfies $|z| \leq 1$ and the roots $|z| = 1$ has multiplicity not exceeding the order of the differential equation which is 4.
To analyse the zero-stability of the block methods, equations (11)-(30) is presented in vector notation form of column vectors $e = \left(e_1,\ldots,e_r\right)^T$, $d = \left(d_1,\ldots,d_r\right)^T$,

$y_m = \left(y_{n+1},\ldots,y_{n+r}\right)^T$, $F\left(y_m\right) = \left(f_{n+1},\ldots,f_{n+r}\right)^T$ and matrices $A = \left(a_{ij}\right)$ and $B = \left(b_{ij}\right)$.

Thus (11)-(30) form the block formula $A^0 y_m = hBF\left(y_m\right) + A^1 y_n + hdf_n$.

The first characteristic polynomial of the block hybrid method is given by $p(z) = \det\left(zA^0 - A^1\right)$, where it has been established that

$$A^0 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad A^1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad (36)$$

Hence, $Z = 0,0,0,0,1$, therefore the method is zero-stable since $|z| = 1$ is simple and the magnitude of other roots are $|z| = 0$.

3.4 Convergence of the Method

According to the theorem of Dahliquist (1983); the necessary and sufficient condition for a LMM to be convergent is to be consistent and zero-stable. Since the method satisfies the two conditions then it is convergent.

4. Numerical Examples

Two test problems are considered to illustrate the method.

Problem 1:

$$y^{iv}(x)=\cos(x)-\sin(x)$$

$$y'''(0)=7, y''(0)=y'(0)=-1, y(0)=0, h=\frac{1}{320}$$

Exact solution: $y(x)=\cos(x)-\sin(x)+x^3-1$

Source: Olabodeand Omole (2015)

Problem 2: An application problem

$$y^{iv}(t)+3y''(t)+y(t)(2+\varepsilon\cos(\Omega)t)=0,\ t>0$$

$$y'''(0)=0, y''(0)=0, y'(0)=0, y(0)=1, h=\frac{1}{320}$$

Exact solution: $y(t)=2\cos(t)-\cos(\sqrt{2}t)$

Source:Familua and Omole (2017)

Results

Table 1. Comparison of results of Problem 1 with the exact solution.

X	Exact	New Result
0.003125	-0.00312984720468769600	-0.00312984720468769600
0.006250	-0.00626924635577210114	-0.00626924635577210114
0.009375	-0.00941798368752841945	-0.0094179836875284194

		4
0.012500	-0.01257584533946248273	-0.01257584533946248273
0.015625	-0.01574261735661109244	-0.01574261735661109244
0.018750	-0.01891808568984328399	-0.01891808568984328399
0.021875	-0.02210203619616251069	-0.02210203619616251069
0.025000	-0.02529425463900974441	-0.02529425463900974441
0.028125	-0.02849452668856748983	-0.02849452668856748983
0.031250	-0.03170263792206470950	-0.03170263792206470950

Table 2. Comparison of errors in Problem 1.

X	Error in new Method	Error in Olabodeand Omole(2017)
0.003125	0.0000E+00	5.8350E-18
0.006250	0.0000E+00	4.6708E-17
0.009375	0.0000E+00	5.2460E-17
0.012500	1.0000E+20	9.3430E-17
0.015625	0.0000E+00	9.9220E-17
0.018750	0.0000E+00	1.4019E-16
0.021875	0.0000E+00	1.4613E-16
0.025000	0.0000E+00	1.8712E-16

0.028125	0.0000E+00	1.9324E-16
0.031250	0.0000E+00	5.8350E-18

Table 3. Comparison of results of Problem 2 with the exact solution.

X	Exact	New Result
0.003125	0.99999999999205272181	0.99999999999205272179
0.006250	0.99999999987284392123	0.99999999987284392118
0.009375	0.99999999935627549414	0.99999999935627549419
0.012500	0.99999999796552658062	0.99999999796552658060
0.015625	0.99999999503306753347	0.99999999503306753344
0.018750	0.99999998970067947569	0.99999998970067947573
0.021875	0.99999998091947944412	0.99999998091947944411
0.025000	0.99999996744995111889	0.99999996744995111889
0.028125	0.99999994786198113959	0.99999994786198113955
0.031250	0.99999992053490100516	0.99999992053490100514

Table 4. Comparison of errors in Problem 2.

X	Error in new Method	Error in Familuaand Omole(2017)
0.003125	2.0000E-19	6.685763E-13
0.006250	5.0000E-19	1.458489E-11
0.009375	5.0000E-20	1.082968E-10
0.012500	2.0000E-20	3.917803E-10
0.015625	3.0000E-20	1.025145E-09
0.01875	4.0000E-20	2.217319E-09

0		
0.02187 5	1.0000E-20	4.2260686E-09
0.02500 0	0.0000E-20	7.3580196E-09
0.02812 5	4.0000E-20	1.1968686E-08
0.03125 0	2.0000E-20	1.8462490E-08

Conclusion

A more accurate hybrid block linear multistep method for direct solution of initial value problems of fourth order ordinary differential equations incorporating the use of two off-step points at both collocation and interpolation has been presented. The methods were applied to solve two problems adapted from Olabode and Omole (2015) and Familua and Omole (2017), errors were computed and the proposed method was proven to produce approximation that are closer to the exact solution than the reviewed works.

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