METHOD OF LINES ANALYSIS OF MHD EFFECT ON CONVECTVE FLOW OF DUSTY FLUID IN THE PRESENCE OF VISCOUS ENERGY DISSIPATION

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Abstract
This paper investigates the effects of Magnetohydrodynamics (MHD) on convective flow of dusty fluid with viscous energy dissipation. The governing partial differential equations (PDEs) in dimensionless form are solved numerically and analyzed using semi-discretization method known as Method of Lines (MOL). The effects of different flow parameters on the velocity profiles of both the fluid and the particles as well as the temperature profile of fluid are examined and presented graphically. It is observed that the velocity of the fluid and the dust particle increases with increase in Grashof number. Also, the velocity of the fluid and the particle decreases with increase in magnetic parameter. Moreso, increase in Prandtl number reduce temperature of the fluid.

Keywords: Dusty fluid, Method of Lines (MOL), MHD, Viscous dissipation.

Introduction
The need for the study of dusty flow of an incompressible and electrically conducting fluid through various cross sections had rapidly increased in recent years as the efficiency of the devices used in industries and engineering depends on the particles suspended in the fluid under the effect of magnetic field. Therefore, great efforts have been made by several authors to analyse the effect of magnetic field on the velocity of fluid in the presence of dust particles. Khare and Singh (2010) studied MHD flow of a dusty viscous Incompressible fluid confined between two vertical walls with volume fraction of dust. Saxena and Dubey (2011) studied unsteady MHD heat and mass transfer free convection flow of polar fluids past a vertical moving porous plate in a porous medium with heat generation and thermal diffusion. Khare and Singh (2012) studied the flow of an unsteady conducting dusty fluid through an inclined circular channel. Gireesha *et al.* (2012) critically analysed the Magnetohydrodynamics (MHD) flow and heat transfer of a dusty fluid over a stretching sheet using numerical technique. Mohammed *et al.* (2015) presented an analytical method to describe the heat and mass transfer in the flow of an incompressible viscous fluid past an infinite vertical plate with the governing equations accounting for viscous dissipation effect and mass transfer with chemical reaction of constant reaction rate. Durojaye *et al.* (2020) used Method of Lines (MOL) in studying and analysing the effects of some flow parameters on unsteady MHD fluid flow past a moving vertical plate embedded in porous medium in the presence of Hall Current and Rotating system.

Olayiwola (2016) presented an analytical method for studying chemically reacting flow in a laminar premixed flame of carbon monoxide/oxygen mixture in the region of the stagnation point. Saidu et al. (2010) studied the laminar convective flow of a dusty viscous fluid through a porous medium of non-conducting walls in the presence of uniform transverse magnetic field with volume fraction and considering porous parameter.

In this paper, the effects of Magnetohydrodynamics (MHD) and other pertinent flow parameters on convective flow of dusty fluid with viscous dissipation are investigated using Method of Lines (MOL).

Model Formulation

We consider unsteady laminar flow of a dusty, incompressible, Newtonian, electrically – conducting and viscous fluid through a porous medium of uniform cross section h, so that when one wall of the channel is fixed, and the other is oscillating in time about a constant non-zero mean. At $t \leq 0$, the channel wall as well as the fluid are assumed to be at the same temperature T_0 . When $t > 0$, the temperature of the channel walls is instantaneously raised to T_w which oscillate with time, then, maintained constant. Let x-axis be along the flow of liquid at the fixed wall and y-axis perpendicular to it. A uniform magnetic field of strength $B_0\left(=\mu_c H_0\right)$ is applied perpendicular to the flow region.

In formulating the governing equations, the following assumptions are considered:

(i) The dust particles are solid, spherical, non-conducting equal in size and uniformly distributed in the flow region i.e., the dust particles gain heat energy from the fluid by conduction through their spherical surface.
(ii) The number density of dust particles is constant with uniform temperature between the particles throughout the motion.
(iii) The interactions between the particles, chemical reaction and radiation between the particles and liquid have not been considered. This is necessary in order to avoid multiple equations.
(iv) The buoyancy force induced magnetic field and Hall effects have been neglected. This means that the flow region has uniform temperature, uniform applied magnetic field and a Cartesian coordinate.
(v) The volume occupied by the particles per unit volume of the mixture and mass concentration has been taken into consideration.
(vi) The magnetic Reynolds number is taken to be very small so that induced magnetic field is negligible. This means that a uniform magnetic field B_0 is applied in the positive y-direction and is the only magnetic field in the problem.
(vii) The dust concentration is so small that it is not disturbing the continuity and hydro magnetic effects. This means that the continuity equation is satisfied.
(viii) Viscous heat dissipation is taken into consideration.

Based on the assumptions (i) – (viii), we have the governing equations:

$$(1-\varphi)\frac{\partial u_f}{\partial t}=(1-\varphi)\left[\upsilon\frac{\partial^2 u_f}{\partial y^2}+g\beta(T-T_0)\right]+\frac{KN_0}{\rho}(u_p-u_f)-\frac{\sigma\mu_c^2 H_0^2}{\rho}u_f-\frac{\mu}{K_1}u_f \tag{1}$$

$$N_0 m\frac{\partial u_p}{\partial t}=\varphi\left[\mu\frac{\partial^2 u_p}{\partial y^2}+\rho g\beta(T-T_0)\right]-KN_0\left(u_p-u_f\right) \tag{2}$$

$$\frac{\partial T}{\partial t}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{\upsilon}{C_p}\left[\left(\frac{\partial u_f}{\partial y}\right)^2+\left(\frac{\partial u_p}{\partial y}\right)^2\right] \tag{3}$$

Subject to the initial and boundary conditions

$$u_f(y,0)=0, \qquad u_f(0,t)=0, \qquad u_f(h,t)=U \tag{4}$$

$$u_p(y,0)=0, \qquad u_p(0,t)=0, \qquad u_p(h,t)=U \tag{5}$$

$$T(y,o)=T_0, \qquad T(0,t)=T_0, \qquad T(h,t)=T_1 \tag{6}$$

where, υ = kinematic viscosity, u_f = The velocity of the fluid, u_p = Velocity of the dust Particles, m = mass of each dust particles, N_0 = number density of dust particles, T = Temperature of the fluid, T_0 = initial temperature fluid and wall, β = volumetric coefficient of thermal expansion, C_p = specific heat at constant pressure, φ = volume fraction of dust particles, K = stokes resistance coefficient, H_0 = magnetic field induction, μ_c = magnetic permeability, σ = electric conductivity of the liquid, k = thermal conductivity, K_1 = porous parameter.

Method of Solution

Non-dimensionalisation

Introducing the following dimensionless quantities as used by Saidu *et al.* (2010):

$$t'=\frac{\upsilon t}{h^2}, \quad \psi=\frac{u_p h}{\upsilon}, \quad y'=\frac{y}{h}, \quad \theta=\frac{T-T_0}{T_1-T_0}, \quad \phi=\frac{u_f h}{\upsilon}, \tag{7}$$

Equations (1) – (6) in dimensionless form become:

$$\frac{\partial\phi}{\partial t}=\frac{\partial^2\phi}{\partial y^2}+Gr\theta+\varepsilon_1(\psi-\phi)-\varepsilon_2 M\phi-\varepsilon_3\phi \tag{8}$$

$$f\frac{\partial\psi}{\partial t}=\varphi\frac{\partial^2\psi}{\partial y^2}+\varphi Gr\theta-\alpha(\psi-\phi) \tag{9}$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{Pr}\frac{\partial^2\theta}{\partial y^2}+E_c\left[\left(\frac{\partial\phi}{\partial y}\right)^2+\left(\frac{\partial\psi}{\partial y}\right)^2\right] \tag{10}$$

Subject to initial and boundary conditions:

$$\phi(y,0)=0; \qquad \phi(0,t)=0; \qquad \phi(1,t)=b_1 \tag{11}$$

$$\psi(y,0)=0; \qquad \psi(0,t)=0; \qquad \psi(1,t)=b_2 \tag{12}$$

$$\theta(y,o)=0; \qquad \theta(0,t)=0; \qquad \theta(1,t)=1 \tag{13}$$

Where $M=\frac{h^2\sigma\mu_c^2H_0^2\upsilon}{\mu}$ (Magnetic Parameter), $\varepsilon_1=\frac{h^2KN_0\upsilon}{\mu(1-\varphi)}$;(Constant) $\varepsilon_2=\frac{1}{(1-\varphi)}$; (Constant), $\varepsilon_3=\frac{h^2\mu}{K_1\upsilon(1-\varphi)}$ (Porous parameter), $f=\frac{\upsilon N_0 m}{\mu}$ (Mass concentration of dust particles), $Gr=\frac{h^3 g\beta(T_1-T_0)}{\upsilon^2}$ (Grashof number), $\alpha=\frac{Kh^2}{m\upsilon}$; (Concentration resistance ratio), $\Pr=\frac{\mu c_p}{k}$ (Prandtl number); $E_c=\frac{\upsilon^2}{h^2 C_p(T_1-T_0)}$ (Eckert number).

Method of Lines (MOL)

The basic idea of the MOL is to replace the spatial (boundary value) derivatives in the PDE with algebraic approximations. Once this is done, only the initial value variable, typically time in a physical problem, remains as seen in Biazar and Nomidi (2013). Then, with only one remaining independent variable, we have a system of ODEs that approximates the original PDE. Any suitable integration algorithm for initial value ODEs can now be used to compute an approximate numerical solution to the PDE as seen in Schiesser (1991) and Knapp (2008).

In linearizing and explicitly decoupling equations (8) – (10), we adopt the following approximations by Chung (2002) considering θ,ψ in equation (8), ϕ,θ in equation (9), $(\frac{\partial\phi}{\partial y})^2, (\frac{\partial\psi}{\partial y})^2$ in equation (10) to be unity (Constant). In view of the approximation, equations (8) – (10) become:

$$\frac{\partial\phi}{\partial t}=\frac{\partial^2\phi}{\partial y^2}-(\varepsilon_1+\varepsilon_2 M+\varepsilon_3)\phi+Gr+\varepsilon_1 \qquad (14)$$

$$\frac{\partial\psi}{\partial t}=\frac{\varphi}{f}\frac{\partial^2\psi}{\partial y^2}-\frac{\alpha}{f}\psi+\frac{\varphi}{f}Gr+\frac{\alpha}{f}\psi \qquad (15)$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{Pr}\frac{\partial^2\theta}{\partial y^2}+2E_c \qquad (16)$$

subject to initial and boundary conditions (11) – (13). Then, we solve equations (14) – (16) subject to the conditions (11) – (13) by the method of lines (MOL).

Discretizing equation (14) in space variable y while leaving time variable t continuous, we have the system of ODEs:

$$\left(\frac{d\phi}{dt}\right)_i=\frac{\phi_{i+1}-2\phi_i+\phi_{i-1}}{h^2}-(\varepsilon_1+\varepsilon_2 M+\varepsilon_3)\phi_i+Gr+\varepsilon_1 \qquad (17)$$

$$=\alpha_1\phi_{i-1}+\alpha_2\phi_i+\alpha_3\phi_{i+1}+\alpha_4 \qquad (18)$$

Where

$$\alpha_1=\frac{1}{h^2},\alpha_2=-\left(\varepsilon_1+\varepsilon_2 M+\varepsilon_3+\frac{2}{h^2}\right),\alpha_3=\frac{1}{h^2},\alpha_4=Gr+\varepsilon_1 \qquad (19)$$

Now, equation (17) – (19) can be solved iteratively using the boundary conditions $\phi(0,t)=0$ and $\phi(1,t)=1$ in equation (11). For $i=1,2,3,\ldots,N$ $\phi(0,t)=\phi_0(y,t)=0$ $\phi(1,t)=\phi_0(N+1,t)=b_1=1$ eq. (18) is written as a system of ODEs:

$$\dot{\phi}_1 = \alpha_1\phi_0 + \alpha_2\phi_1 + \alpha_3\phi_2 + \alpha_4$$
$$\dot{\phi}_2 = \alpha_1\phi_1 + \alpha_2\phi_2 + \alpha_3\phi_3 + \alpha_4$$
$$\dot{\phi}_3 = \alpha_1\phi_2 + \alpha_2\phi_3 + \alpha_3\phi_4 + \alpha_4 \tag{20}$$
....
$$\dot{\phi}_N = \alpha_1\phi_{N-1} + \alpha_2\phi_N + \alpha_3\phi_{N+1} + \alpha_4$$

The system of equations in (20), in matrix form is given as:

$$\begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \\ \vdots \\ \dot{\phi}_{N-1} \\ \dot{\phi}_N \end{bmatrix} = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & 0 & 0 & \cdots & 0 & 0 \\ 0 & \alpha_1 & \alpha_2 & \alpha_3 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \alpha_1 & \alpha_2 & \alpha_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & \alpha_1 & \alpha_2 \end{bmatrix} \begin{bmatrix} 0 \\ \phi_1 \\ \phi_2 \\ \vdots \\ \phi_N \\ \phi_{N+1} \end{bmatrix} + \begin{bmatrix} \alpha_4 \\ \alpha_4 \\ \alpha_4 \\ \vdots \\ \alpha_4 \\ \alpha_4 \end{bmatrix} \tag{21}$$

where the coefficients $\alpha_1, \alpha_2, \alpha_3$ and α_4 are given by equation (19) and $\dot{\phi}_i = \left(\frac{d\phi}{dt}\right)_i$,

In a similar way, equation (15) becomes:

$$\left(\frac{d\psi}{dt}\right)_i = \frac{\varphi}{f}\left(\frac{\psi_{i+1} - 2\psi_i + \psi_{i-1}}{h^2}\right) - \frac{\alpha}{f}\psi_i + \frac{\varphi}{f}Gr + \frac{\alpha}{f} \tag{22}$$
$$= \beta_1\psi_{i-1} + \beta_2\psi_i + \beta_3\psi_{i+1} + \beta_4 \tag{23}$$

where

$$\beta_1 = \frac{\varphi}{h^2 f}, \beta_2 = \frac{1}{f}\left(\alpha - \frac{2\varphi}{h^2}\right), \beta_3 = \frac{\varphi}{h^2 f}, \beta_4 = \frac{1}{f}(Gr + \alpha) \tag{24}$$

Now, equation (22) – (23) can be solved iteratively using the boundary conditions $\psi(0,t) = 0$ and $\psi(1,t) = 1$ in equation (12). For $i = 1,2,3,\dots,N$ $\psi(0,t) = \psi_0(y,t) = 0$ $\psi(1,t) = \psi(N+1,t) = b_2 = 1$ eq. (23) can be written as a system of ODE:

$$\dot{\psi}_1 = \beta_1\psi_0 + \beta_2\psi_1 + \beta_3\psi_2 + \beta_4$$
$$\dot{\psi}_2 = \beta_1\psi_1 + \beta_2\psi_2 + \beta_3\psi_3 + \beta_4$$
$$\dot{\psi}_3 = \beta_1\psi_2 + \beta_2\psi_3 + \beta_3\psi_4 + \beta_4 \tag{25}$$
....
$$\dot{\psi}_N = \beta_1\psi_{N-1} + \beta_2\psi_N + \beta_3\psi_{N+1} + \beta_4$$

The system of equation (25), in matrix form is given as

$$\begin{bmatrix} \dot{\psi}_1 \\ \dot{\psi}_2 \\ \vdots \\ \dot{\psi}_{N-1} \\ \dot{\psi}_N \end{bmatrix} = \begin{bmatrix} \beta_1 & \beta_2 & \beta_3 & 0 & 0 & \cdots & 0 & 0 \\ 0 & \beta_1 & \beta_2 & \beta_3 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \beta_1 & \beta_2 & \beta_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & \beta_1 & \beta_2 \end{bmatrix} \begin{bmatrix} 0 \\ \psi_1 \\ \psi_2 \\ \vdots \\ \psi_N \\ \psi_{N+1} \end{bmatrix} + \begin{bmatrix} \beta_4 \\ \beta_4 \\ \beta_4 \\ \vdots \\ \beta_4 \\ \beta_4 \end{bmatrix} \tag{26}$$

where the coefficients $\beta_1, \beta_2, \beta_3$ and β_4 are given by equation (24) and $\dot{\psi}_i = \left(\frac{d\psi}{dt}\right)_i$

Similarly, equation (16) becomes:

$$\left(\frac{d\theta}{dt}\right)_i = \frac{1}{\Pr}\left(\frac{\theta_{i+1} - 2\theta_i + \theta_{i-1}}{h^2}\right) + 2Ec \quad (27)$$

$$= \gamma_1\theta_{i-1} + \gamma_2\theta_i + \gamma_3\theta_{i+1} + \gamma_4 \quad (28)$$

where $\gamma_1 = \frac{1}{h^2 \Pr}, \gamma_2 = \frac{-2}{h^2 \Pr}, \gamma_3 = \frac{1}{h^2 \Pr}, \gamma_4 = 2Ec$ (29)

Thus, equation (27) – (25) can be solved iteratively using the boundary conditions $\theta(0,t) = 0$ and $\theta(1,t) = 1$ in equation (13).

For $i = 1,2,3,...,N$ and $\theta(0,t) = \theta(y,t) = 0$ and $\theta(1,t) = \theta(N+1,t) = 1$ can be written as a system of ODEs:

$$\begin{aligned}
\dot{\theta}_1 &= \gamma_1\theta_0 + \gamma_2\theta_1 + \gamma_3\theta_2 + \gamma_4 \\
\dot{\theta}_2 &= \gamma_1\theta_1 + \gamma_2\theta_2 + \gamma_3\theta_3 + \gamma_4 \\
\dot{\theta}_3 &= \gamma_1\theta_2 + \gamma_2\theta_3 + \gamma_3\theta_4 + \gamma_4 \\
&\dots\dots\dots\dots \\
\dot{\theta}_N &= \gamma_1\theta_{N-1} + \gamma_2\theta_N + \gamma_3\theta_{N+1} + \gamma_4
\end{aligned} \quad (30)$$

The system of equation (26), in matrix form is given as

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \vdots \\ \dot{\theta}_{N-1} \\ \dot{\theta}_N \end{bmatrix} = \begin{bmatrix} \gamma_1 & \gamma_2 & \gamma_3 & 0 & 0 & \cdots & 0 & 0 \\ 0 & \gamma_1 & \gamma_2 & \gamma_3 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \gamma_1 & \gamma_2 & \gamma_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & \gamma_1 & \gamma_2 \end{bmatrix} \begin{bmatrix} 0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_N \\ \theta_{N+1} \end{bmatrix} + \begin{bmatrix} \gamma_4 \\ \gamma_4 \\ \gamma_4 \\ \vdots \\ \gamma_4 \\ \gamma_4 \end{bmatrix} \quad (31)$$

where the coefficients $\gamma_1, \gamma_2, \gamma_3$ and γ_4 are given by equation (25) and $\dot{\theta}_i = \left(\frac{d\theta}{dt}\right)_i$

Results and Discussion

In this paper, the effects of concentration resistance ratio, volume fraction and mass concentration of dust particle, porous parameter, and other pertinent flow parameters on convective flow of dusty fluid with viscous dissipation are considered. In the analysis, Method of lines (MOL) is used as a numerical method to solve the governing equations of the flow model. For numerical computations, the values of the flow parameters: $G_r = 1.0, M = 1.0, E_c = 1.0, P_r = 0.7, \varepsilon_3 = 0.1, f = 0.5, \alpha = 0.5, \varphi = 0.1$. Also, the MATLAB code is used in obtaining solutions of systems of ODEs in equations (21), (26) and (31), as well as stimulating the graphs. Figure 1 shows the effects of magnetic parameter M, on velocity profile of the fluid flow, it can be seen that as parameter increases, velocity of the flow decreases. Figure 2 shows the effect of Grashof number, G_r, on the velocity profile of the fluid flow and as it increases, the velocity of the flow increases. Figure 3 shows the effect of porous parameter, ε_3, on velocity profile of the flow and as it increases, the velocity of the fluid flow decreases. Figure 4 – Figure 5 show the effects Grashof number G_r and volume fraction of dust particles φ, on velocity profile of the dust particle and as they increase, the velocity of the dust particle flow increase. Figure 6 shows the effect of mass concentration of dust particle, f, on the velocity of the dust particle flow and as the parameter increases, the velocity of the dust particle flow decreases. Figure 7 shows the effect of concentration resistance ratio, α, on the velocity of the dust particle flow and as it increases, the velocity of the dust particle flow increases. Figure 8 shows the effect of Eckert number, E_c, on the temperature profile of the flow and as it increases, the temperature profile of the flow increases. Figure 9 shows the effect of Prandtl number, P_r, on the temperature profile of the flow and as it increases, the temperature profile of the flow decreases.

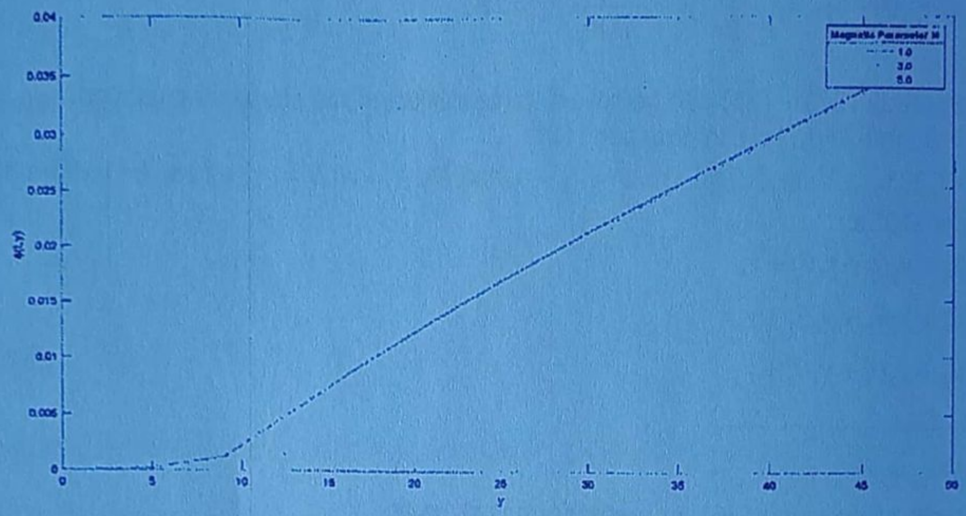

Figure 1: Velocity profile of the fluid flows for various values of M

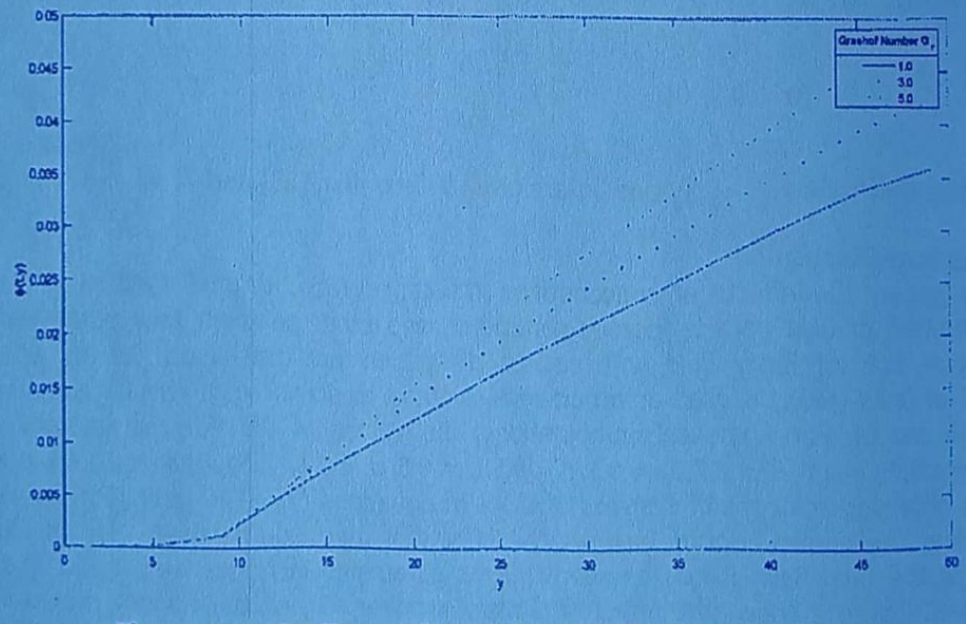

Figure 2: Velocity profile of the fluid flows for various values of G_r

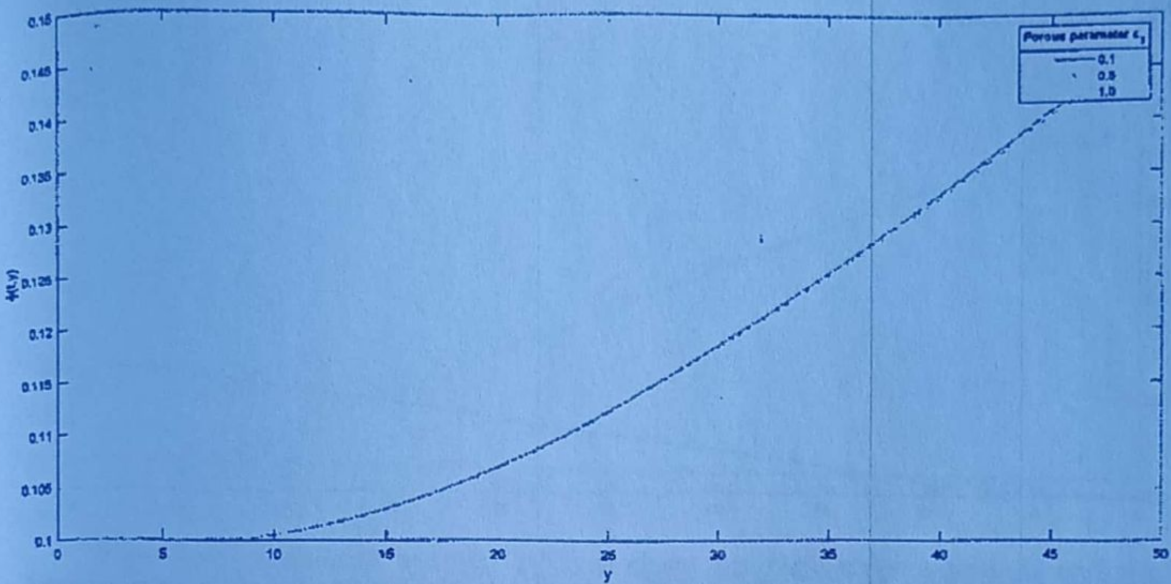

Figure 3: Velocity profile of the fluid flows for various values of ε_3

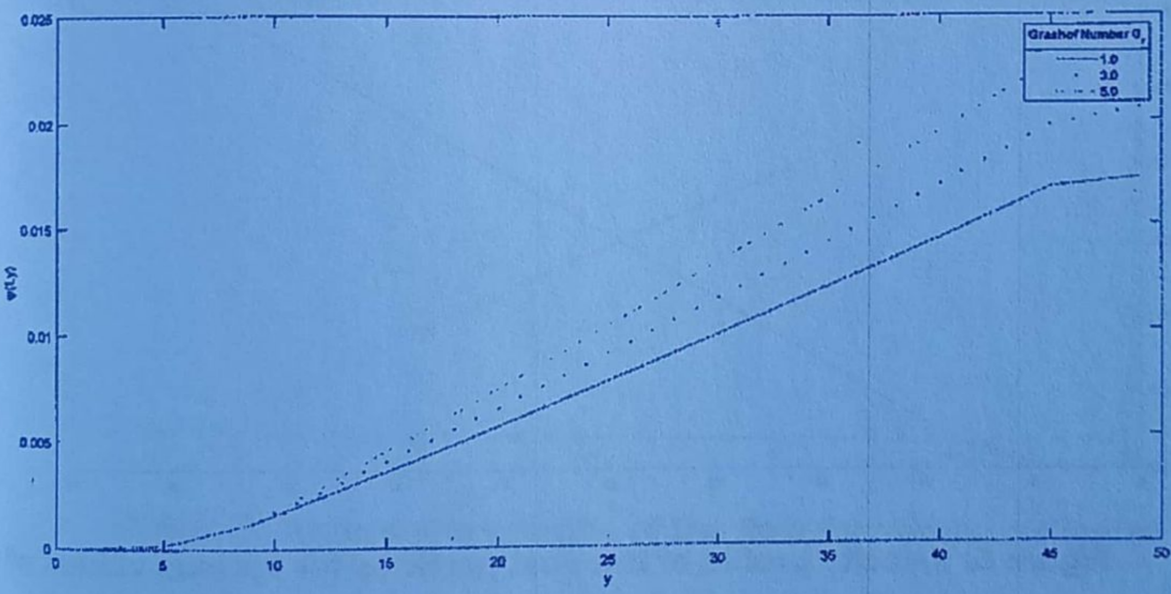

Figure 4: Velocity profile of the dust particles for various values of G_r

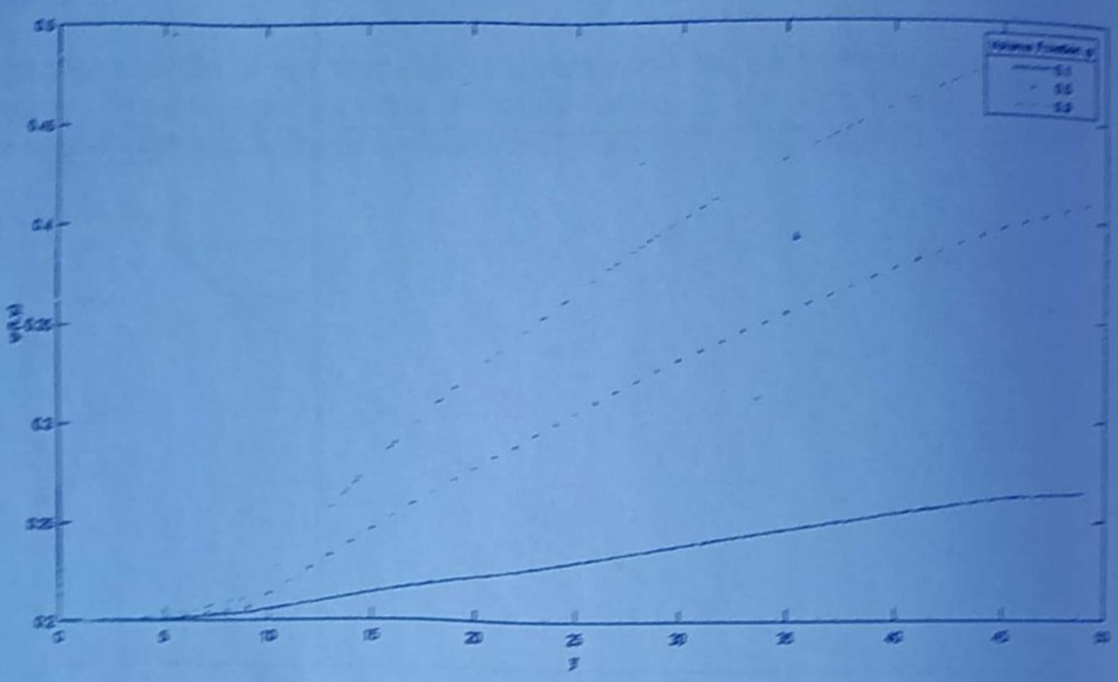

Figure 5: Velocity profile of the dust particles for various values of φ

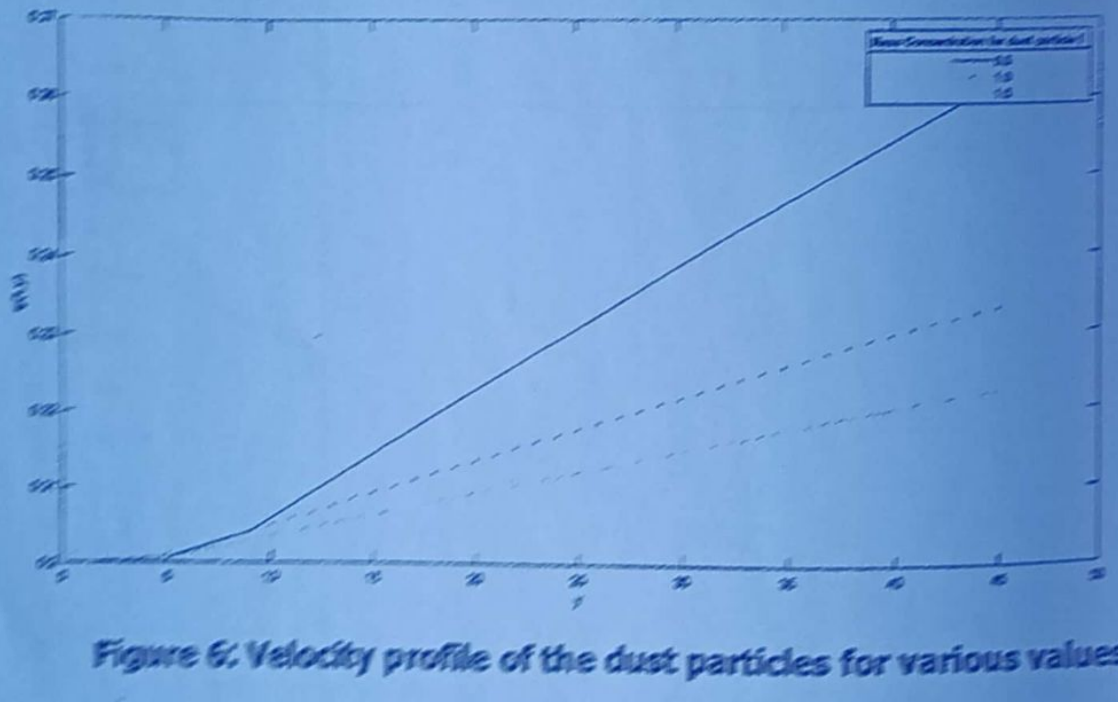

Figure 6: Velocity profile of the dust particles for various values of f

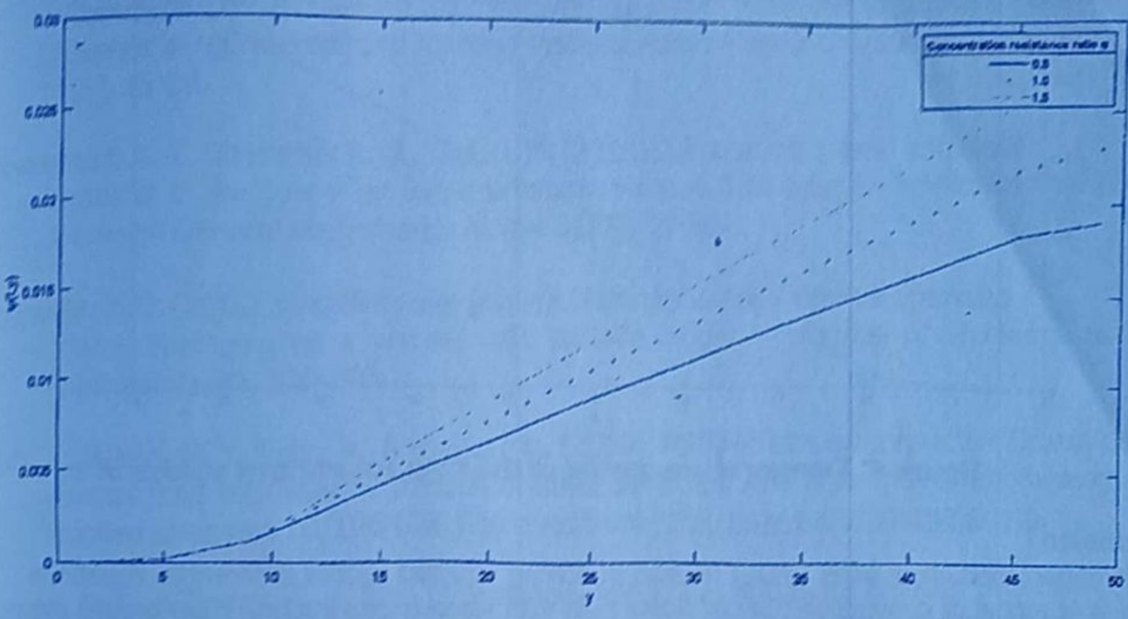

Figure 7: Velocity profile of the dust particles for various values of α

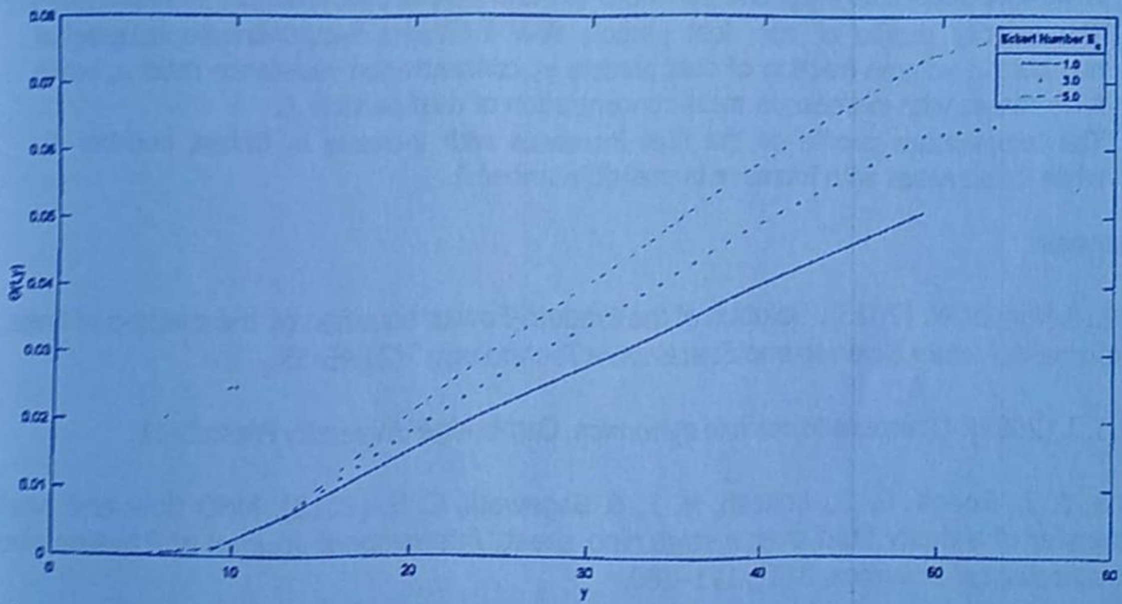

Figure 8: Temperature profile of the flow for various values of E_c

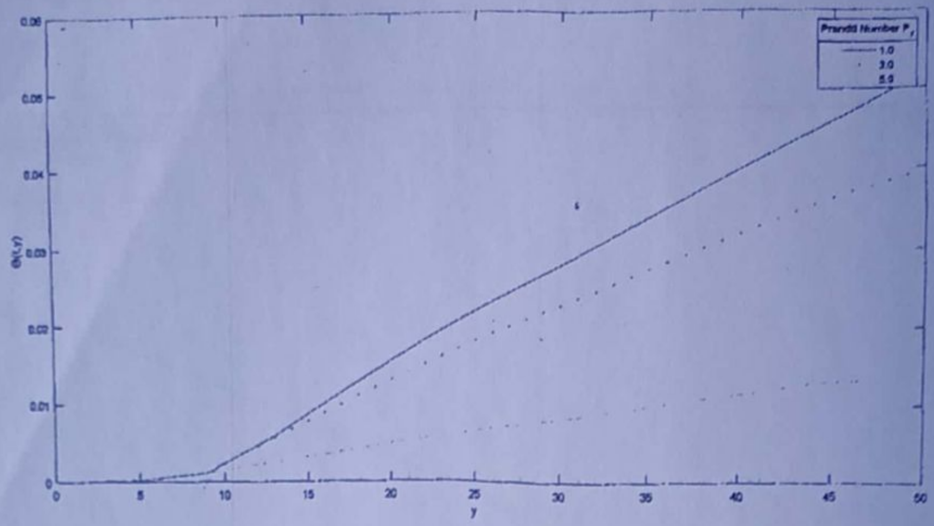

Figure 9: Temperature profile of the flow for various values of P_r

Conclusion

In this paper, method of lines (MOL) is used in solving coupled partial differential equations of the flow model of convective flow of dusty fluid with viscous dissipation. Then graphs are also obtained to examine the effects of different physical parameters on velocity profile of the fluid flow, velocity profile of the dust particle and the temperature profile. It is found that:

(1) The velocity profile of the fluid flow increases with increase in grashof number G_r while it decreases as magnetic parameter , M and porous parameter ε_3 increase.

(2) The velocity profile of the dust particle flow increases with increase in grashof number G_r, volume fraction of dust particle φ, concentration resistance ratio α, while it decreases with increase in mass concentration of dust particle f.

(3) The temperature profile of the flow increases with increase in Eckert number E_c while it decreases with increase in prandtl number P_r.

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