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ANALYSIS OF LEAKAGE OF NON-VISCOUS FLOW IN A PIPE USING MASS FLOW RATE

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ABSTRACT

A lot of work has been done on leakages in pipe. However, this study presents the analysis of transient flow parameters of non-viscous fluid within a pipe using mass flow rate. The model equation evolved was solved and considered under two situations of non-viscous flow with leakage and the case of no leakage in the pipe. The method of direct integration was applied and the results obtained were simulated and presented graphically using Maple 17 software. The analyses of the graphs show that the parameters considered in the simulation some of which are flow velocity, pressure, density, measured inlet mass flow, measured outlet mass flow and Reynolds number are useful tools in analysing leakages in a pipe notwithstanding the degree of leakage.

Keywords: Non-Viscous Flow, Fluid, Leakage and Mass Flow Rate.

INTRODUCTION

In today's world, pipeline system is one of the reliable, efficient and safest means of transporting hazardous fluids from where they are produced to where they are stored before being released to the end users (White *et al*., 2019). However, it is important to be cautious and guard against leakages because no matter how carefully the pipeline is designed and built, there is always the probability of leakage. A leakage is described as the amount of fluid which escapes from the pipeline systems by means other than through a controlled action (Ajao *et al*., 2018).

Leakage in distribution systems can be caused by a number of different factors. Some examples include – bad pipe connections, internal or external pipe corrosion or mechanical damage caused by excessive pipe load which could be traffic on the road above the pipes (Xiao *et al*., 2018). Other common factors that influence leakages are ground movement, high system pressure damage due to excavation, age of pipe, winter temperature, defects in pipes, ground conditions and poor quality of workmanship. The presence of leakage may damage the infrastructure, cause financial and energy losses, health risks and other havoc of monumental proportions (Puust *et al.,* 2010).

In their work, Oyedeko and Balogun (2015) presented a transient flow imbalance in the continuity and momentum equations. Measurements of flow parameters at inlet and outlet of pipeline were used in developing the model. With the model, leakages in pipeline were easy to detect during start-up and shut-down. However, the method depends greatly on the performance of the pipeline observer which is a demerit to the method and therefore unreliable. A system is considered to be reliable if actual leaks are detected in consistency without false alarms. Reliability is described as a measure of the capacity of the system of leak detection to give accurate decisions about the possible existence of a leak on the pipeline (Chinwuko *et al*., 2016). According to Geiger (2006), it follows that reliability is directly related to the probability to detect a leak, given that a leak truly exists, and to incorrectly declare a leak given that no leak has occurred.

Due to the inflammable nature of the fluids being transported through pipelines, efforts have been made and are continuously being made on various methods that can be applied in monitoring and detecting accurately leakage in fluid pipeline as this plays a key role in the possible prevention or minimization of the impact of its eventual occurrence (Mutiu *et al.,* 2019). This is the reason why this research work is being undertaken as part of efforts to review and refine the method used by Chinwuko *et al.* (2016). Some parameters which could be simulated to depict early and quick detection of very minute leakage were considered, varied and examined.

In this study, we solved and extended the model from Chinwuko *et al*. (2016) for leakage in pipeline. The model was modified to include no leak and leak situations for non-viscous fluids. The model equations governing the two situations were solved analytically using the method of direct integration. Graphs of the resulting solutions were subsequently plotted and analysed.

MATHEMATICAL FORMULATION

In formulating the model equations, relevant assumptions in line with Chinwuko *et al.* (2016) were made. The equations were formulated to depict two different cases for non-viscous fluids. These are: (i) when there is no leakage and (ii) when there is leakage in the pipe.

Model Equations

The following assumptions were made:

(i) Pipe cross-sectional area remains constant;
(ii) Isothermal and adiabatic flow;
(iii) One-dimensional flow (unidirectional)
(iv) No chemical reaction between the transporting fluid and internal wall of the pipe;
(v) Constant density throughout the pipeline segments
(vi) Homogenous fluid (i.e. either oil or gas) being transported in the pipeline.
(vii) Energy conservation equation is neglected, since the leak considered in the transient model affects only the downstream temperature at the fluid flowing velocity.

Based on the above assumptions, the continuity equation also known as the mass balance equation which is based on the law of conservation of mass for a one-dimensional flow is expressed as:

$$\frac{\partial(\rho)}{\partial t}+\frac{\partial(\rho u)}{\partial x}=0 \qquad (1)$$

When there is leakage in the pipe, the continuity equation becomes

$$\frac{\partial(\rho)}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\dot{M}_{leak}=0 \qquad (2)$$

where the $\dot{M}_{leak}$ is the leak rate (Tetzner, 2003) defined as:

$$\dot{M}_{leak}=M_I-\dot{M}_I-(M_0-\dot{M}_0) \qquad (3)$$

The leak position is also given as

$$x_{leak}=-\frac{(M_0-\dot{M}_0)}{\dot{M}_{leak}}L \qquad (4)$$

The momentum equation describes the force balance on the fluid within a segment of the pipeline.

From the Navier-Stokes equations, the conservation of momentum in one-dimensional flow (Perry *et al.*, 1999; Oyedeko and Balogun, 2015) is:

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}\right)=-\frac{\partial \rho}{\partial x}+\mu\frac{\partial^2 u}{\partial x^2} \qquad (5)$$

Substituting $p=P+\rho gH$ (Chinwuko *et al.*, 2016) into (3.6) and introducing the leak term gives

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}\right)=-\frac{\partial P}{\partial x}-\rho g\frac{\partial H}{\partial x}+\mu\frac{\partial^2 u}{\partial x^2}+\rho u_{leak} \qquad (6)$$

where u is the one-dimensional velocity of fluid, t is time, x is the spatial space, p is the static pressure, H is the elevation of the pipeline, ρ is the density of the fluid, A is the cross sectional area of the pipe, $\dot{M}_{Leak}$ is the leak rate, x_{Leak} is the leak position, L is the length of the pipe, g is the acceleration due to gravity, u_{Leak} is the leak velocity, U_0 is the reference flow velocity, M_0 is the estimated outlet mass flow rate, M_I is the estimated inlet mass flow rate, $\dot{M}_0$ is the measured outlet mass flow rate, $\dot{M}_I$ is the measured inlet mass flow rate. The following initial and boundary conditions are applied:

$$\left.\begin{aligned}u(x,0)&=0\\u(0,t)&=U_0\\u(L,t)&=0\end{aligned}\right\} \qquad (7)$$

Method of Solution

Assume the pattern of flow $\frac{\partial H}{\partial x}$ to be parabolic, then it implies that

$$\frac{\partial H}{\partial x}=\frac{H_0}{L}x\left(1-\frac{x}{L}\right) \qquad (8)$$

Then, equations (2) and (6) satisfying the boundary conditions in (7) become (9) and (10) respectively

$$\frac{\partial \rho}{\partial t}+\frac{\partial \rho u}{\partial x}+\dot{M}_{leak}=0 \qquad (9)$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}\right)=\frac{\partial P}{\partial x}-\frac{\rho gH_0}{L}x\left(1-\frac{x}{L}\right)+\mu\frac{\partial^2 u}{\partial x^2}+\rho u_{leak} \qquad (10)$$

Solution to the Equations

Changing equations (9) and (10) satisfying (7) to dimensionless equations, we have:

$$u' = \frac{u}{U_0}, t' = \frac{U_0 t}{L}, x' = \frac{x}{L}, P' = \frac{P}{\rho_0 U_0^2}, \rho' = \frac{\rho}{\rho_0} \quad (11)$$

The dimensionless equations (12) & (13) with initial and boundary conditions in (14) are obtained

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + R_{leak} = 0 \quad (12)$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) = -\frac{\partial P}{\partial x} - \beta x(1-x) + \frac{1}{R_\ell}\frac{\partial^2 u}{\partial x^2} + V_{leak} \quad (13)$$

$$\left.\begin{aligned} u(x,0) &= 0 \\ u(0,t) &= 1 \\ u(1,t) &= 0 \end{aligned}\right\} \quad (14)$$

where R_{leak} is the leak rate; V_{leak} is the leak velocity; β is the elevation of the pipeline and R_e is the Reynolds number for the flow.

Equations (12) and (13) using (14) will now be considered under two (2) cases or subheadings namely - non-viscous flow without leakage and non-viscous flow with leakage. It is important to note the following:

1. If there is no leak, the flow is steady and the transient pressure drop per unit length and the mass flow rate along the pipeline is constant i.e., no leak implied, therefore

$$\frac{\partial P}{\partial x} = f\left(\dot{M}_I\right) = \dot{M}_I = \text{constant} \quad (15)$$

2. If a leak occurs, the mass flow rate upstream of the leak will be greater than the mass flow rate downstream of the leak. Therefore, the pressure drop per unit length upstream of the leak will also be greater than pressure drop downstream of the leak. This implies:

$$\left.\frac{\partial P}{\partial x}\right|_{0\le x\le x_{leak}} - \left.\frac{\partial P}{\partial x}\right|_{x_{leak}<x\le L} = f(\dot{M}_I) - f(\dot{M}_0) = \dot{M}_I + (\dot{M}_0 - \dot{M}_I)x > 0 \quad (16)$$

Case 1: Non-viscous flow without leakage

In this case, equations (12) and (13) using (14) reduce to:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} = 0 \quad (17)$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right) = -\dot{M}_I - \beta x(1-x) \quad (18)$$

$$\left.\begin{aligned} u(x,0) &= 0 \\ u(0,t) &= 1 \\ u(1,t) &= 0 \end{aligned}\right\} \quad (19)$$

Analytical solution of case 1

Eliminating equation (17) by means of streamlines function (Olayiwola, 2011), we have

$$\eta(x,t) = (\rho^2)^{-\frac{1}{2}} \int_0^x \rho(s,t)\, ds \quad (20)$$

therefore, $x = x(t)$ becomes $\eta = \eta(x,t)$

Let $u = u(x,t)$ such that $\eta = \eta(x,t)$ and $x = x(t)$ then by chain rule, we have:

$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial t} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial t} \quad (21)$$

and

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial x} + \frac{\partial u}{\partial t}\frac{\partial t}{\partial x} \quad (22)$$

From the continuity equation (17), we have:

$$u = -\frac{1}{\rho}\int_0^x \frac{\partial \rho}{\partial t} ds \quad (23)$$

Differentiating equation (20) with respect to t, we have:

$$\frac{\partial \eta}{\partial t} = -u \quad (24)$$

and with respect to x, we have $\frac{\partial \eta}{\partial x}=1$ (25)

Then, the coordinate transformations become:

$$\frac{\partial}{\partial x}\rightarrow\frac{\partial}{\partial \eta}.\frac{\partial \eta}{\partial x}=\frac{\partial}{\partial \eta} \quad (26)$$

$$\frac{\partial}{\partial t}\rightarrow\frac{\partial}{\partial \eta}.\frac{\partial \eta}{\partial t}+\frac{\partial}{\partial t}=\frac{\partial}{\partial t}-u\frac{\partial}{\partial \eta} \quad (27)$$

Using equations (26) and (27), then equations (18) can be simplified to:

$$\rho\frac{\partial u}{\partial t}=-\dot{M}_I-\beta\eta(1-\eta) \quad (28)$$

Integrating (28) with respect to t, we have

$$u(\eta,t)=-\frac{1}{\rho}\left(\dot{M}_I+\beta\eta(1-\eta)\right)t \quad (29)$$

Case 2: Non-viscous flow with leakage

In this case, equations (12) and (13) become

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+R_{Leak}=0 \quad (30)$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}\right)=-\left(\dot{M}_I+\left(\dot{M}_0-\dot{M}_I\right)x\right)-\beta x(1-x)+V_{Leak} \quad (31)$$

Analytical solution of case 2

Using equations (20)-(22) and considering the continuity equation (30), we obtain

$$u=-\left(\frac{1}{\rho}\int_0^x\frac{\partial \rho}{\partial t}ds+\frac{R_{Leak}}{\rho}\eta\right) \quad (32)$$

This implies $-\left(u+\frac{R_{Leak}}{\rho}\eta\right)=\frac{1}{\rho}\int_0^x\frac{\partial \rho}{\partial t}ds$ (33)

Recall (23) and (24), then $\frac{\partial \eta}{\partial t}=-\left(u+\frac{R_{Leak}}{\rho}\eta\right)$ (34)

Also, recall from (25) that $\frac{\partial \eta}{\partial x}=1$ (35)

Then, the coordinates transformation become

$$\frac{\partial}{\partial x}=\frac{\partial}{\partial \eta} \quad (36)$$

$$\frac{\partial}{\partial t}=\frac{\partial}{\partial t}-\left(u+\frac{R_{Leak}}{\rho}\eta\right)\frac{\partial}{\partial \eta} \quad (37)$$

Using (36) and (37), then equations (30)-(31) can be simplified as

$$\rho\left(\frac{\partial u}{\partial t}-\frac{R_{Leak}}{\rho}\eta\frac{\partial u}{\partial \eta}\right)=-\left(\dot{M}_I+\left(\dot{M}_0-\dot{M}_I\right)\eta\right)-\beta\eta(1-\eta)+V_{Leak} \quad (38)$$

Suppose the solution $u(\eta,t)$ can be expressed as

$$u(\eta,t)=u_0(\eta,t)+R_{Leak}u_1(\eta,t) \quad (39)$$

Then, equation (38) becomes

$$\rho\left(\frac{\partial}{\partial t}(u_0+R_{Leak}u_1)-\frac{R_{Leak}}{\rho}\eta\frac{\partial}{\partial \eta}(u_0+R_{Leak}u_1)\right)=-\left(\dot{M}_I+\left(\dot{M}_0-\dot{M}_I\right)\eta\right)-\beta\eta(1-\eta)+V_{Leak} \quad (40)$$

Collecting the like powers of R_{Leak}, we have

$R_{Leak}{}^0$:

$$\rho\frac{\partial u_0}{\partial t}=-\left(\dot{M}_I+\left(\dot{M}_0-\dot{M}_I\right)\eta\right)-\beta\eta(1-\eta)+V_{Leak} \quad (41)$$

$$\left.\begin{array}{l} u_0(\eta,0)=0 \\ u_0(0,t)=1 \\ u_0(1,t)=0 \end{array}\right\} \quad (42)$$

$$R_{Leak}{}^1:\ \rho\left(\frac{\partial u_1}{\partial t}-\frac{1}{\rho}\eta\frac{\partial u_0}{\partial \eta}\right)=0 \quad (43)$$

$$\left.\begin{array}{l} u_1(\eta,0)=0 \\ u_1(0,t)=0 \\ u_1(1,t)=0 \end{array}\right\} \quad (44)$$

Solving equations (41)-(44) we obtain

$$u(\eta,t)=\frac{1}{\rho}\left(\dot{M}_i+\left(\dot{M}_0-\dot{M}_i\right)\eta\right)t-\frac{\beta}{\rho}\eta(1-\eta)t+V_{Leak}t+R_{Leak}\left(\frac{1}{\rho^2}\eta\left(\dot{M}_0-\dot{M}_i\right)\frac{t^2}{2}-\frac{\beta}{\rho^2}\eta(1-2\eta)\frac{t^2}{2}\right) \quad (45)$$

RESULTS AND DISCUSSION

In analysing the solution, the measured inlet mass flow rate (m_i), measured outlet mass flow rate (m_0), elevation, the leak rate (R_{leak}) and leak velocity (V_{leak}) were varied and examined. Their effects on the flow distribution were as presented in the graphs.

4.1 Analysis of Results

The graphs of some of the different cases are given in the following illustrations.

CASE 1: Non-viscous flow without leakage

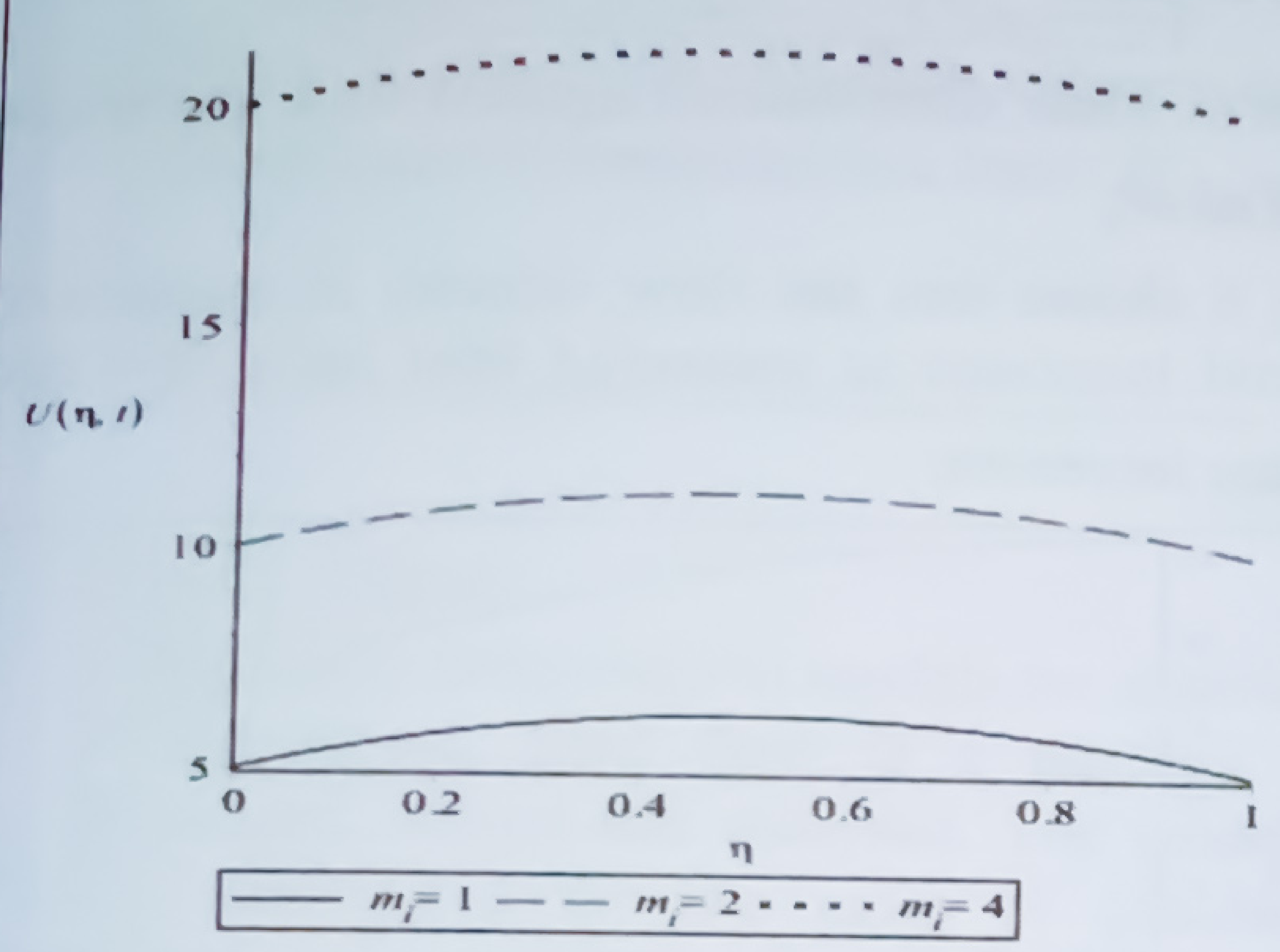

Figure 1: Flow distribution against distance at various values of m_i

Graph of flow velocity against distance for various values of measured inlet mass flow rate m_i. It is observed that the flow velocity increases and later decreases with the distance.

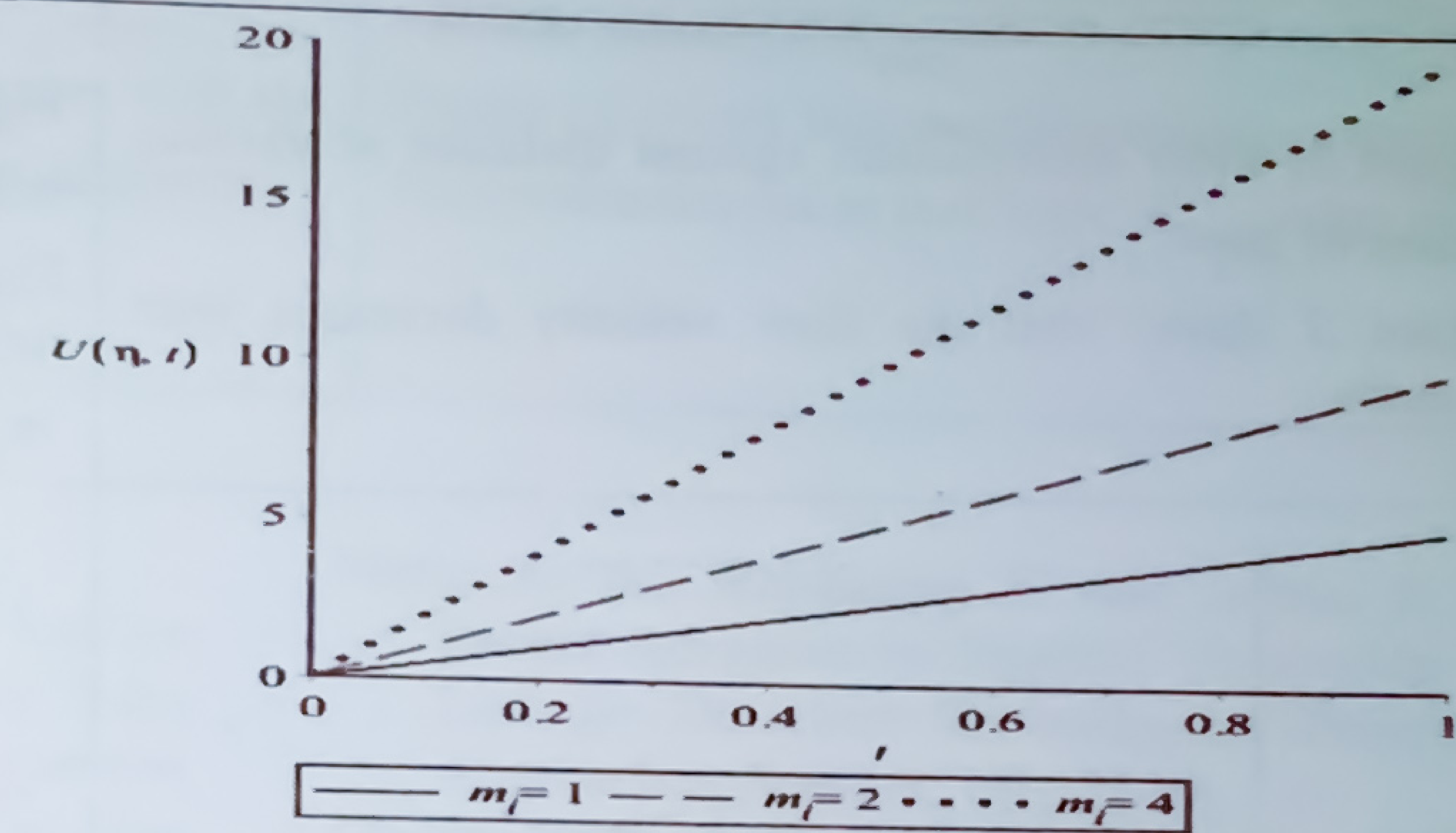

Figure 2: Flow distribution against time at various values of m_i

Figure 2 shows that the flow velocity increases with time and measured inlet mass flow rate m_i

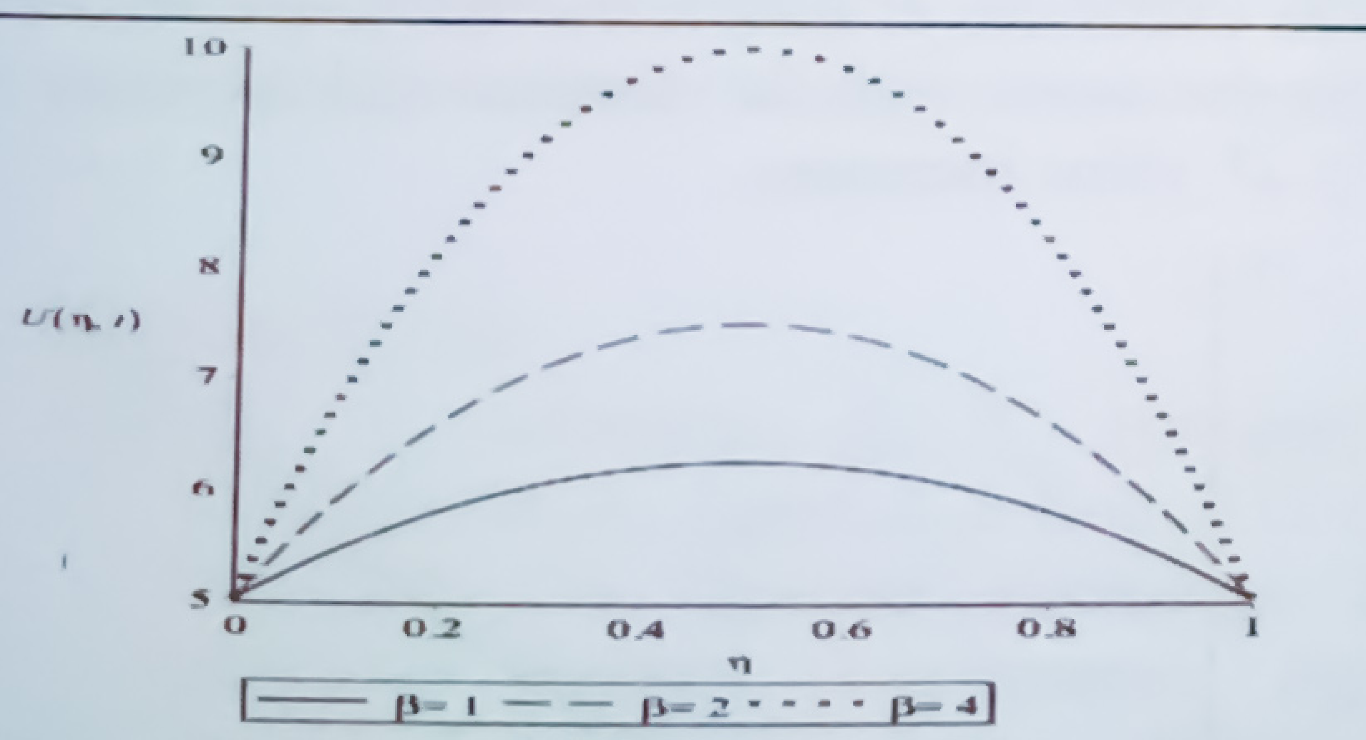

Figure 3: Flow distribution against distance at various values of β

Observation: The flow velocity increases as elevation value increases and later decreases with distance.

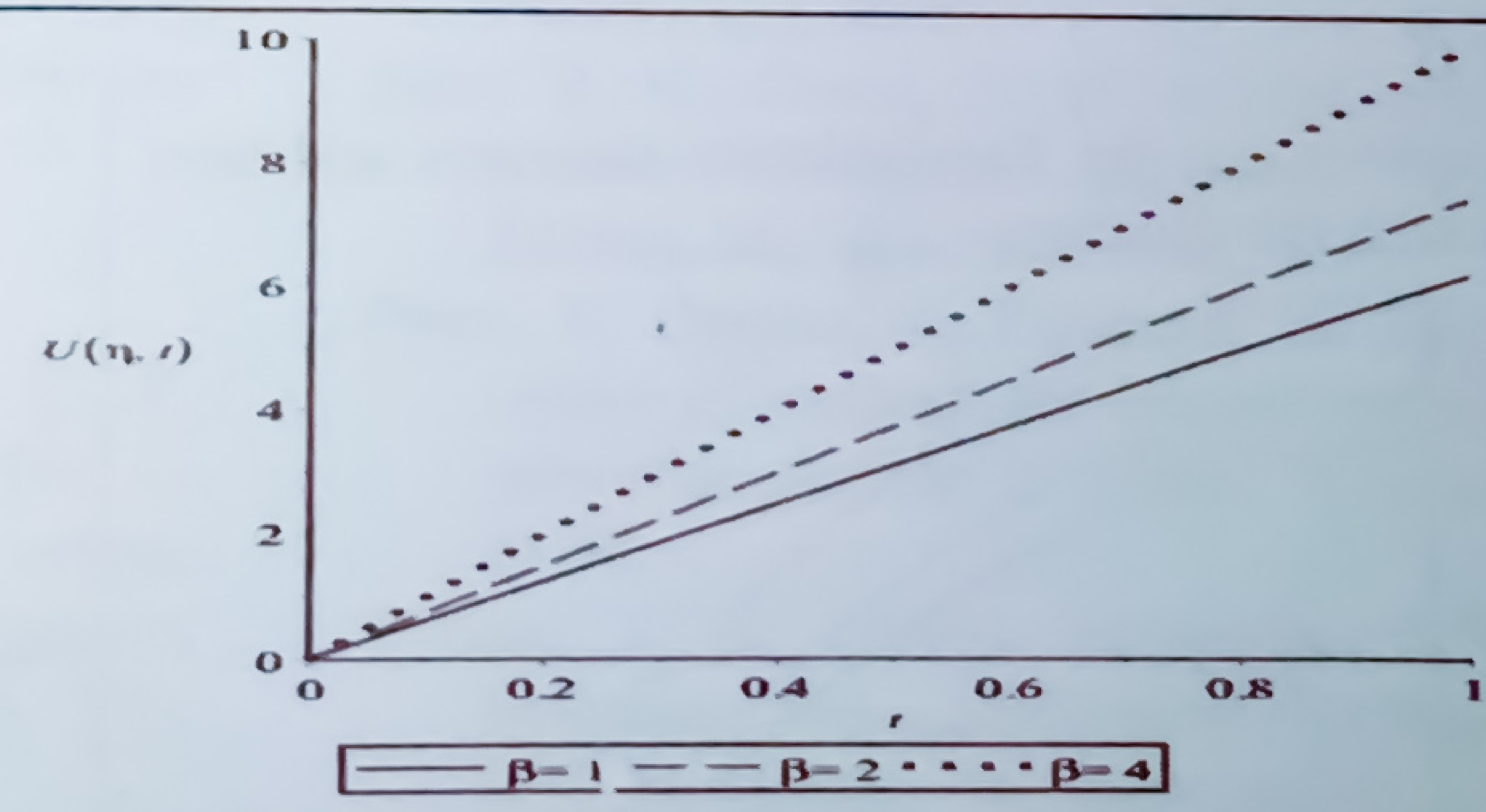

Figure 4: Flow distribution against time at various values of β

Observation: The flow velocity increases with time and elevation value.

CASE 2: Non-viscous flow with leakage

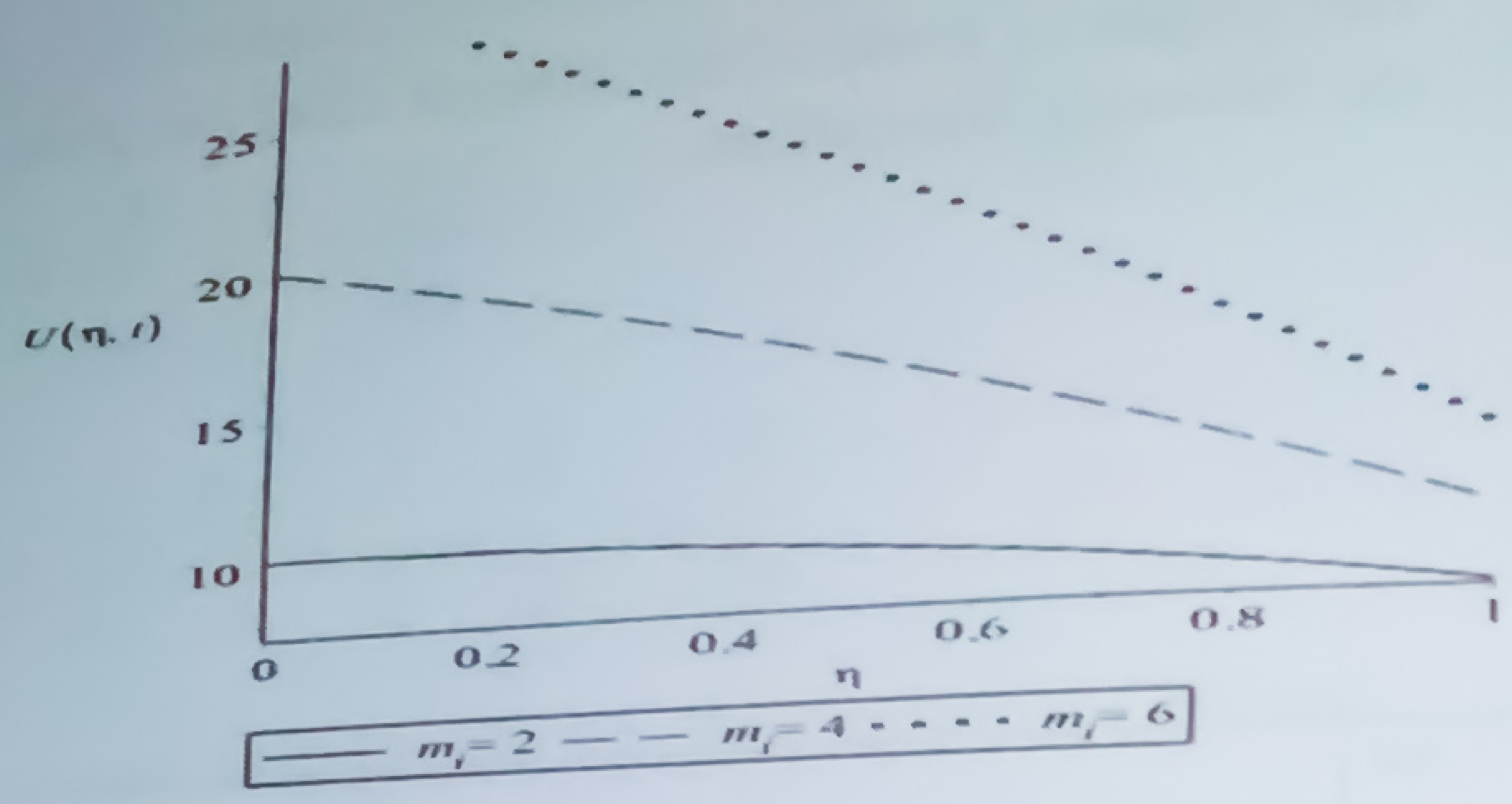

Figure 5: Flow distribution against distance at various values of m_i

Figure 5 shows that the flow velocity decreases with distance.

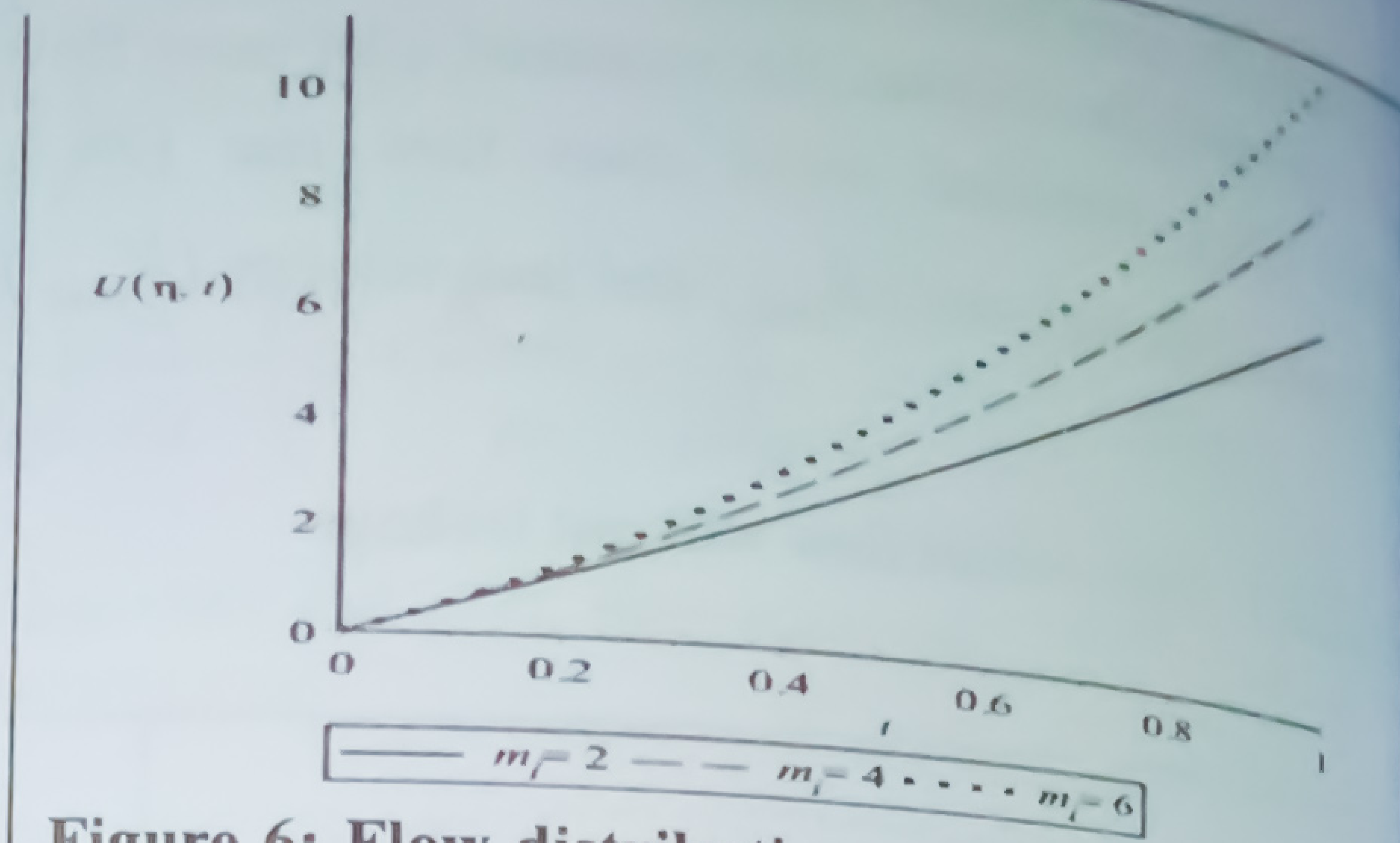

Figure 6: Flow distribution against time at various values of m_i

Figure 6 shows that the flow velocity increases with time and increases as measured inlet mass flow rate m_i value increases.

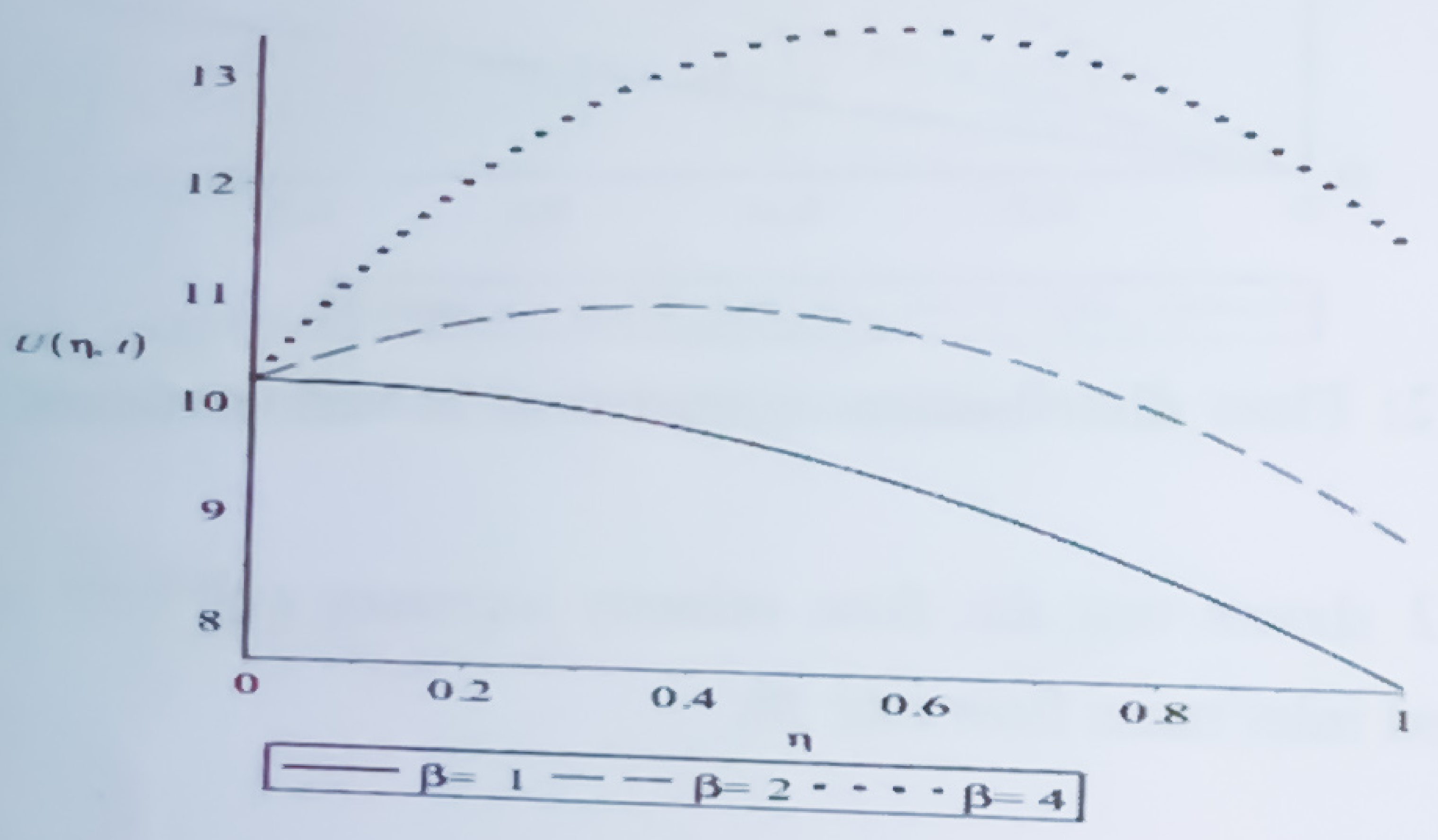

Figure 7: Flow distribution against distance at various values of β

Figure 7 shows that the flow velocity increases and later decreases with the distance.

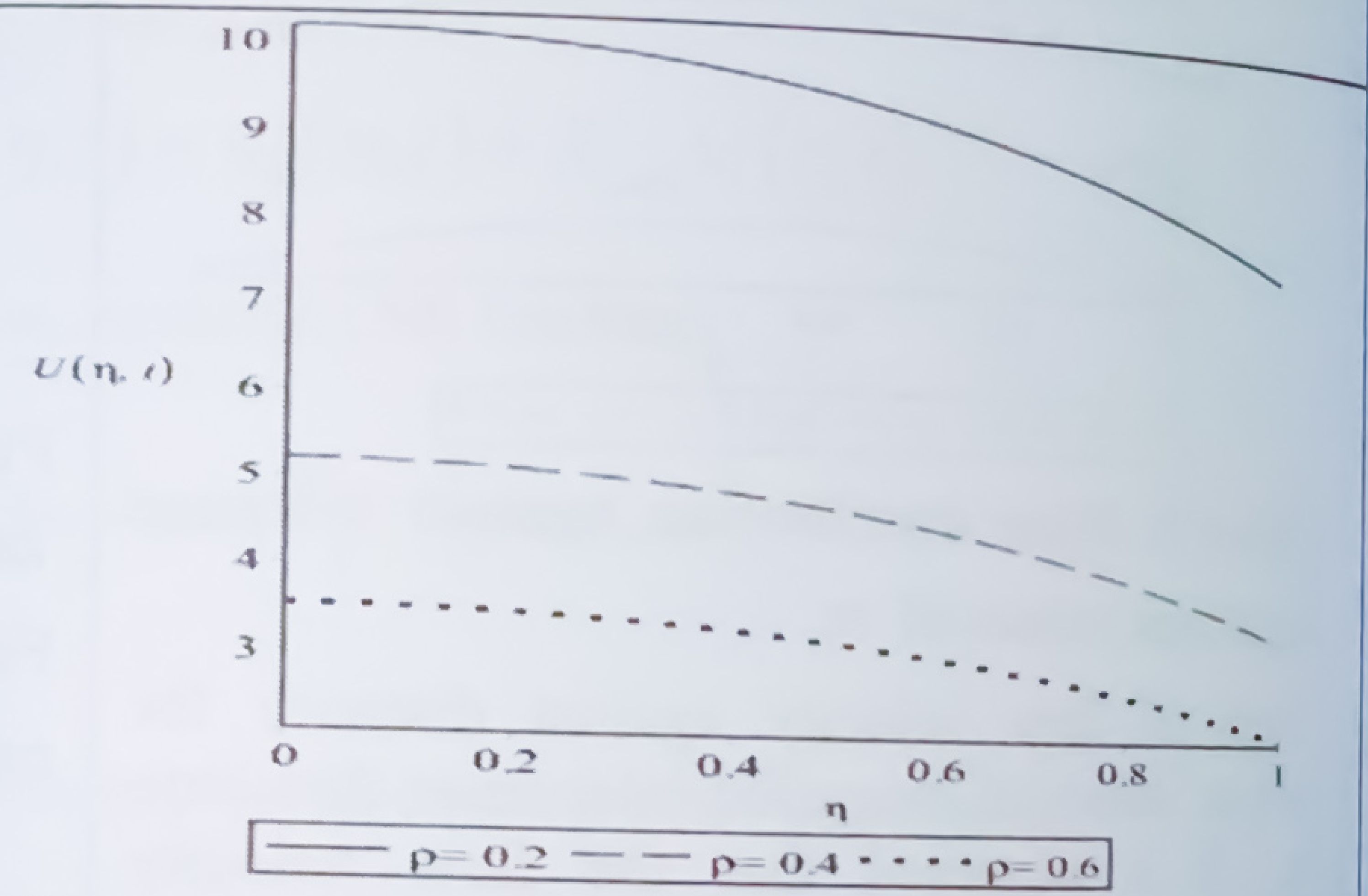

Figure 8: Flow distribution against distance at various values of ρ Figure 8 shows that the flow velocity decreases with the distance and decreases as density ρ value increases.

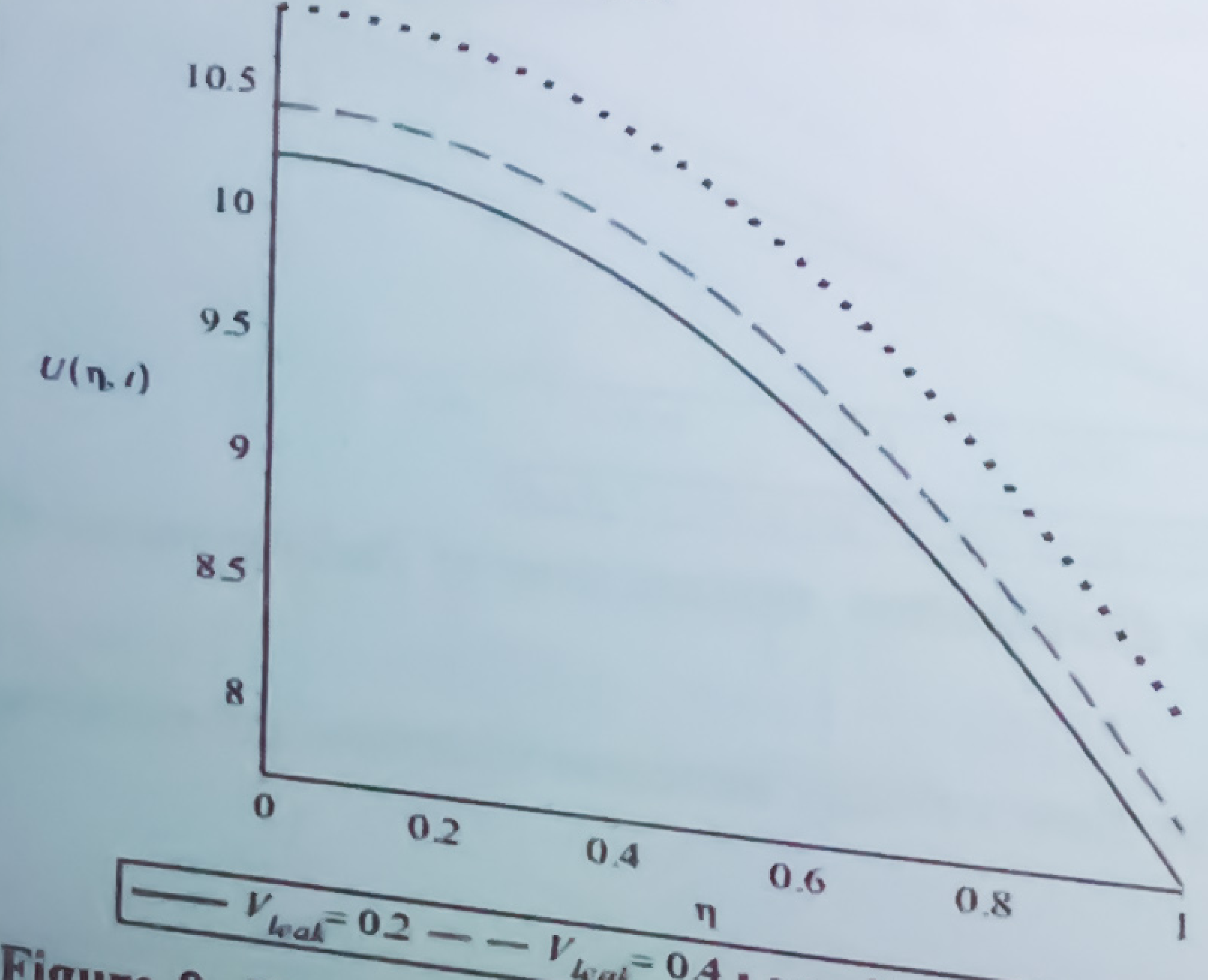

Figure 9: Flow distribution against distance at various values of V_{leak}

Figure 9 shows that the flow velocity decreases with the distance.

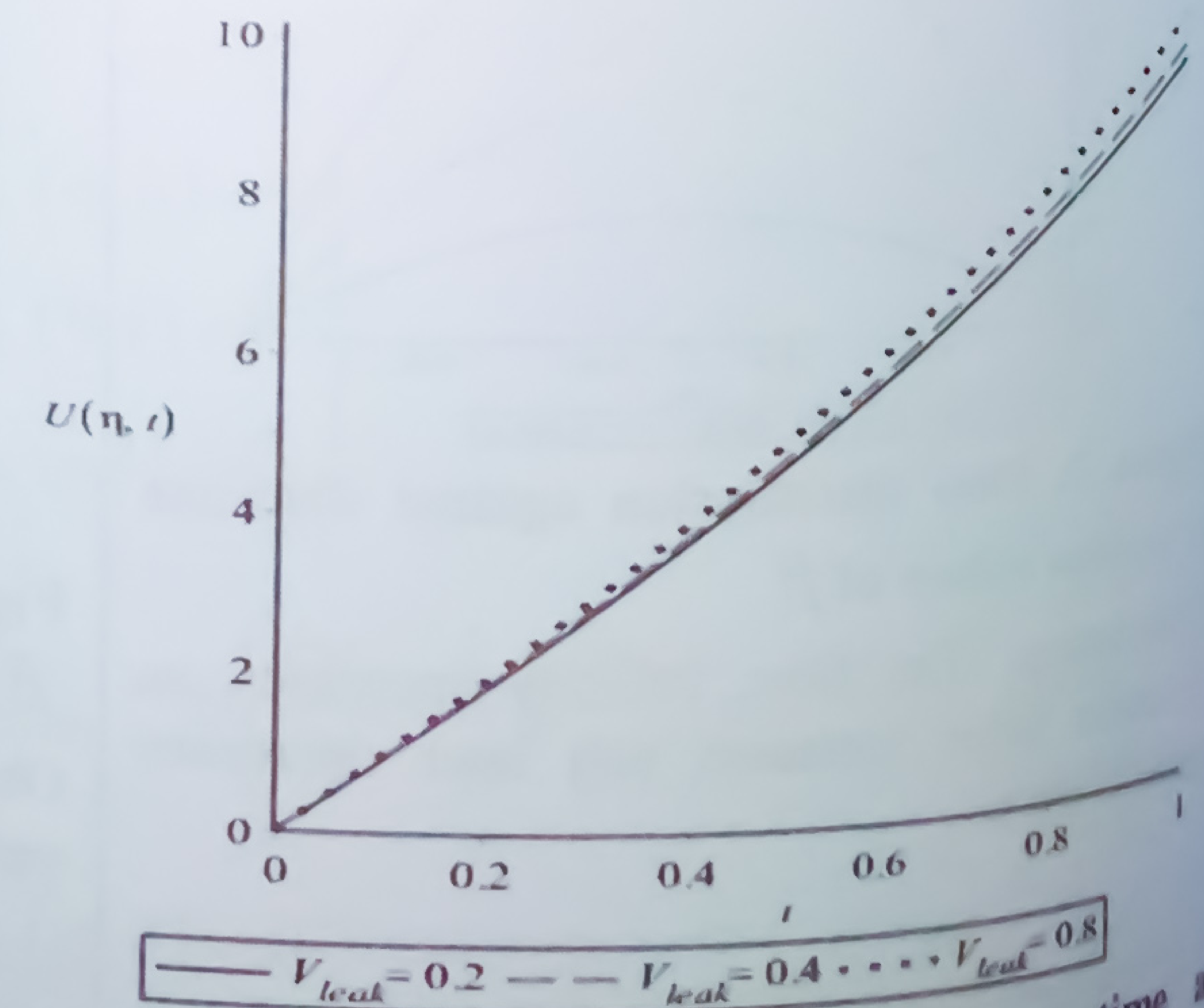

Figure 10: Flow distribution against time at various values of V_{leak}

Figure 10 shows that the flow velocity increases with time and increases as leak velocity V_{leak} value increases.

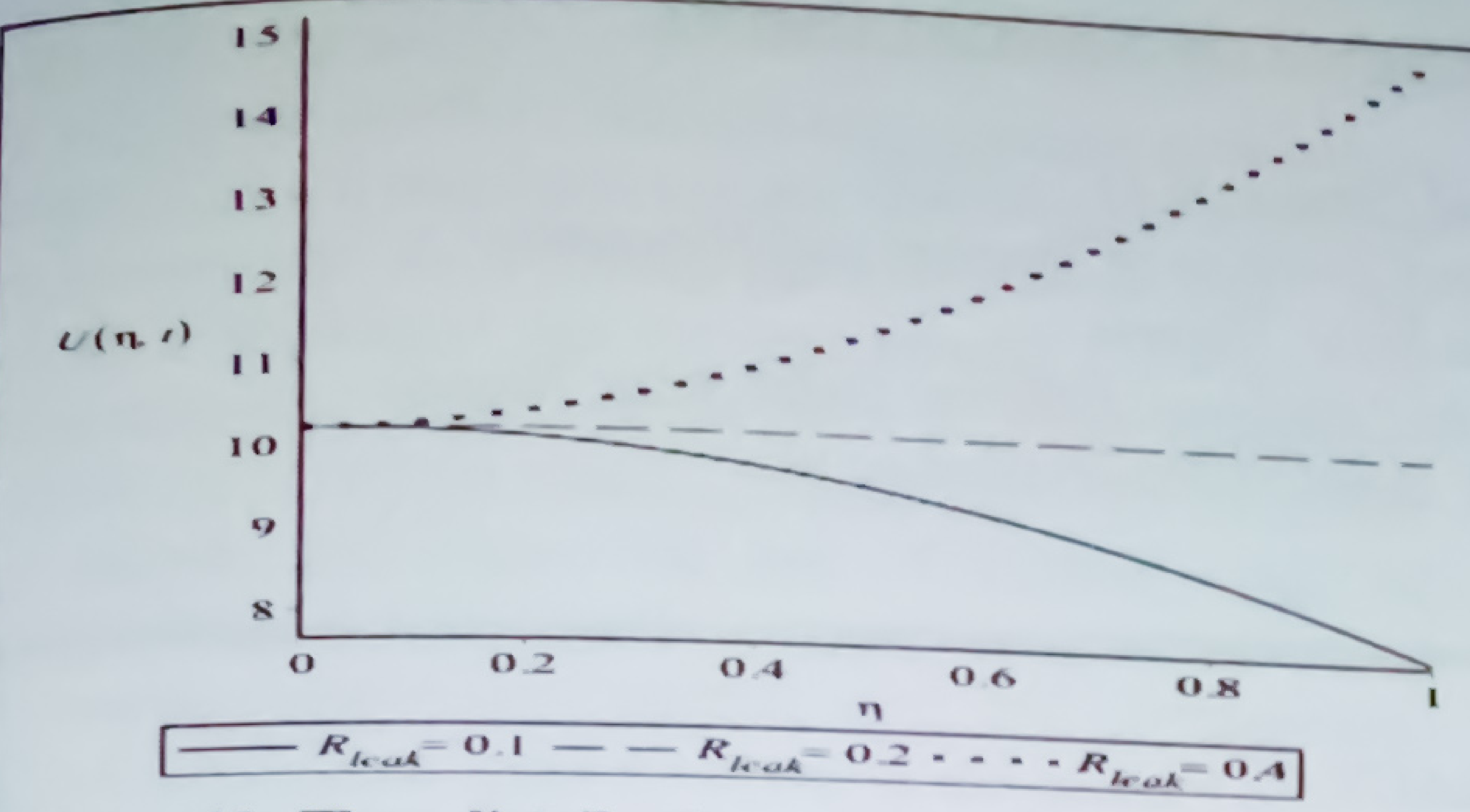

Figure 11: Flow distribution against distance at various values of R_{leak}

Figure 11 shows that the flow velocity decreases with the distance and increases as leak Rate R_{leak} value increases.

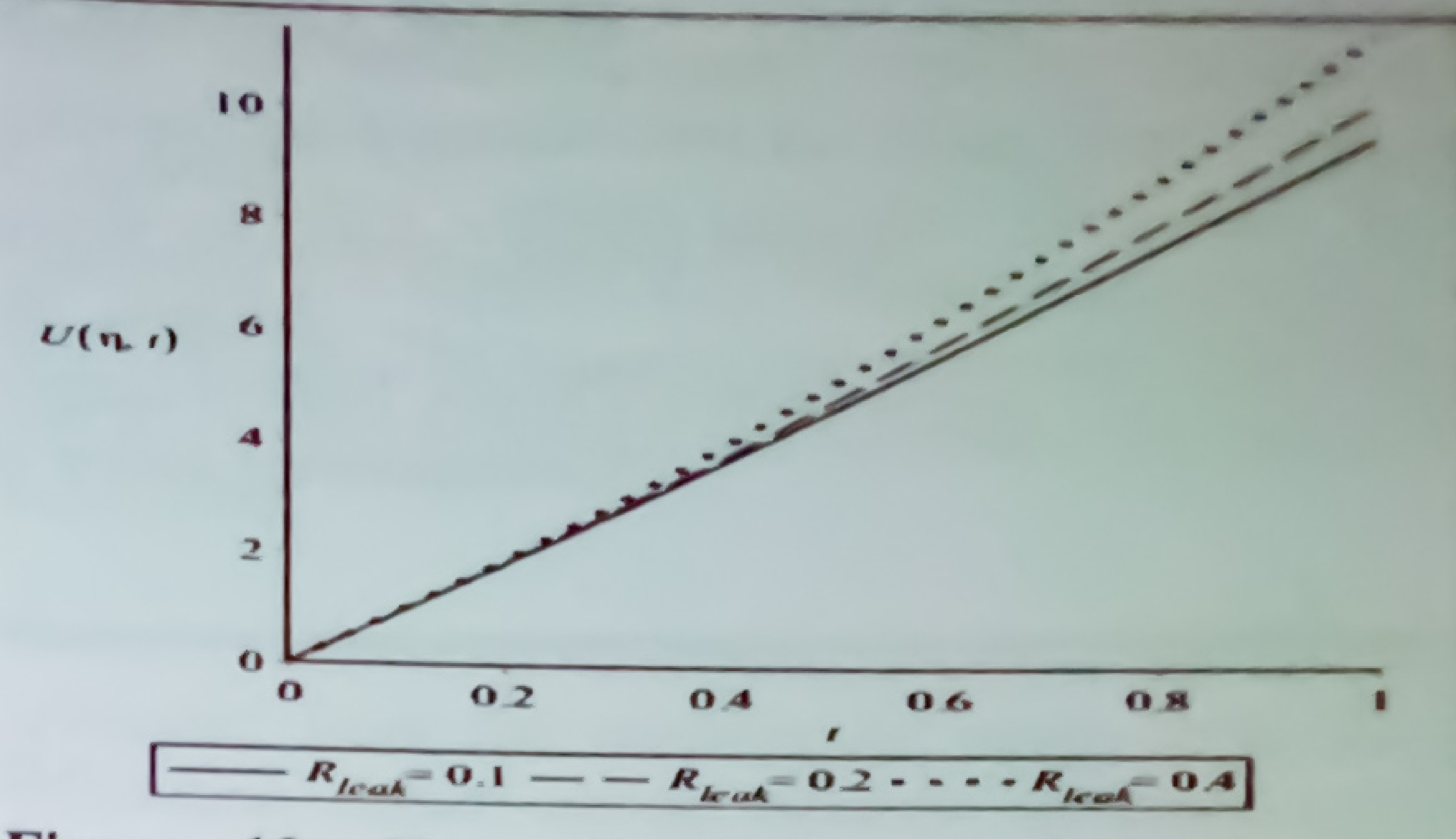

Figure 12: Flow distribution against time at various values of R_{leak}

Figure 12 shows that the flow velocity increases with time and increases as leak Rate R_{leak} value increases.

CONCLUSION

In this study, mathematical models for detecting leakage of non-viscous fluid flow in a pipeline have been formulated, solved and analysed. The model equations were considered for two cases of non-viscous flow without leakage and non-viscous flow with leakage in a pipe. From the solutions obtained, it could be observed that the parameters representing density, flow velocity, inlet mass flow rate, measured outlet mass flow rate, elevation and leak velocity in the equations gave the pattern of variation of each parameter. The information has shown that those parameters are not only fundamental but useful tools in describing and analysing the type and position of leakage of non-viscous fluid in a pipe.

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