FEDERAL UNIVERSITY OF TECHNOLOGY MINNA
Technology for Empowerment

SPCBIC 2021

25th – 28th October, Minna Nigeria

3RD SCHOOL OF PHYSICAL SCIENCES BIENNIAL INTERNATIONAL CONFERENCE (SPSBIC 2021)

PROCEEDINGS

THEME:
THE ROLE OF SCIENCE AND TECHNOLOGY IN THE REALIZATION OF RESEARCH AND DEVELOPMENT IN THE ERA OF GLOBAL PANDEMIC

FEDERAL UNIVERSITY OF TECHNOLOGY MINNA,
NIGER STATE, NIGERIA

Analysis of T_1 *and* T_2 Relaxation Times from Bloch Equation for the Estimation of Age of Human Organs

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Abstract
As one of the preferred diagnostic imaging tools, Magnetic Resonance Imaging (MRI) has become a diagnostic modality which has made an in-road into age estimation. However, many works in this area were carried out using observed statistical data to classify and analyze findings. In this research work, a time-independent non-homogenous linear differential equation from the Bloch Nuclear Magnetic Resonance (NMR) equations is evolved. The equation is solved under the influence of radio frequency magnetic field $[rf B_1(x,t) \neq 0]$ *and in the absence of radio frequency magnetic field* $[rf B_1(x,t) = 0]$. T_1 *and* T_2 *relaxation times were varied with a view to analyze the signals as it relates to the age of any human organ.*

Keywords: T_1 relaxation time, T_2 relaxation time, radio frequency field, Magnetic resonance fingerprinting, magnetization.

1.0 Introduction

Relaxation usually means the return of a perturbed system into equilibrium. Tissue can be characterized by two different relaxation times – T_1 *and* T_2 . T_1(Longitudinal relaxation time) is the time constant which determines the rate at which excited protons return to equilibrium. It is a measure of the time taken for spinning protons to realign with the external magnetic field. The T_1 relaxation time, also known as the spin-lattice relaxation time, is a measure of how quickly the net magnetization vector (NMV) recovers to its ground state. The return of excited nuclei from the high energy state to the low energy or ground state is associated with loss of energy to the surrounding nuclei. Nuclear magnetic resonance (NMR) was originally used to examine solids in the form of lattices, hence the name "spin-lattice" relaxation. Two other forms of relaxation are the T_2relaxation time (spin-spin relaxation) and T_2* relaxation (Rock, 2021).

These two tissue parameters, T_1 *and* T_2 represent different tissue information that is largely independent of each other. Nevertheless, T_1 *and* T_2 information is not fully independent of each other because all spin–lattice interactions that cause T_1 recovery also contribute to T_2decay (Suzuki *et al.*, 2006). In a study carried out by Yusuf (2010), the two different relaxation times – T_1 *and* T_2 were used to carry out the general analysis of physiological flow in human body. The results show that the two relaxation parameters are very vital in describing and analyzing the flow of fluids in human body and the surrounding tissues.

Magnetic resonance fingerprinting (MRF) is a method that simultaneously and rapidly measures multiple tissue properties, with initial application in measuring T_1 *and* T_2. This technique is based on the premise that acquisition parameters can be varied in a pseudorandom manner such that each combination of tissue properties will have a unique signal evolution. Using the Bloch equations, a dictionary of all possible signal evolutions can be created that includes all known acquisition parameters and all possible range of values and combination of the properties of interest (Badve *et al.*, 2015).

Badve *et al.* (2015) presented simultaneous quantification of regional brain $T_1 and\ T_2$ relaxation times in healthy volunteers using MRF and assess differences in tissue properties resulting from age, sex, and laterality of hemispheres. They further compare different best-fit options for regression analysis of age and brain relaxometry and assess how age-sex interactions affect these findings in the context of the known literature on relaxometry measurements with aging.

The study of the living human brain has shown that at low nuclear MR frequencies, age-related factors constitute an important influence on the T_1relaxation time. The findings demonstrate the importance of understanding the variation of $T_1 and\ T_2$ in relation to age and to the localization in the brain. For instance, in the investigation of disease, the contrast resolution between the tissues changed by disease and the normal tissue might be different, depending on the normal tissue variation in relaxation times (Agartz *et al.*, 1991).

It is in line with these that this research work is aim at investigating further the effects of $T_1$and T_2 on the estimation of age of human organs.

2.0 Mathematical Formulation

Bloch equations explain the magnetization properties of matters using relaxation times. They are given as:

$$\frac{dM_x}{dt} = -\frac{M_x}{T_2}$$

2.1

$$\frac{dM_y}{dt} = \gamma M_z B_1(t) - \frac{M_y}{T_2}$$

2.2

$$\frac{dM_z}{dt} = -\gamma M_y B_1(t) - \frac{M_z - M_o}{T_1}$$

2.3

where M_o= equilibrium magnetization

M_x= component of transverse magnetization along the x-axis

M_y= component of transverse magnetization along y-axis

M_z= component of magnetization along the field (z -axis)

γ = gyro-magnetic ratio of fluid spins

$B_1(t)$ = radio-frequency (RF) magnetic field

T_1 = Longitudinal or spin lattice relaxation time

T_2 = Transverse or spin-spin relaxation time

From the kinetic theory of moving fluids, given a property M of fluid then the rate at which this property changes with respect to a point moving along with the fluid be total derivative

$$\frac{dM}{dt} = \frac{\partial M}{\partial t} + \frac{\partial M}{\partial x} v_x + \frac{\partial M}{\partial y} v_y + \frac{\partial M}{\partial z} v_z \quad 2.4$$

$$\frac{dM}{dt} = \frac{\partial M}{\partial t} + v.\nabla M \quad 2.5$$

Therefore, from Yusuf *et al. (2019)* the three Bloch equations become:

$$\frac{dM_x}{dt} = \frac{\partial M_x}{\partial t} + v.\nabla M_x = -\frac{M_x}{T_2} \quad 2.6$$

$$\frac{dM_y}{dt} = \frac{\partial M_y}{\partial t} + v.\nabla M_y = \gamma M_z \beta_1 - \frac{M_y}{T_2} \quad 2.7$$

$$\frac{dM_x}{dt} = \frac{\partial M_x}{\partial t} + v.\nabla M_x = -\gamma M_y \beta_1 - \frac{(M_z - M_o)}{T_1} \quad 2.8$$

If the flow is considered only along x-direction. The partial derivative along y and z direction is zero. This implies that the flow is constant along y and z directions. Therefore,

$$v.\nabla M_x = [v_i].\left[\frac{\partial}{\partial x} i\right] M = v\frac{\partial M_x}{\partial x} \quad 2.9$$

Equations 2.6 to 2.8 become

$$\frac{dM_x}{dt} = \frac{\partial M_x}{\partial t} + v\frac{\partial M_x}{\partial x} = -\frac{M_x}{T_2} \quad 2.10$$

$$\frac{dM_y}{dt} = \frac{\partial M_y}{\partial t} + v\frac{\partial M_y}{\partial x} = \gamma M_z \beta_1 - \frac{M_y}{T_2} \quad 2.11$$

$$\frac{dM_z}{dt} = \frac{\partial M_z}{\partial t} + v\frac{\partial M_z}{\partial x} = -\gamma M_y \beta_1 - \frac{(M_z - M_o)}{T_1} \quad 2.12$$

By making M_z the subject from *2.12* and simplifying the terms, we have

$$v^2\frac{\partial^2 M_y}{\partial x^2} + \frac{\partial^2 M_y}{\partial t^2} + 2v\frac{\partial^2 M_y}{\partial x \partial t} + \left[\frac{1}{T_1} + \frac{1}{T_2}\right]\frac{\partial M_y}{\partial t} + v\left[\frac{1}{T_1} + \frac{1}{T_2}\right]\frac{\partial M_y}{\partial x} + \left[\frac{1}{T_1 T_2} + \gamma^2 \beta_1^2\right] M_y = \frac{\gamma \beta_1 M_o}{T_1}$$

- Awojoyogbe *et al.* (2009)

2.13

2.1 Time-independent Bloch-flow equation

For time-independent Bloch-flow equation, we have.

$$v^2\frac{\partial^2 M_y}{\partial x^2} + v\left[\frac{1}{T_1} + \frac{1}{T_2}\right]\frac{\partial M_y}{\partial x} + \left[\frac{1}{T_1 T_2} + \gamma^2 \beta_1^2\right] M_y = \frac{\gamma \beta_1 M_o}{T_1} \quad 2.14$$

Let $\left[\frac{1}{T_1}+\frac{1}{T_2}\right] = T_o$ and $\frac{1}{T_1T_2} = k$ 2.15

$$\frac{\partial^2 M_y}{\partial x^2} + \frac{1}{v}T_o\frac{\partial M_y}{\partial x} + \frac{1}{v^2}[k+\gamma^2\beta_1^2]M_y = \frac{\gamma\beta_1 M_o}{T_1 v^2} \quad 2.16$$

Solving *2.16*,

The homogeneous part implies that $\frac{\gamma\beta_1 M_o}{T_1 V^2} = 0$ 2.17

Assuming $M_y = y$; $\frac{1}{V}T_o = p$; $\frac{1}{V^2}[k+\gamma^2\beta_1^2] = q$ then 2.18

$$\frac{d^2y}{dx^2} + p\frac{dy}{dx} + qy = 0 \quad 2.19$$

It can be seen that *2.16* has been changed to *2.19* which is a system of second order Ordinary Differential Equation (ODE).

The complementary solutions with the different cases are given as:

Case 1: $\frac{\sqrt{b^2-4ac}}{2} > 0$; $y_c = c_1e^{m_1x} + c_2e^{m_2x}$ where c_1, c_2 are constants 2.20

Case 2: $\frac{\sqrt{b^2-4ac}}{2} = 0$; $y_c = A(1+x)e^{mx}$ where $A =$ constant 2.21

Case 3: $\frac{\sqrt{b^2-4ac}}{2} < 0$; $y_c = c_1e^{(a+ib)x} + c_2e^{(a-ib)x}$ where c_1, c_2 are constants

2.22

$$m_1 = -\frac{p}{2} + \frac{\sqrt{p^2-4q}}{2} \quad 2.23$$

Recall we set $\frac{1}{v}T_o = p, \frac{1}{v^2}[k+\gamma^2\beta_1^2] = q m_1 = -\frac{\frac{1}{v}T_o}{2} + \frac{1}{2}\sqrt{\frac{1}{v^2}T_o^{\,2} - \frac{4}{v^2}[k+\gamma^2\beta_1^2]}$
2.24

Simplifying

$$m_1 = -\frac{1}{2v}\left(T_o - \sqrt{T_o^{\,2} - 4[k+\gamma^2\beta_1^2]}\right) \quad 2.25$$

Such that

$$m_2 = -\frac{1}{2v}\left(T_o + \sqrt{T_o^{\,2} - 4[k+\gamma^2\beta_1^2]}\right) \quad 2.26$$

2.1.1 Case 1

For case 1: $\frac{\sqrt{b^2-4ac}}{2} > 0$; 2.27

$$y_c = c_1e^{-\frac{1}{2v}\left(T_o - \sqrt{T_o^{\,2}-[k+\gamma^2B_1^2]}\right)x} + c_2e^{-\frac{1}{2v}\left(T_o + \sqrt{T_o^{\,2}-[k+\gamma^2B_1^2]}\right)x} \quad 2.28$$

Solving for the particular solution, using the technique of variation of parameters.

$$y_p = u_1 y_1 + u_2 y_2 \qquad 2.29$$

From *2.28* we have the identifiers as y_1 and y_2 respectively given as;

$$y_1 = e^{-\frac{x}{2v}\left(T_o - \sqrt{T_o^2 - [k+\gamma^2\beta_1^2]}\right)} \qquad 2.30$$

$$y_2 = e^{-\frac{x}{2v}\left(T_o + \sqrt{T_o^2 - [k+\gamma^2\beta_1^2]}\right)} \qquad 2.31$$

$$y_1' = -\frac{1}{2v}\left(T_o - \sqrt{T_o^2 - [k+\gamma^2\beta_1^2]}\right) e^{-\frac{x}{2v}\left(T_o - \sqrt{T_o^2 - [k+\gamma^2\beta_1^2]}\right)} \qquad 2.32$$

$$y_2' = -\frac{1}{2v}\left(T_o + \sqrt{T_o^2 - [k+\gamma^2\beta_1^2]}\right) e^{-\frac{x}{2v}\left(T_o + \sqrt{T_o^2 - [k+\gamma^2\beta_1^2]}\right)} \qquad 2.33$$

Computing the Wrouskian matrix for y_1 and y_2, we have;

$$W(y_1 y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \qquad 2.34$$

$$W(y_1 y_2) = \begin{vmatrix} e^{-\frac{x}{2v}\left(T_o - \sqrt{T_o^2 - [k+\gamma^2\beta_1^2]}\right)} & e^{-\frac{x}{2v}\left(T_o + \sqrt{T_o^2 - [k+\gamma^2\beta_1^2]}\right)} \\ -\frac{1}{2v}\left(T_o - \sqrt{T_o^2 - [k+\gamma^2\beta_1^2]}\right) y_1 & -\frac{1}{2v}\left(T_o + \sqrt{T_o^2 - [k+\gamma^2\beta_1^2]}\right) y_2 \end{vmatrix}$$

2.35

$$W(y_1, y_2) = -\frac{1}{2v}\left(T_o + \sqrt{T_o^2 - [k+\gamma^2\beta_1^2]}\right) e^{-\frac{T_o}{v}x} + \frac{1}{2v}\left(T_o - \sqrt{T_o^2 - [k+\gamma^2\beta_1^2]}\right) e^{-\frac{T_o}{v}x}$$

2.36

$$W(y_1, y_2) = -\frac{\left(\sqrt{T_o^2 - 4[k+\gamma^2\beta_1^2]}\right)}{v} e^{-\frac{T_o}{v}x} \qquad 2.37$$

Computing the Wrouskian matrix for f_x and y_2, we have;

$$W_1(f_x, y_2) = \begin{vmatrix} 0 & e^{-\frac{x}{2v}\left(T_o + \sqrt{T_o^2 - [k+\gamma^2\beta_1^2]}\right)} \\ \frac{\gamma\beta_1^2 M_o}{T_1 v^2} & -\frac{1}{2v}\left(T_o + \sqrt{T_o^2 - [k+\gamma^2\beta_1^2]}\right) e^{-\frac{x}{2v}\left(T_o + \sqrt{T_o^2 - [k+\gamma^2\beta_1^2]}\right)} \end{vmatrix} \qquad 2.38$$

$$W_1 = -\frac{\gamma\beta_1^2 M_o}{T_1 v^2} e^{-\frac{x}{2v}\left(T_o + \sqrt{T_o^2 - 4[k+\gamma^2\beta_1^2]}\right)} \qquad 2.39$$

Computing the Wrouskian matrix for y_1 and f_x, we have;

$$W_2 = \begin{vmatrix} e^{-\frac{x}{2v}\left(T_o - \sqrt{T_o{}^2 - [k+\gamma^2\beta_1^2]}\right)} & 0 \\ -\frac{1}{2v}\{T_o - \sqrt{T_o{}^2 - [k+\gamma^2\beta_1^2]}\}e^{-\frac{x}{2v}\left(T_o - \sqrt{T_o{}^2 - [k+\gamma^2\beta_1^2]}\right)} & \frac{\gamma\beta_1^2 M_o}{T_1 v^2} \end{vmatrix} \quad 2.40$$

$$W_2 = \frac{\gamma\beta_1^2 M_o}{T_1 v^2} e^{-\frac{x}{2v}\left(T_o - \sqrt{T_o{}^2 - 4[k+\gamma^2\beta_1^2]}\right)} \quad 2.41$$

Dividing equation *2.39* with equation *2.37* and then integrate -

$$u_1 = \int \frac{W_1}{W} dx = \int \frac{-\frac{\gamma\beta_1^2 M_o}{T_1 v^2} e^{-\frac{x}{2v}\left(T_o + \sqrt{T_o{}^2 - [k+\gamma^2\beta_1^2]}\right)}}{-\frac{\left(\sqrt{T_o{}^2 - 4[k+\gamma^2\beta_1^2]}\right)}{v} e^{-\frac{T_o}{v}x}} dx \quad 2.42$$

Simplifying;

$$u_1 = \frac{\gamma\beta_1^2 M_o}{T_1 v\sqrt{T_o{}^2 - [k+\gamma^2\beta_1^2]}} \int e^{\frac{x}{2v}\left(T_o - \sqrt{T_o{}^2 - [k+\gamma^2\beta_1^2]}\right)} dx \quad 2.43$$

Integrating with respect to x

$$u_1 = \frac{\gamma\beta_1^2 M_o}{T_1 v\sqrt{T_o{}^2 - [k+\gamma^2\beta_1^2]}} \cdot \frac{2v}{\left(T_o + \sqrt{T_o{}^2 - [k+\gamma^2\beta_1^2]}\right)} e^{\frac{x}{2v}\left(T_o - \sqrt{T_o{}^2 - [k+\gamma^2\beta_1^2]}\right)} \quad 2.44$$

Simplifying

$$u_1 = \frac{2\gamma\beta_1^2 M_o}{T_o T_1\sqrt{T_o{}^2 - [k+\gamma^2\beta_1^2]} + T_1(T_o{}^2 - [k+\gamma^2\beta_1^2])} e^{\frac{x}{2v}\left(T_o - \sqrt{T_o{}^2 - [k+\gamma^2\beta_1^2]}\right)} \quad 2.45$$

$$u_1 y_1 = \frac{2\gamma\beta_1^2 M_o}{T_o T_1\sqrt{T_o{}^2 - [k+\gamma^2\beta_1^2]} + T_1(T_o{}^2 - [k+\gamma^2\beta_1^2])} e^{\frac{x}{2v}\left(T_o - \sqrt{T_o{}^2 - [k+\gamma^2\beta_1^2]}\right)} e^{-\frac{x}{2v}\left(T_o - \sqrt{T_o{}^2 - [k+\gamma^2\beta_1^2]}\right)}$$

2.46

$$u_1 y_1 = \frac{2\gamma\beta_1^2 M_o}{T_o T_1\sqrt{T_o{}^2 - [k+\gamma^2\beta_1^2]} + T_1(T_o{}^2 - [k+\gamma^2\beta_1^2])} \quad 2.47$$

Also, dividing the Wrouskian, W_2 with the Wrouskian, W and then integrate.

$$u_2 = \int \frac{W_2}{w} dx = \int \frac{\frac{\gamma\beta_1^2 M_o}{T_1 v^2} e^{-\frac{x}{2v}\left(T_o - \sqrt{T_o{}^2 - [k+\gamma^2\beta_1^2]}\right)}}{-\frac{\left(\sqrt{T_o{}^2 - [k+\gamma^2\beta_1^2]}\right)}{v} e^{-2\frac{T_o}{v}x}} dx$$

2.48

Simplifying

$$u_2 = -\frac{\gamma\beta_1^2 M_o}{T_1 v\sqrt{T_o{}^2-[k+\gamma^2\beta_1^2]}} \int e^{\frac{x}{2v}\left(T_o+\sqrt{T_o{}^2-[k+\gamma^2\beta_1^2]}\right)}\, dx \qquad 2.49$$

Integrating with respect to x

$$u_2 = -\frac{\gamma\beta_1^2 M_o}{T_1 v\sqrt{T_o{}^2-[k+\gamma^2\beta_1^2]}} \cdot \frac{2v}{\left(T_o-\sqrt{T_o{}^2-[k+\gamma^2\beta_1^2]}\right)} e^{\frac{x}{2v}\left(T_o+\sqrt{T_o{}^2-[k+\gamma^2\beta_1^2]}\right)} \qquad 2.50$$

Simplifying

$$u_2 = -\frac{2\gamma\beta_1^2 M_o}{T_o T_1\sqrt{T_o{}^2-[k+\gamma^2\beta_1^2]}-T_1(T_o{}^2-[k+\gamma^2\beta_1^2])} e^{\frac{x}{2v}\left(T_o+\sqrt{T_o{}^2-[k+\gamma^2\beta_1^2]}\right)} \qquad 2.51$$

$$u_2 y_2 = -\frac{\gamma\beta_1^2 M_o}{T_o T_1\sqrt{T_o{}^2-[k+\gamma^2\beta_1^2]}-T_1(T_o{}^2-[k+\gamma^2\beta_1^2])} e^{\frac{x}{2v}\left(T_o+\sqrt{T_o{}^2-[k+\gamma^2\beta_1^2]}\right)} \cdot e^{-\frac{x}{2v}\left(T_o+\sqrt{T_o{}^2-[k+\gamma^2\beta_1^2]}\right)}$$

2.52

$$u_2 y_2 = -\frac{\gamma\beta_1^2 M_o}{T_o T_1\sqrt{T_o{}^2-[k+\gamma^2\beta_1^2]}-T_1(T_o{}^2-[k+\gamma^2\beta_1^2])} \qquad 2.53$$

Recall $y_p = u_1 y_1 + u_2 y_2$ 2.54

$$y_p = \frac{\gamma\beta_1^2 M_o}{2T_o T_1\sqrt{T_o{}^2-[k+\gamma^2\beta_1^2]}+2T_1(T_o{}^2-[k+\gamma^2\beta_1^2])} - \frac{\gamma\beta_1^2 M_o}{2T_o T_1\sqrt{T_o{}^2-[k+\gamma^2\beta_1^2]}-2T_1(T_o{}^2-[k+\gamma^2\beta_1^2])}$$

2.55

Simplifying

$$y_p = \gamma\beta_1^2 M_o\left(\frac{1}{2T_o T_1\sqrt{T_o{}^2-[k+\gamma^2\beta_1^2]}+2T_1(T_o{}^2-4[k+\gamma^2\beta_1^2])} - \frac{1}{2T_o T_1\sqrt{T_o{}^2-[k+\gamma^2\beta_1^2]}-2T_1(T_o{}^2-[k+\gamma^2\beta_1^2])}\right) \qquad 2.56$$

$$y_p = \gamma\beta_1^2 M_o\left(\frac{T_o T_1\sqrt{T_o{}^2-[k+\gamma^2\beta_1^2]}-T_1(T_o{}^2-4[k+\gamma^2\beta_1^2])-\left(T_o T_1\sqrt{T_o{}^2-[k+\gamma^2\beta_1^2]}+T_1(T_o{}^2-[k+\gamma^2\beta_1^2])\right)}{\left(T_o T_1\sqrt{T_o{}^2-[k+\gamma^2\beta_1^2]}+T_1(T_o{}^2-[k+\gamma^2\beta_1^2])\right)\left(T_o T_1\sqrt{T_o{}^2-[k+\gamma^2\beta_1^2]}-T_1(T_o{}^2-[k+\gamma^2\beta_1^2])\right)}\right) \qquad 2.57$$

$$y_p = \gamma\beta_1^2 M_o\left(\frac{-2T_1(T_o{}^2-[k+\gamma^2\beta_1^2])}{\left(T_1{}^2 T_o{}^2(T_o{}^2-[k+\gamma^2\beta_1^2])-T_1{}^2(T_o{}^2-[k+\gamma^2\beta_1^2])\right)}\right) \qquad 2.58$$

Comparing common factors in numerator with denominator, we have.

$$y_p = -\frac{2\gamma\beta_1^2 M_o}{(T_1 T_o{}^2-T_1)} \qquad 2.59$$

Hence the general solution to Case 1: $\frac{\sqrt{b^2-4ac}}{2} > 0$; 2.60

$$y = y_p + y_c \qquad 2.61$$

$$y = c_1 e^{\frac{1}{2v}\left(T_o - \sqrt{T_o{}^2 - [k+\gamma^2\beta_1^2]}\right)x} + c_2 e^{-\frac{1}{2v}\left(T_o + \sqrt{T_o{}^2 - [k+\gamma^2\beta_1^2]}\right)x} - \frac{2\gamma\beta_1^2 M_o}{(T_1 T_o{}^2 - T_1)} \qquad 2.62$$

2.1.2 Case 2

Similarly, for Case 2: $\frac{\sqrt{b^2-4ac}}{2} = 0$, the general solution is

$$y = A(1+x)e^{-\frac{T_o}{v}x} + \frac{4\gamma\beta_1 M_o}{T_0{}^2 T_1} \qquad 2.63$$

2.1.3 Case 3

For case 3: $\frac{\sqrt{b^2-4ac}}{2} < 0$;we have the general solution as $y = y_c + y_p$

$$y = e^{-\frac{T_o x}{2v}}\left(c_1 \cos\frac{x\sqrt{T_o{}^2 - [k+\gamma^2\beta_1^2]}}{2v} + c_2 \sin\frac{x\sqrt{T_o{}^2 - [k+\gamma^2\beta_1^2]}}{2v}\right) + \frac{4\gamma\beta_1 M_o}{T_1(4v^2h^2 + T_o{}^2)} \qquad 2.64$$

$$\text{Recall } \frac{\sqrt{T_o{}^2 - [k+\gamma^2\beta_1^2]}}{2v} = h \qquad 2.65$$

$$y = e^{-\frac{T_o x}{2v}}\left(c_1 \cos\frac{x\sqrt{T_o{}^2 - [k+\gamma^2\beta_1^2]}}{2v} + c_2 \sin\frac{x\sqrt{T_o{}^2 - [k+\gamma^2\beta_1^2]}}{2v}\right) + \frac{4\gamma\beta_1 M_o}{T_1(2T_o{}^2 - [k+\gamma^2\beta_1^2])} \qquad 2.66$$

3.0 Results and Discussion

The graphs displayed in figures 3.1 – 3.10 show the plots for case 1, for $h > 0$; when radiofrequency $[\gamma^2\beta^2 \ll k]$ is negligible and T_2 varied while keeping T_1constant. It can be seen that before the magnetization relaxes, it is able to detect signals between when T_2 is 0.002 and 0.027. This implies that different organs in human body can be distinguished and in turn be related to their length of time (age).

Similarly, the graphs displayed in figures 3.11 – 3.20 show the plots for case 1, for $h > 0$; when radiofrequency $[\gamma^2\beta^2 \gg k]$ is greater than k and T_2 varied while keeping T_1constant. It can be seen that before the magnetization relaxes, it is able to detect signals between when T_2 is 0.002 and 0.047. This implies that different organs in human body can be distinguished and in turn be related to their length of time (age).

Case 1: For $h > 0$; when radiofrequency $[\gamma^2\beta^2 \ll k]$ is negligible.

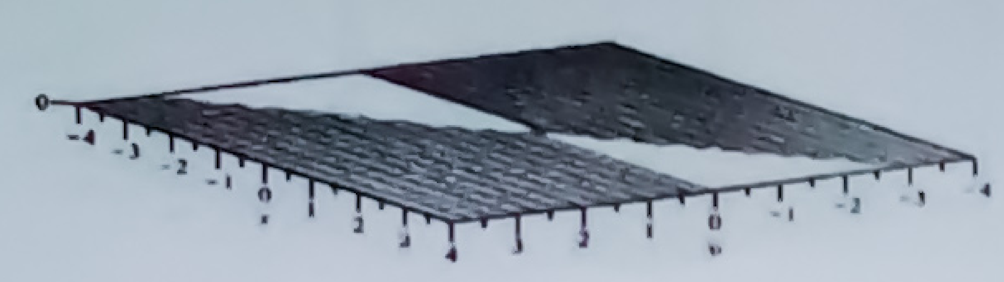

Figure 3.1 shows the plot of magnetization against velocity and distance when $T_1 = 1$ *and* $T_2 = 0.002$

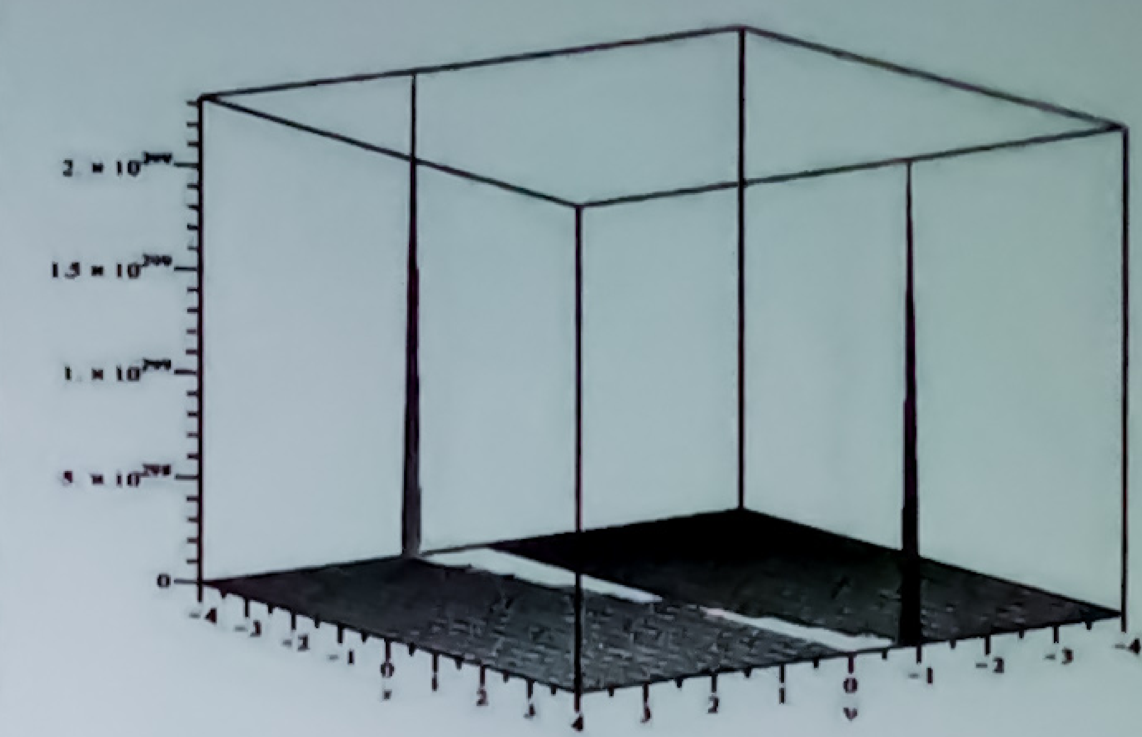

Figure 3.2 shows the plot of magnetization against velocity and distance when $T_1 = 1$ *and* $T_2 = 0.007$

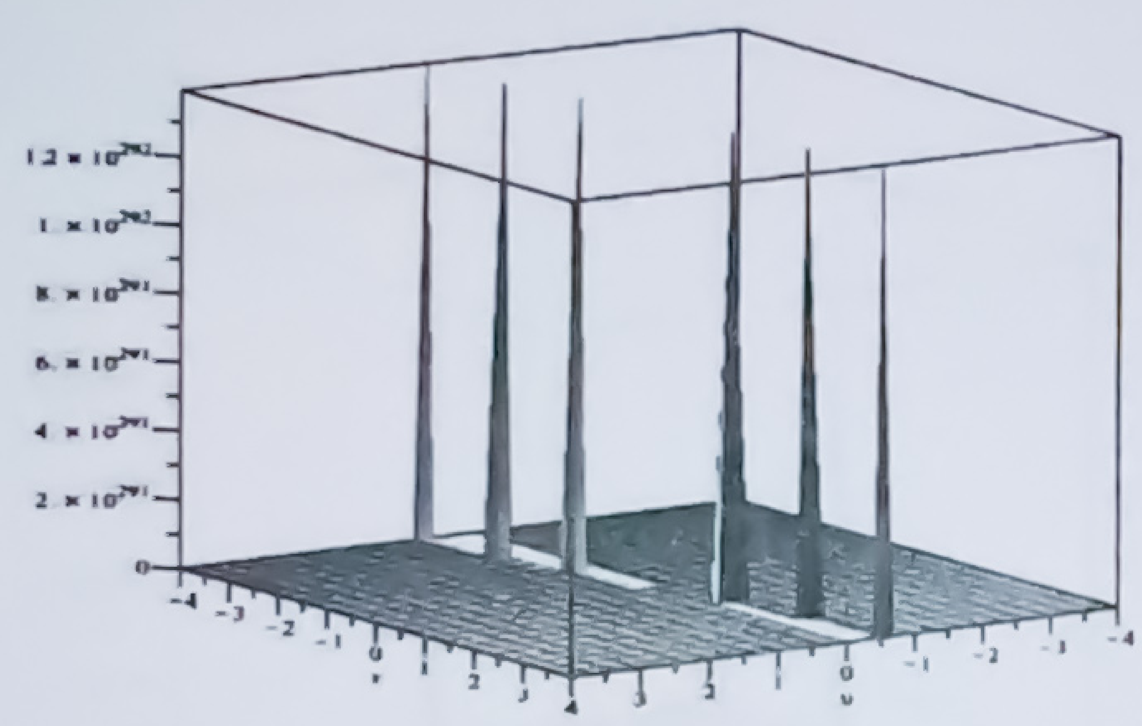

Figure 3.3 shows the plot of magnetization against velocity and distance when $T_1 = 1$ *and* $T_2 = 0.012$

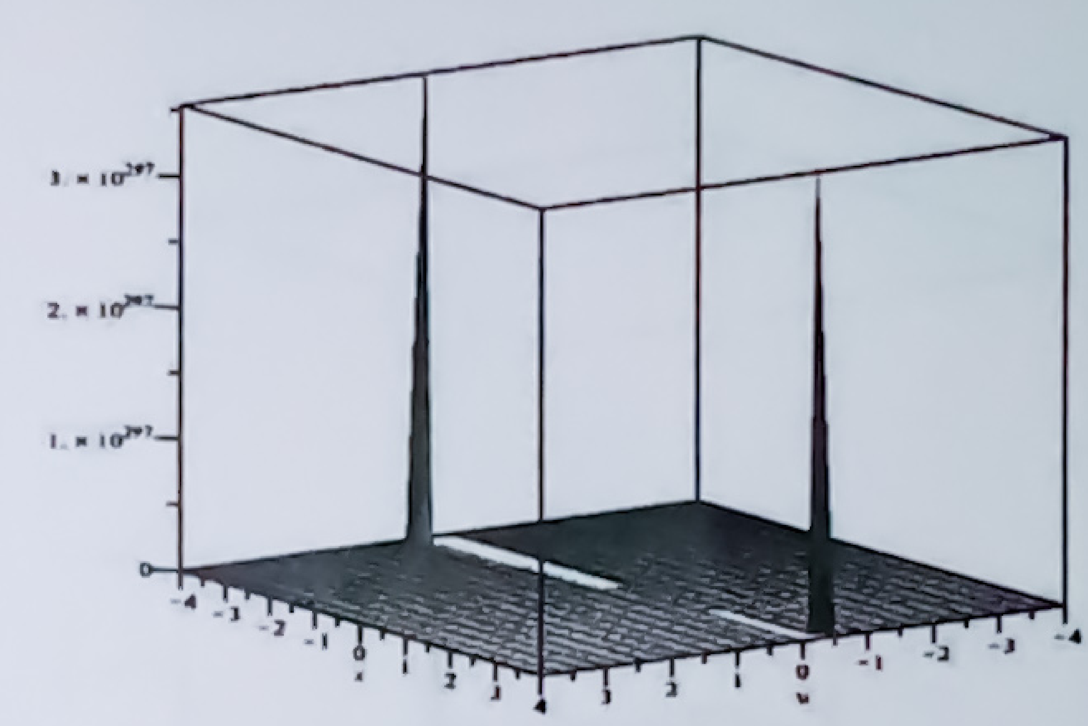

Figure 3.4 shows the plot of magnetization against velocity and distance when $T_1 = 1$ *and* $T_2 = 0.017$

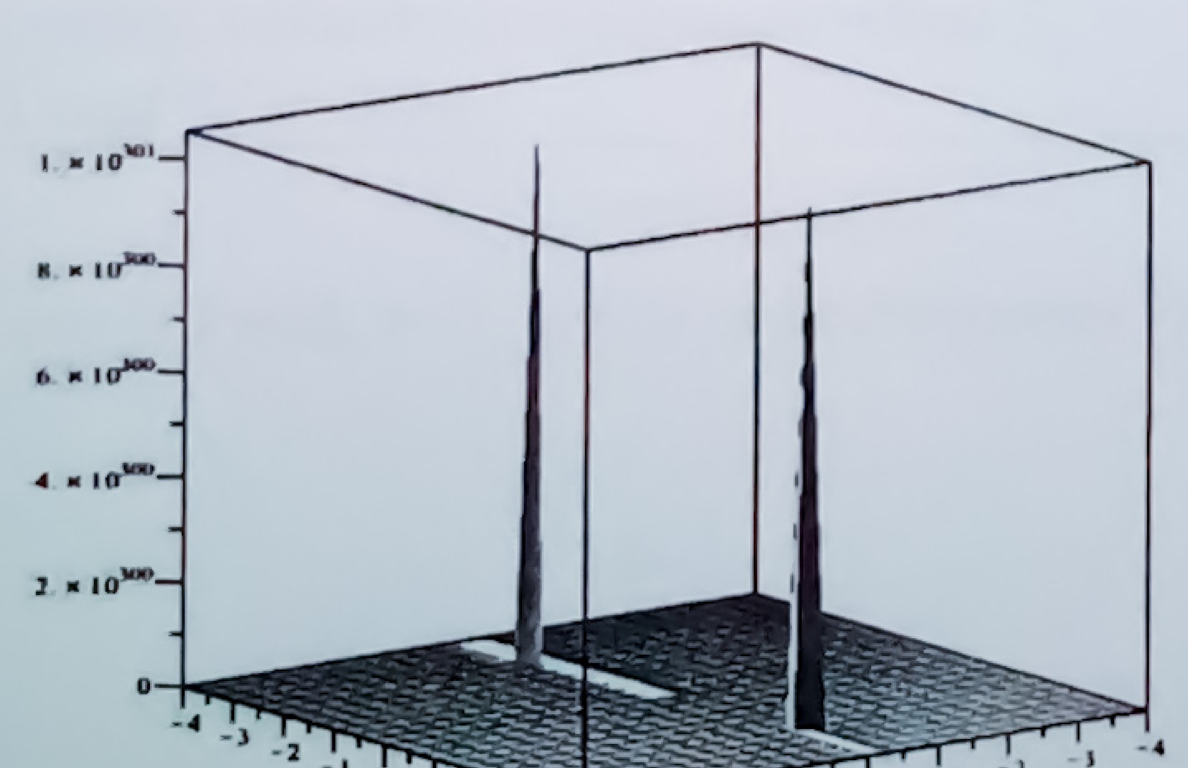

Figure 3.5 shows the plot of magnetization against velocity and distance when $T_1 = 1$ *and* $T_2 = 0.022$

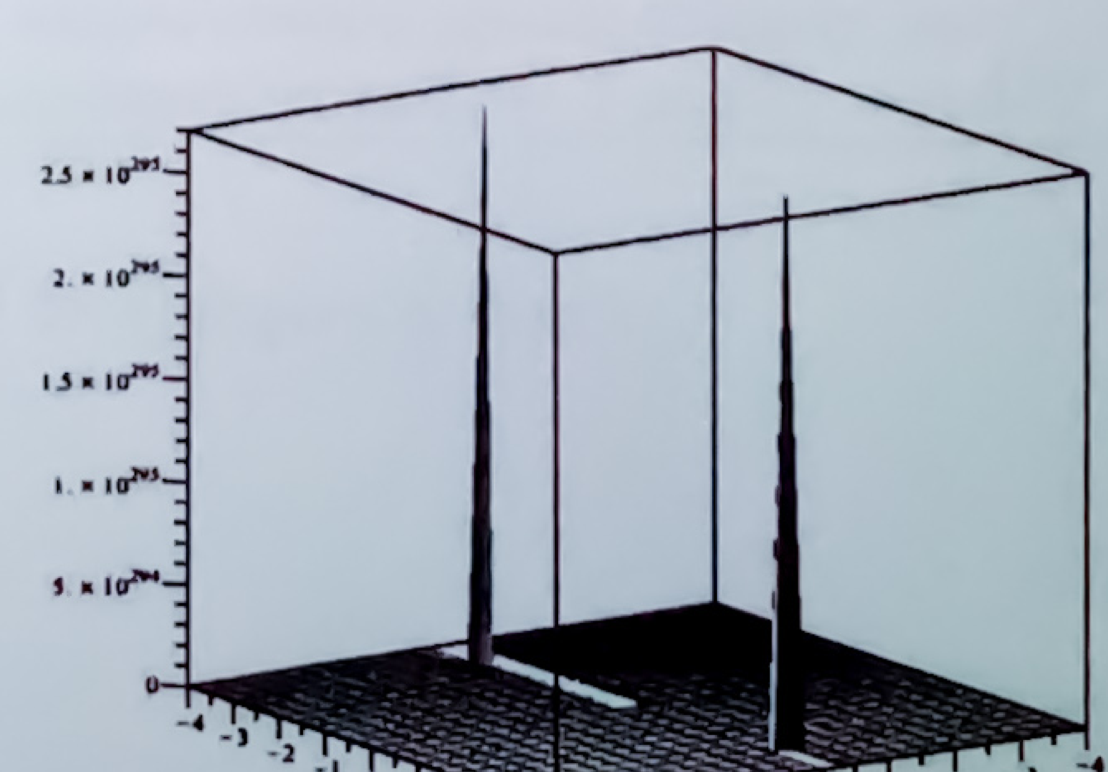

Figure 3.6 shows the plot of magnetization against velocity and distance when $T_1 = 1$ *and* $T_2 = 0.027$

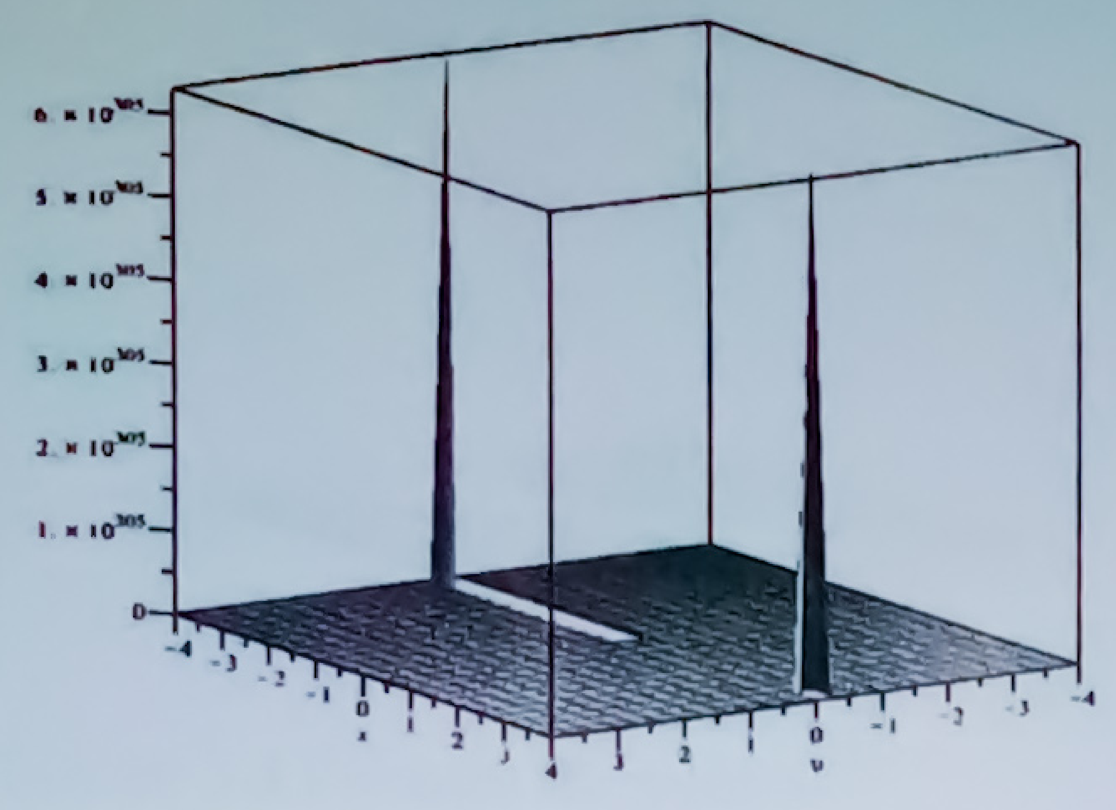

Figure 3.7 shows the plot of magnetization against velocity and distance when $T_1 = 1\ and\ T_2 = 0.032$

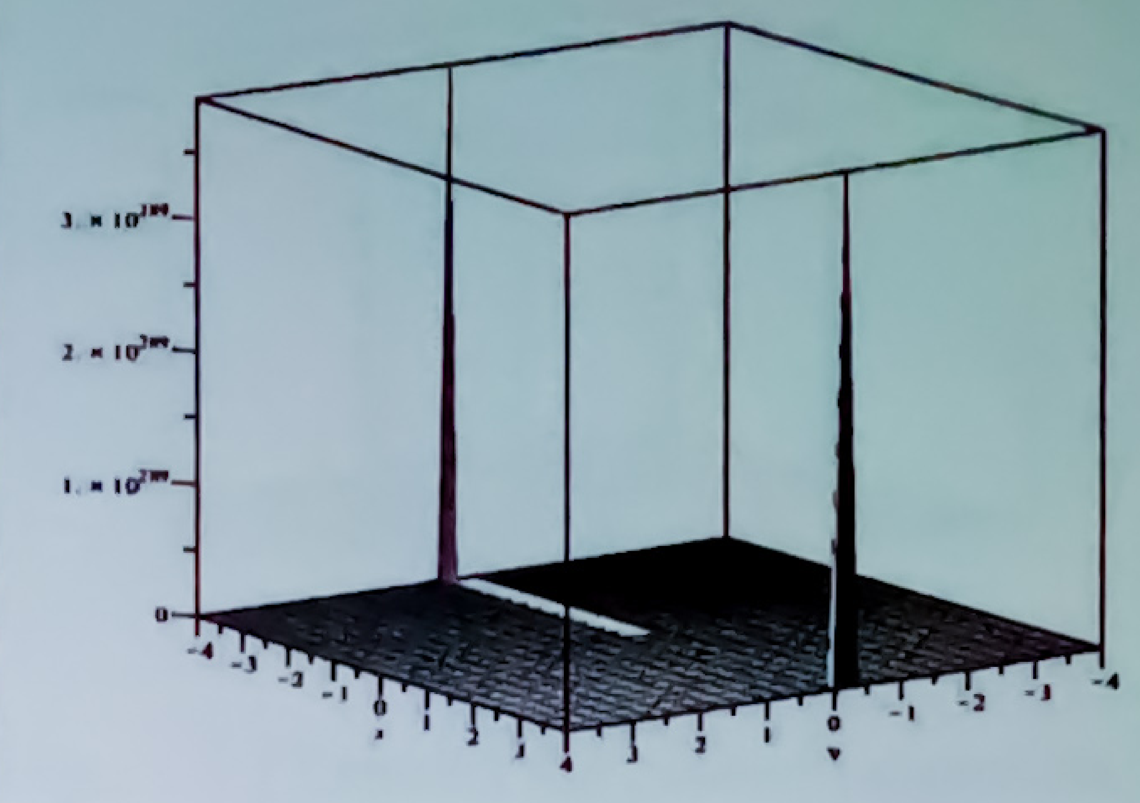

Figure 3.8 shows the plot of magnetization against velocity and distance when $T_1 = 1\ and\ T_2 = 0.037$

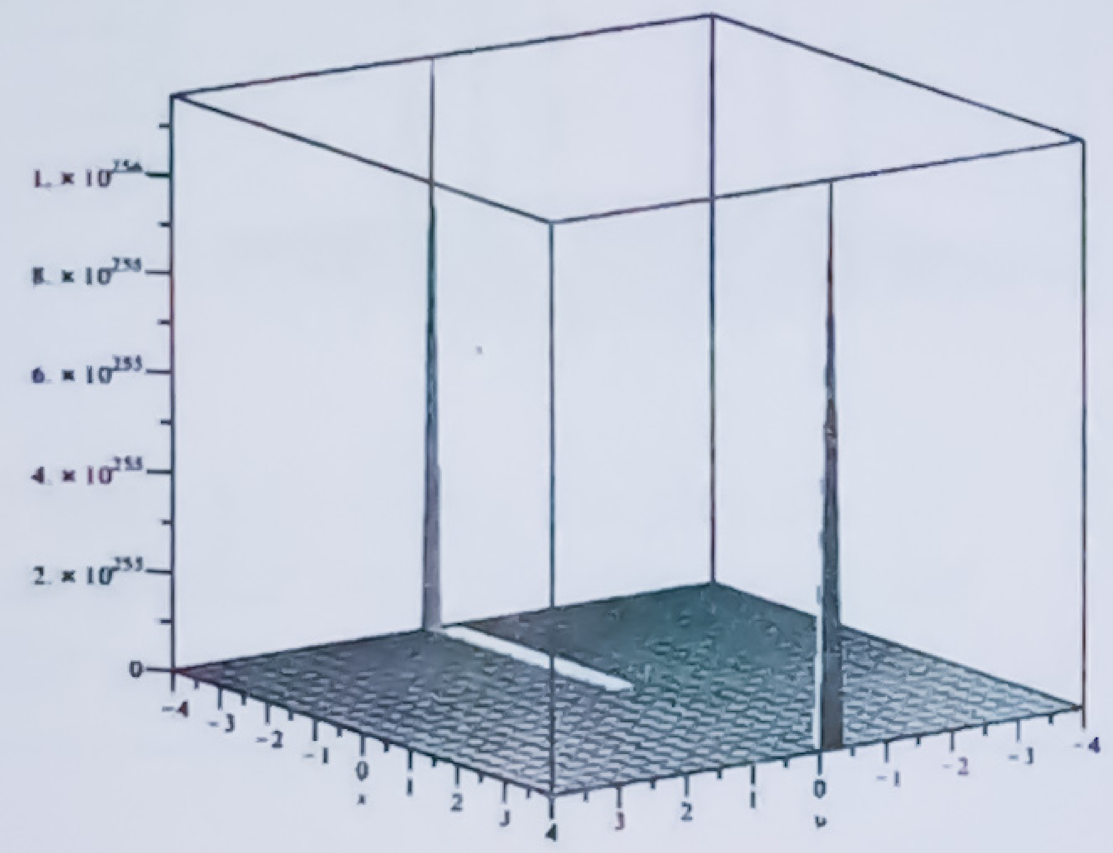

Figure 3.9 shows the plot of magnetization against velocity and distance when $T_1 = 1\ and\ T_2 = 0.042$

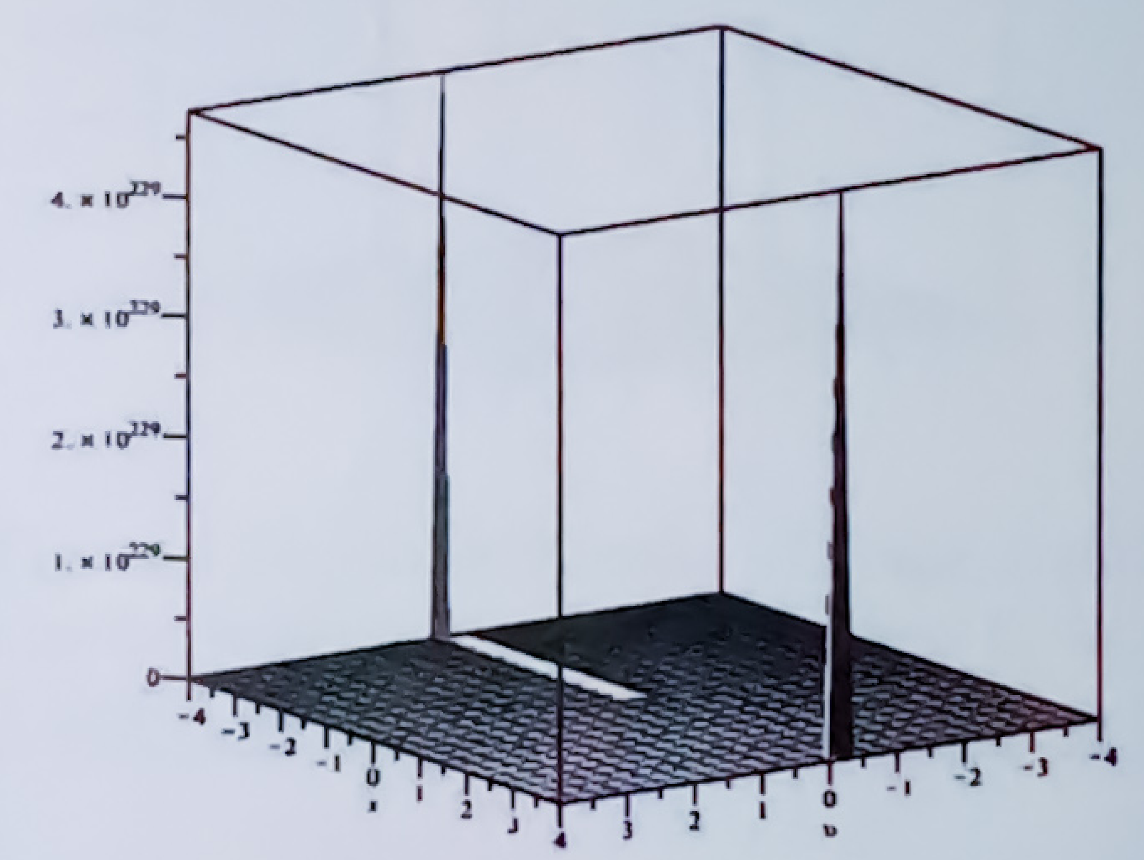

Figure 3.10 shows the plot of magnetization against velocity and distance when $T_1 = 1\ and\ T_2 = 0.047$

Case 1: For $h > 0;\ when$ radiofrequency $[\gamma^2\beta^2 \gg k]$ is greater than k.

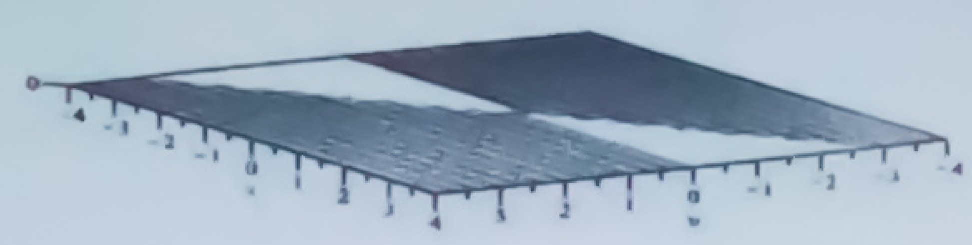

Figure 3.11 shows the plot of magnetization against velocity and distance when $T_1 = 1\ and\ T_2 = 0.002$

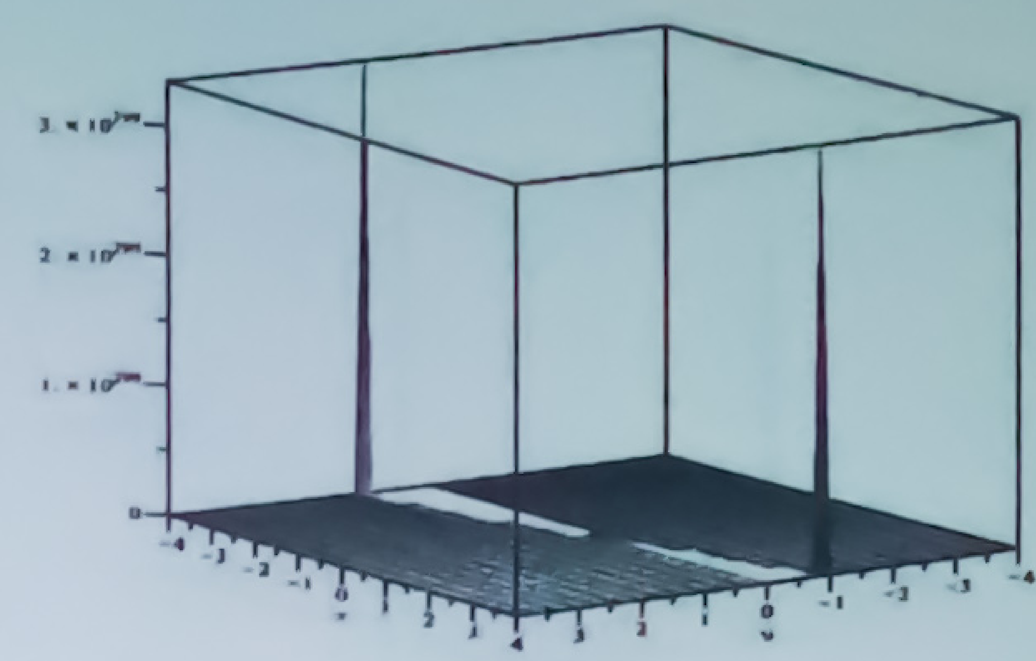

Figure 3.12 shows the plot of magnetization against velocity and distance when $T_1 = 1\ and\ T_2 = 0.007$

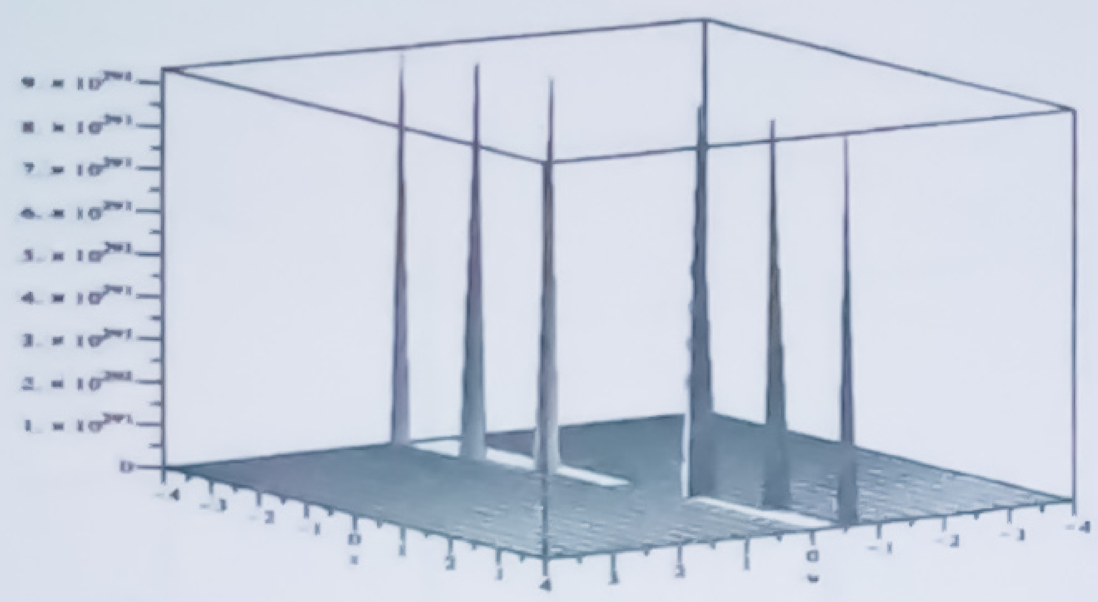

Figure 3.13 shows the plot of magnetization against velocity and distance when $T_1 = 1\ and\ T_2 = 0.012$

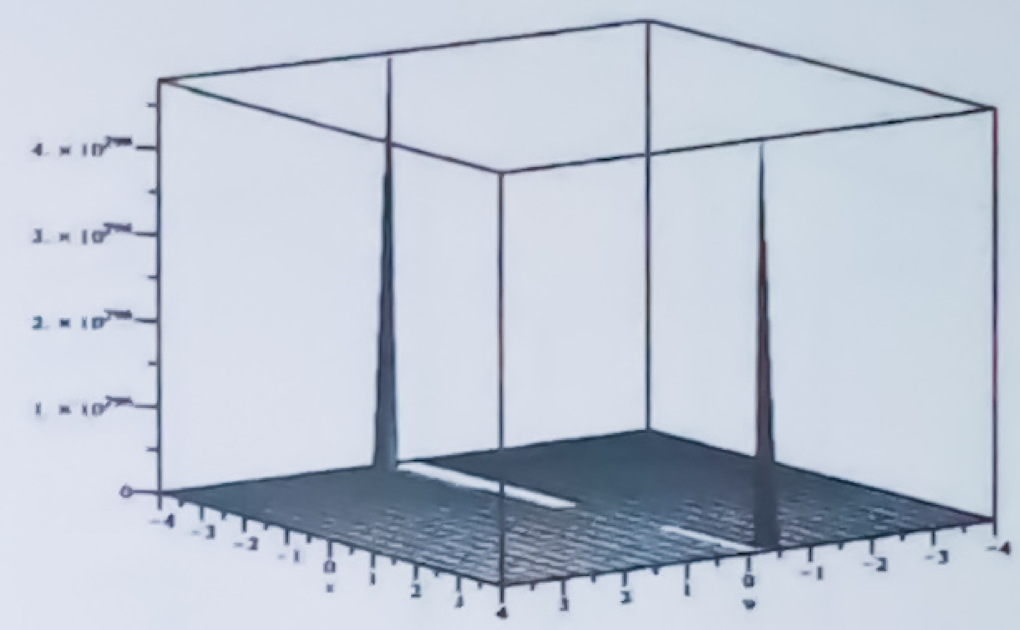

Figure 3.14 shows the plot of magnetization against velocity and distance when $T_1 = 1\ and\ T_2 = 0.017$

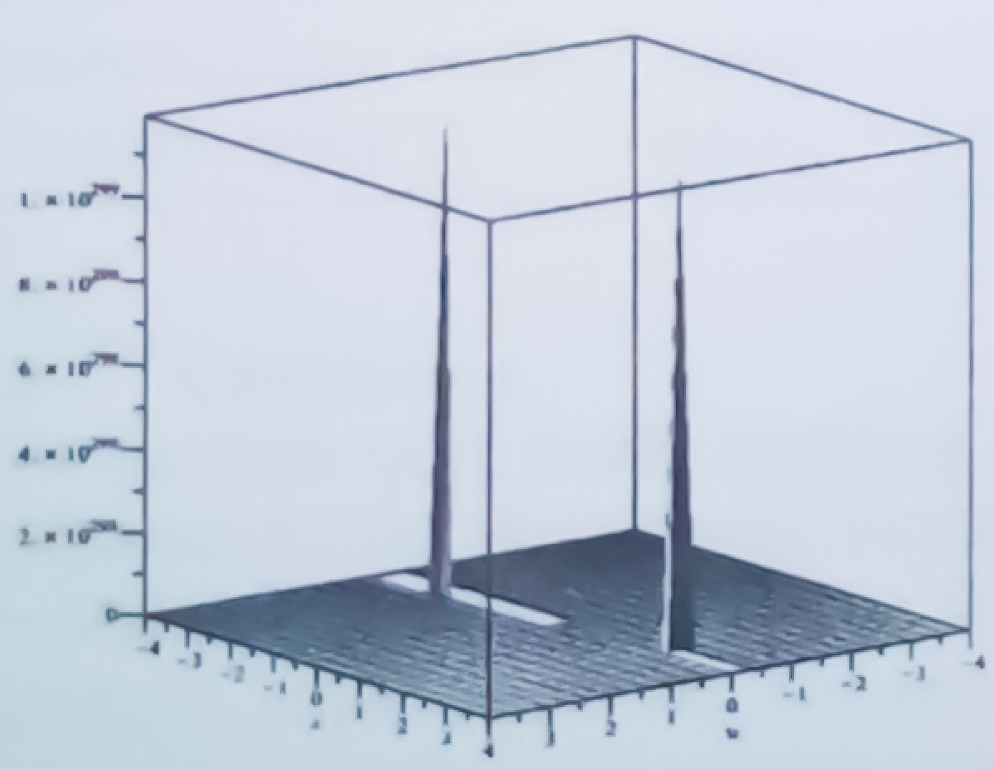

Figure 3.15 shows the plot of magnetization against velocity and distance when $T_1 = 1\ and\ T_2 = 0.022$

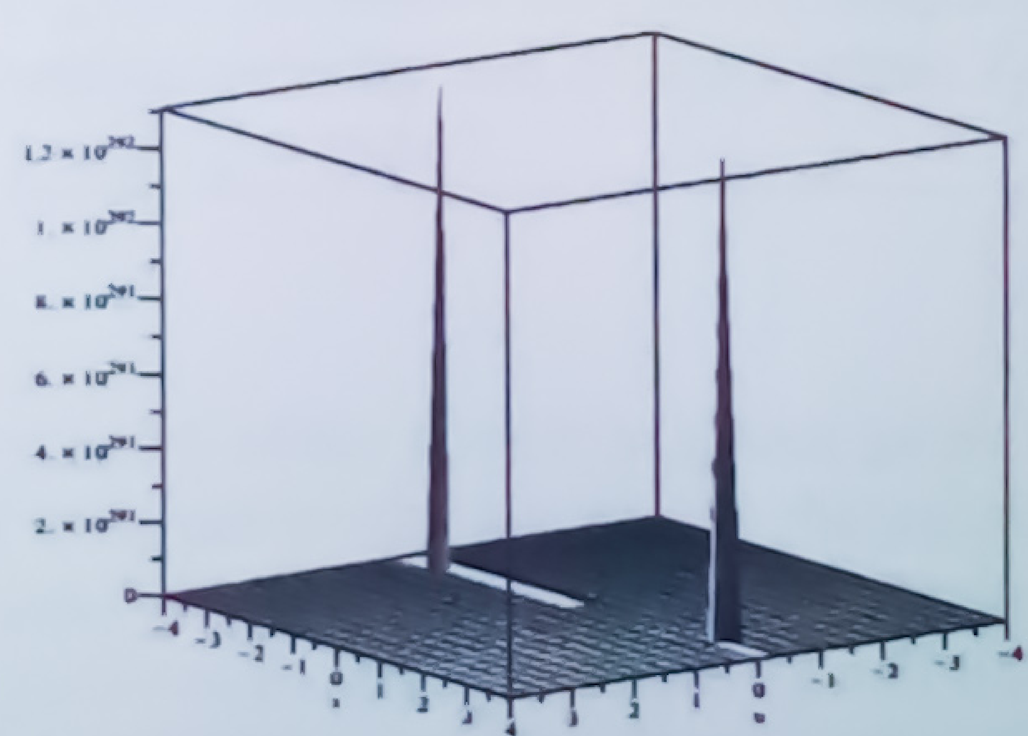

Figure 3.16 shows the plot of magnetization against velocity and distance when $T_1 = 1\ and\ T_2 = 0.027$

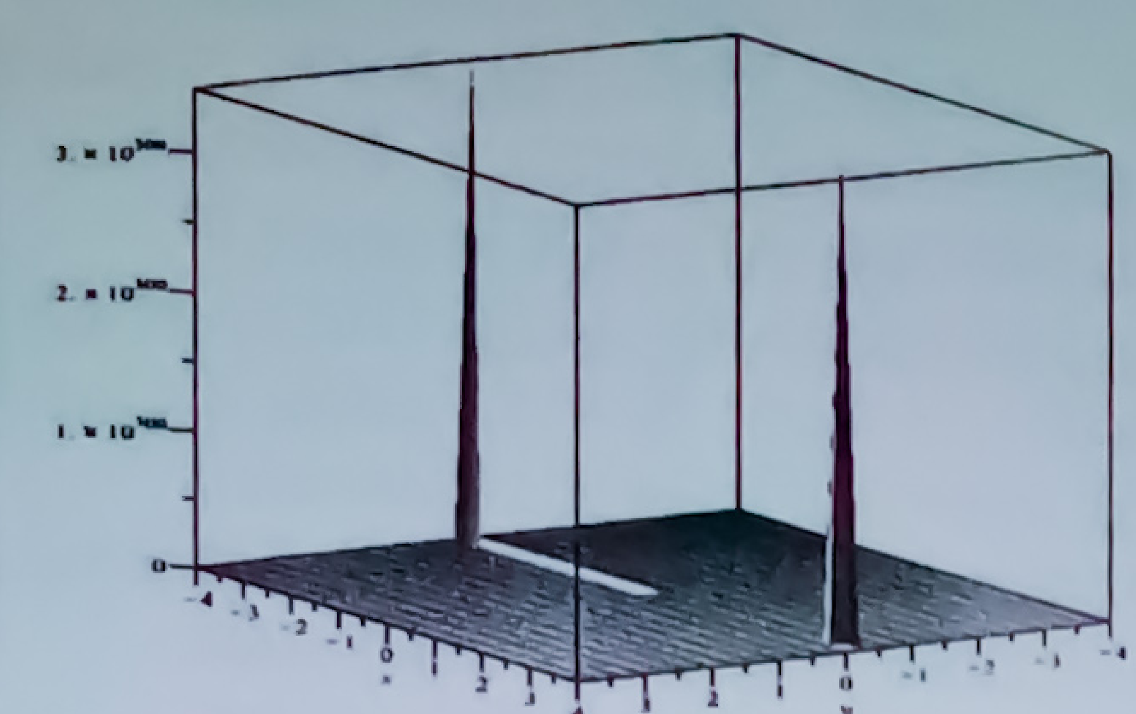

Figure 3.17 shows the plot of magnetization against velocity and distance when $T_1 = 1$ *and* $T_2 = 0.032$

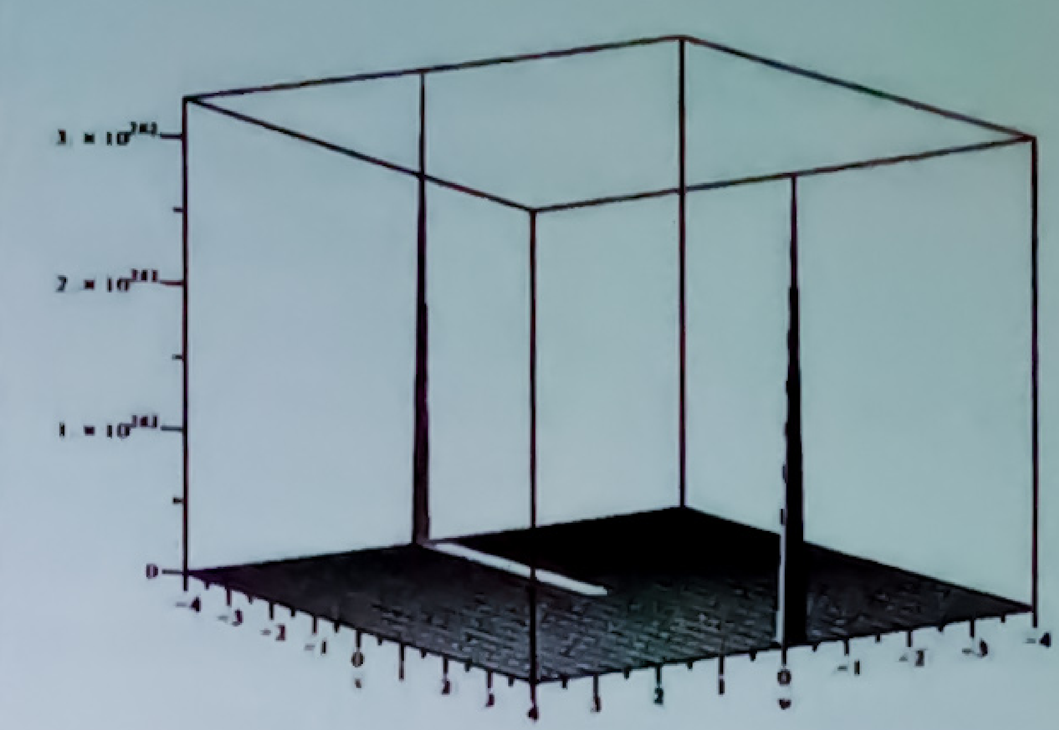

Figure 3.18 shows the plot of magnetization against velocity and distance when $T_1 = 1$ *and* $T_2 = 0.037$

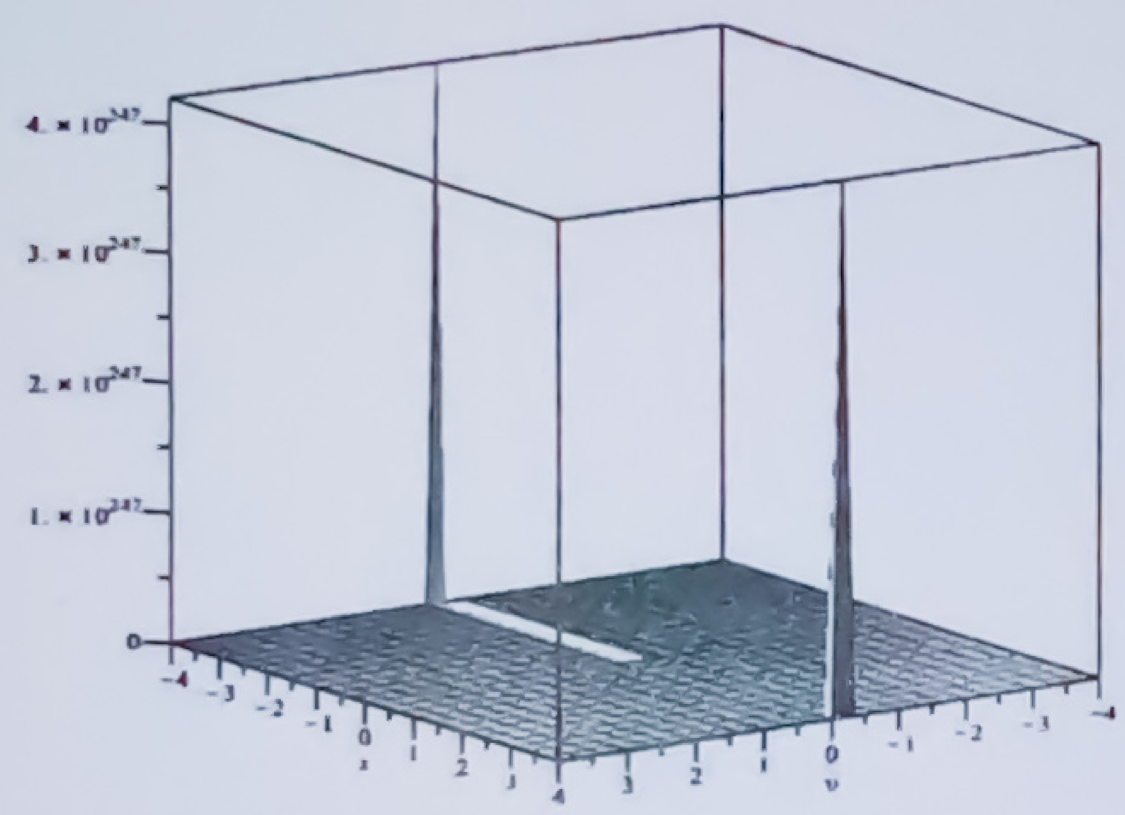

Figure 3.19 shows the plot of magnetization against velocity and distance when $T_1 = 1$ *and* $T_2 = 0.042$

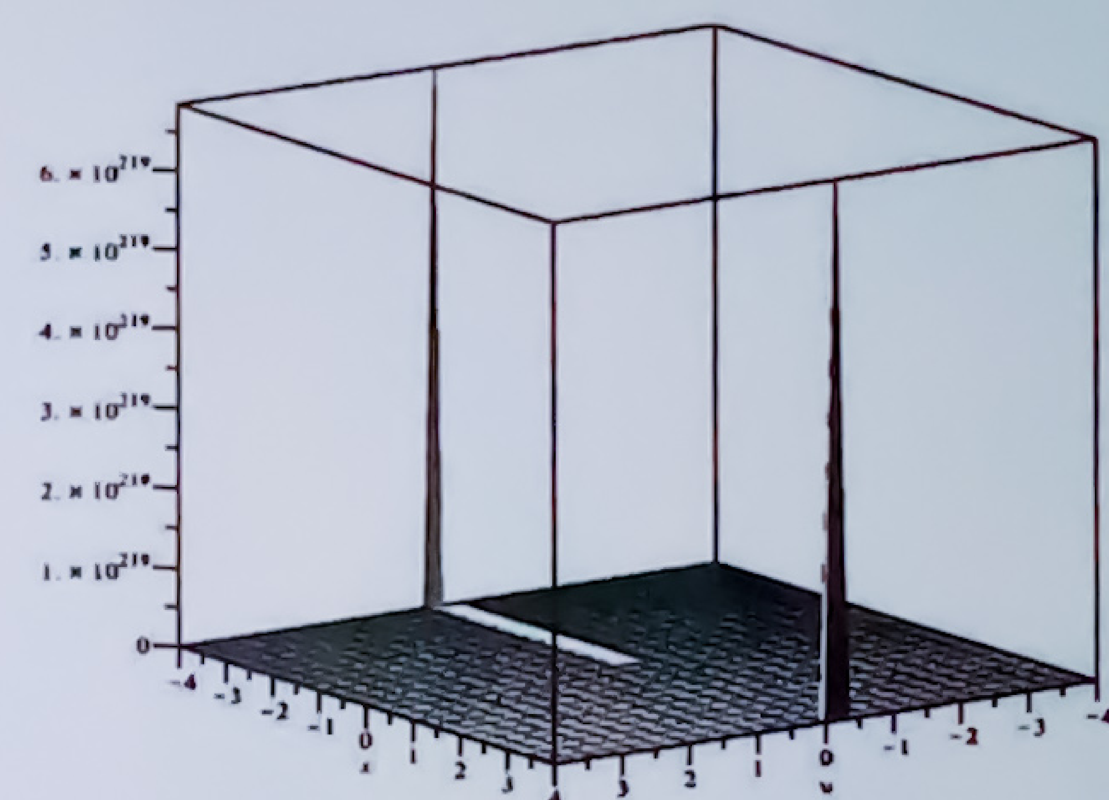

Figure 3.20 shows the plot of magnetization against velocity and distance when $T_1 = 1$ *and* $T_2 = 0.047$

4.0 Conclusion

This research work is still on-going. Hence only a part of the results has been generated and presented. So far, preliminary results generated from the work have shown the possibility of determining age of human organs from the T_1*and* T_2relaxation parameters as obtained from the Bloch NMR equations. The remaining cases ($h = 0$ *and when* $h < 0$) will eventually be examined and results generated compared and analyzed in relation to how relaxation rates could be used to estimate or determine the length of time of existence (age) of the human organ under consideration.

References

Awojoyogbe, O. B. (2009). Analytical solution of the time – dependent Bloch NMR flow equations: A Translational Mechanical Analysis. *Physica A*, 339, 437-460.

AgartzI., Saaf J., Wahlund L. and Wetterberg L.(1991). T1 and T2 Relaxation Time Estimates in the Normal Human Brain.

Badve C., Yu A.,Rogers M., Ma D., Liu Y., Schluchter M., Sunshine J., Griswold M. and Gulani V.,(2015). Simultaneous T1 and T2 Brain Relaxometry in Asymptomatic Volunteers Using MagneticResonance Fingerprinting.

Rock P. J.(2021) *https://radiopaedia.org/articles/t1-relaxation-time*.

Suzuki S.,Sakai O. and Jara4 H., (2006). Combined volumetric T1, T2 and secular-T2 quantitative MRI of the brain: age-related global changes.

Yusuf, S. I. (2010). A Study of Longitudinal and Transverse Relaxation Rates in Magnetic Resonance Imaging – MRI. A paper presented at the 4th National Conference of School of Science and Science Education, Federal University of Technology, Minna, Page 136 -151, November, 2010.

Yusuf, S. I., Aiyesimi, Y. M., Jiya M., Awojoyogbe, O. B. and Dada, M. (2019). Magnetic Resonance

Imaging of Plaques in a Cylindrical Channel. *Journal of Science, Technology, Mathematics and Education.* 15(2), 71 – 78.