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Mixed Convective Mmagnetohydrodynamics Micropolar Boundary Layer Flow Past a Stretching Sheet with Heat Generation

[1]Yusuf, A., [1]Ishaq, M. A., [1]Salihu, N. O., [2]Salisu, A. and [1]Bolarin, G.

[1]Department of Mathematics, Federal University of Technology, Minna, Niger State, Nigeria
[2]Department of Computer Science, Niger State Polytechnic, Zungeru, Niger State, Nigeria

ABSTRACT

The problem of mixed convective magnetohydrodynamics micropolar boundary layer past a stretching sheet with heat generation was presented in rectangular form. The partial differential equations formulated are transformed into nonlinear ordinary differential equations using the stream functions and appropriate similarity variables. The solution to the nonlinear coupled ordinary differential equations is presented via decomposition method. The results are validated with the literatures and there is an agreement. The effects of dimensionless physical parameters which occur in the presented results are graphically studied in the absence of microstructural slip. The micro rotation is found to be a reducing agent of thermal and mass Grashof numbers while the fluid is an increasing agent due to the increase in the temperature which resulted in reduction of the viscousity.

Keywords: Micropolar, Heat generation, Magnetohydrodynamics, Decomposition method, Boundary layer.

INTRODUCTION

Eringen (1972) extend the theory of micropolar fluids by considering the thermomicropolar fluids. Ebert (1973) studied the similarity solution of boundary layer flow close to a stagnation point for micropolar fluids.Micropolar fluids belong to the class of fluids which exhibit some certain microscopic effects arising from the motion of the micro elements. These class of fluid can be non-Newtonian due to the fact that they contain micro-constituents that can undergo rotation which can interfere with the hydrodynamics. Aiyesimi *et al.* (2013) studied hydromagnertic boundary micropolar fluid flow over a stretching surface embedded in a non-Dacian medium with variable permeability and it was observed that the magnetic parameter and the inverse Darcy number are reduction agent of the fluid velocity. Karwe and Jaluria (1988) works on mixed convection on micropolar fluids past a flat, wavy surface has been considered by a number of researcher due to significant of heat transfer on the flow of micropolar fluids.

Si *et al.* (2017) presented the effect of slip-velocity on the flow of magnetohydrodynamics non-Newtonian fluid over a moving surface. The most important aspect of microsystem such as nozzles, micro-pump and hard disc are slip flows. They depicted that temperature -Jump and velocity slip boundary condition can cause the earlier flow to transit from laminar to turbulent. Hosseini *et al.* (2017) numerically studied the Heat transfer and boundary layer unsteady flow in the presence of velocity slip over a porous extending surface. He and Cai (2017) presented the entire effects of slip velocity and temperature Jump on flow of a boundary layer to a flat surface. Daniel *et al.* (2017) theoretically analyzed the slip effect on MHD nanofluid flow over a stretching/shrinking sheet. In their first solutions, the velocity, thermal and solutal boundary layer thickness is lower than that of the second solutions, and the first solution is more robust relative to the second solution. They also found that heat and mass convective boundary conditions are improved by temperature and nanoparticle concentration distributions. Heat transmission in magnetized Newtonian nanoliquids between vertical cylinders was presented by Mebarek-Oudina *et al.* (2017). They claimed that with the growth of porosity, nanoparticle concentration, Rayleigh number, Darcy number, and length of the source, the transferred thermal flux in laminar natural convection increases significantly. A theoretical analysis on heat transfer in MHD mixed convective micropolar fluid flow with velocity slip conditions was presented by Mahmoud and Waheed (2010).

Several Researchers such as Bolarin *et al.* (2019), Suleiman and Yusuf (2020), Yusuf *et al.* (2021a), Yusuf *et al.* (2021b), have all demonstrated in their work the reliability of the Decomposition method in solving boundary layer problems.

The use of decomposition method to study mixed convective magnetohydrodynamics micropolar boundary layer flow past a stretching sheet with heat generation is a new advancement in the literature.

*Corresponding author. E-mail:yusuf.abdulhakeem@futminna.edu.ng (+2347032143075)

PROBLEM FORMULATION AND SOLUTION

Consider steady, laminar and boundary layer flow of a micropolar fluid past a stretching sheet with a second order and microstructural slip condition. The micropolar fluid has a stretching velocity of $u_s = ax$ where *a* is a constant rate of the stretching velocity. Neglecting the slip conditions, following the work of Dawar *et al*. (2021) with heat generation and convective boundary condition, the model formulations are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\frac{\kappa}{\rho}\frac{\partial N}{\partial y}+\frac{(\mu+\kappa)}{\rho}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u+g\beta_T(T-T_\infty)+g\beta_c(C-C_\infty), \tag{2}$$

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}=\frac{\Omega}{\rho j}\left(2N+\frac{\partial u}{\partial y}\right), \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\left(1+\frac{16}{3}\frac{\sigma^* T_\infty^3}{kk^*}\right)\frac{\partial^2 T}{\partial y^2}+\frac{\sigma B_0^2}{\rho c_p}u^2+\tau\left[D_B\frac{\partial T}{\partial y}\frac{\partial C}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right]+\frac{Q}{\rho c_p}(T-T_\infty), \tag{4}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+k_1(C_\infty-C)+\frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right), \tag{5}$$

The corresponding boundary conditions are:

$$\left.\begin{aligned} &u=u_s=ax,\ \ v=0,\ \ \mathrm{T}=\mathrm{T}_s,\ \ N=0,\ \ C=C_s\ \ \text{at}\ \ y=0\\ &u\to 0,\ \ N\to 0,\ \ T\to\infty, \text{as}\ \ y\to 0\end{aligned}\right\} \tag{6}$$

Where *x* and *y* are coordinate along velocity *u* and *v* respectively, *a* stretching constant, A B Slip constant, B_0 magnetic field, C fluid concentration, C_∞ concentration at free stream, C_s concentration near surface, c_p specific heat, D_1 *and* D_2 constants, D_B Brownian diffusion, D_T thermopheric diffusion, Ec Eckert number, j micro inertia density, k thermal conductivity, k_1 chemical reaction, $k_1$reaction, T temperature, T_∞ temperature at free stream, T_s temperature near the surface, Q heat generation, μ dynamic viscosity, τ heat capacity ratio, κ votex viscosity, σ electrical conductivity, σ^* Stefan Boltzmann constant, υ kinematic viscousity, ρ density.

Equations, (1) to (6) are transformed using the following similarity transformation:

$$\left.\psi=\sqrt{a\upsilon}xf(\eta),u=\frac{\partial\psi}{\partial y},v=-\frac{\partial\psi}{\partial x},N=ax\sqrt{\frac{a}{\upsilon}}g(\eta),\eta=\sqrt{\frac{a}{\upsilon}}y\right\} \tag{7}$$

$$\left.T-T_\infty=(T_s-T_\infty)\theta(\eta),\mathrm{C}-C_\infty=(C_s-C_\infty)\varphi(\eta),D_1=\frac{T_s-T_\infty}{x},D_2=\frac{C_s-C_\infty}{x}\right\} \tag{8}$$

Where $\eta, f, g, \theta, \varphi$ are the dimensionless fluid distance, velocity, micro-rotation, temperature, and concentration.

The continuity equation in (1) is satisfied on introducing the transformation in (7) and (8), while equation (2) to (6) becomes:

$$\left.\begin{aligned}
&(1+\alpha) f^{///} + ff^{//} - f^{/2} + \alpha g^{/} - Mf^{/} + \gamma_1\theta + \gamma_2\varphi = 0 \\
&\left(1+\frac{\alpha}{2}\right) g^{//} - f^{/} g + fg^{/} - \alpha\left(2g + f^{//}\right) = 0 \\
&(1+Ra)\theta^{//} + \Pr f\theta^{/} - \Pr\theta f^{/} + \Pr MEcf^{/} + Nb\Pr\theta^{/}\varphi^{/} + Nt\theta^{/2} + \Pr Q_0\theta = 0 \\
&\varphi^{//} + Scf\varphi^{/} - Scf^{/}\varphi - ScKr\varphi + \frac{Nt}{Nb}\theta^{//} = 0 \\
&\textit{with the corresponding boundary conditions:} \\
&f = 0, f^{/} = 1, g = 0, \theta = 1, \varphi = 1, \quad \eta = 0 \\
&f^{/} \to 0, \mathrm{g} \to 0, \theta \to 0, \varphi \to 0, \quad \eta \to \infty
\end{aligned}\right\} \tag{9}$$

Where

$$\alpha = \frac{k}{\rho\upsilon}, M = \frac{\sigma B_0^2}{a\rho}, \gamma_1 = \frac{g\beta_T D_1}{a^2}, \gamma_2 = \frac{g\beta_c D_2}{a^2}$$

$$Ra = \frac{16\sigma^* T_\infty^3}{2kk^*}, Ec = \frac{a^2 x^2}{c_p\left(T_s - T_\infty\right)}, \Pr = \frac{\upsilon\rho c_p}{k}, \mathrm{Q}_0 = \frac{Q}{a\rho c_p}, Nt = \frac{\tau D_T\left(T_s - T_\infty\right)}{\upsilon T_\infty}$$

$$Nb = \frac{\tau D_B\left(C_s - C_\infty\right)}{\upsilon}, Kr = \frac{K_1}{a}, Sc = \frac{\upsilon}{D_B}$$

Corresponding to:

Micro polar parameter, magnetic field, thermal Grashof number, mass Grashof number, radiation parameter, Eckert number, Prandtl number, Heat generation parameter, thermophoresis parameter, Brownian motion, chemical reaction parameter, Schmidt number respectively.

In other to solve the problem in equation (9), the method of Decomposition is introduced with the decomposed equation :

$$\left.\begin{aligned}
&f_n = \frac{1}{(1+\alpha)} L_1^{-1}\left[-\sum_{k=0}^{n} f_{n-k} f_k^{//} + \sum_{k=0}^{n} f_{n-k}^{/} f_k^{/}, -\alpha g_n^{/} + Mf_n^{/} - \gamma_1\theta_n - \gamma_2\varphi_n\right] \\
&g_n = \frac{1}{\left(1+\frac{\alpha}{2}\right)} L_2^{-1}\left[\sum_{k=0}^{n} f_{n-k}^{/} g_k - \sum_{k=0}^{n} g_{n-k}^{/} f_k + \alpha\left(2g_n + f_n^{//}\right)\right] \\
&\theta_n = \frac{\Pr}{(1+Ra)} L_2^{-1}\left[-\sum_{k=0}^{n} f_{n-k}^{/}\theta_k + \sum_{k=0}^{n}\theta_{n-k}^{/} f_k - MEcf_n^{/} - Nb\sum_{k=0}^{n}\theta_{n-k}^{/}\varphi_k^{/} - Nt\sum_{k=0}^{n}\theta_{n-k}^{/}\theta_k^{/} - \Pr Q_0\theta_n\right] \\
&\varphi_n = L_2^{-1}\left[-Sc\sum_{k=0}^{n} f_{n-k}^{/}\varphi_k + Sc\sum_{k=0}^{n}\varphi_{n-k}^{/} f_k + ScKr\varphi_n - \frac{Nt}{Nb}\theta_n^{//}\right]
\end{aligned}\right\} \tag{10}$$

Maple 17 software is used to carry out the integrals in equation (10) and the initial guesses are assumed to be:

$$\left.\begin{aligned} f_0(\eta) &= \frac{1}{\gamma}\left(1-a_1e^{-\eta}\right) \\ g_0(\eta) &= \frac{1}{\gamma}a_2e^{-\eta} \\ \theta_0(\eta) &= a_3e^{-\eta} \\ \varphi_0(\eta) &= a_4e^{-\eta} \end{aligned}\right\} \tag{11}$$

The final solutions are given as

$$\left.\begin{aligned} f(\eta) &= \sum_{n=0}^{3} f_n \\ g(\eta) &= \sum_{n=0}^{3} g_n \\ \theta(\eta) &= \sum_{n=0}^{3} \theta_n \\ \varphi(\eta) &= \sum_{n=0}^{3} \varphi_n \end{aligned}\right\} \tag{12}$$

RESULTS AND DISCUSSION

The nonlinear coupled ordinary differential equations with corresponding boundary conditions in equation (9) are solved using the Adomian Decomposition method to obtain the solution. The results obtained are compared with the existing literature and a good agreement is observed as seen in Table 1.

Table 1: Comparison of Skin friction ($f^{//}(0)$) with slip parameter (γ) when $M=\gamma_1=\gamma_2=0$

γ	**Ibrahim (2016)**	**Khan *et al.* (2020)**	**Dawar *et al.*(2021)**	**Present result**
0	1	1	1	1
0.1	0.872082	0.8719	0.87209	0.8735
0.2	0.776377	0.7762	0.77638	0.7973
0.3	0.701548	0.7014	0.70155	0.7309
0.5	0.591196	0.5922	0.5912	0.5981
1	0.43016	0.4301	0.43017	0.4994
2	0.28398	0.284	0.28399	0.2613
3	0.214055	0.214	0.21406	0.2706
5	0.144841	0.1448	0.14485	0.1378
10	0.081243	0.0812	0.08124	0.0788
20	0.04379	0.0438	0.04379	0.0431

Figures 1 to 4 is the graphical presentation of the variation of micropolar parameter on fluid velocity, micro rotation, temperature and concentration respectively. As the micro rotation parameter increases, the fluid velocity, temperature and concentration get boosted while the micro rotation dropped.

Figures 5 to 8 depict the implication of thermal Grashof number on fluid velocity, micro rotation, temperature and concentration profiles respectively. Grashof number which is the ratio of buoyancy to viscous force is found to enhance the fluid velocity temperature and concentration but decrease the micro rotation. Increase in temperature leads to decrease in the fluid viscousity and when the fluid viscousity dropped, the fluid velocity thickens.

Figures 9 to 12 shows the variation of mass Grashof number to fluid velocity, micro rotation, temperature and concentration respectively. The same effects as in the case of thermal Grashof number are observed.

Figures 13 to 16 present the variation of magnetic parameter on fluid velocity, micro rotation, temperature and concentration. As the magnetic parameter increases the fluid velocity decreases, while the micro rotation, temperature and concentration all rises. The magnetic parameter produces a drag like force which causes the velocity to shrink as it gets boosted.

Figure 17 present the variation of Prandtl number on temperature distributions. Prandtl number is the ratio of momentum diffusion to heat diffusion in the fluid. For higher values of Prandtl number, the fluid temperature drops. Figure 18 displays the distribution of heat generation parameter on temperature profile and is seen that as the parameter increases from negative to positive, the fluid temperature is boosted. The negativity shows absorption while the positivity signifies generation. Figure 19 show the distribution of Eckert number on temperature profile. The Eckert number is found to be an increasing agent of the fluid temperature. Figure 20 present the variation of Schimdt number on concentration profile. As the schimdt number increases, the concentration of the fluid reduces.

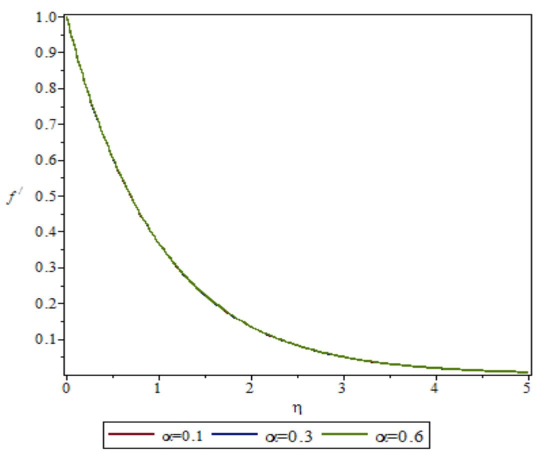

Figure 1: Variation of α on $f^{/}$

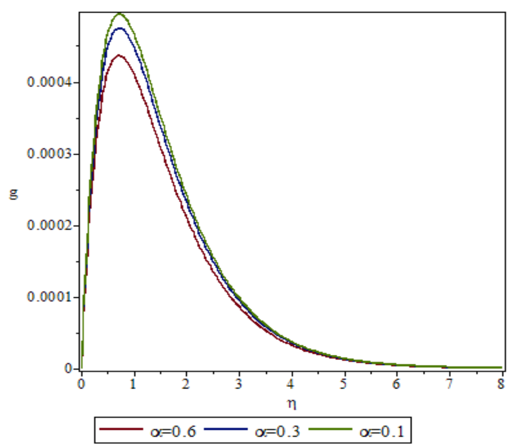

Figure 2: Variation of α on g

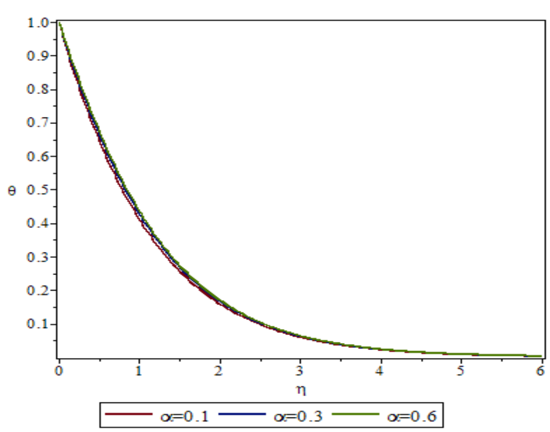

Figure 3: Variation of α on θ

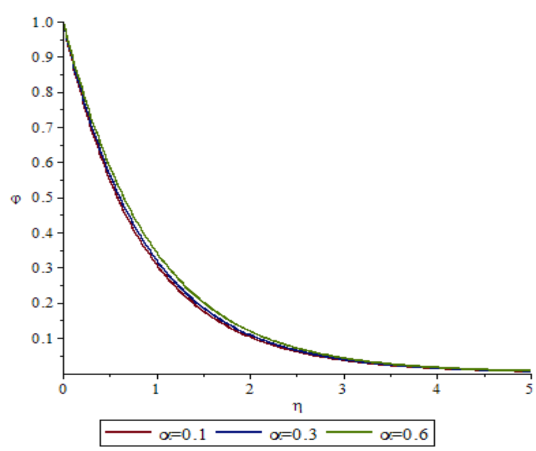

Figure 4: Variation of α on φ

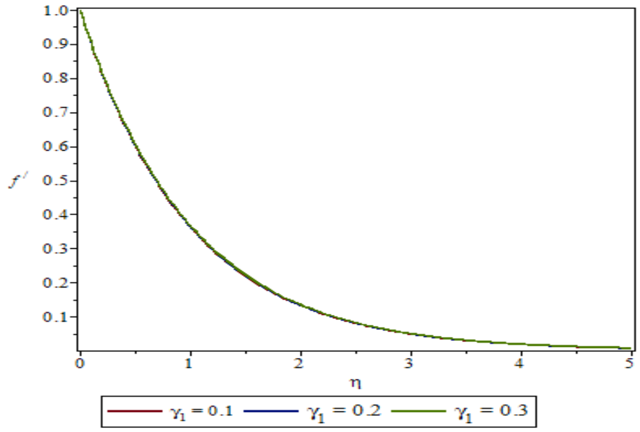

Figure 5: Variation of γ_1 on f'

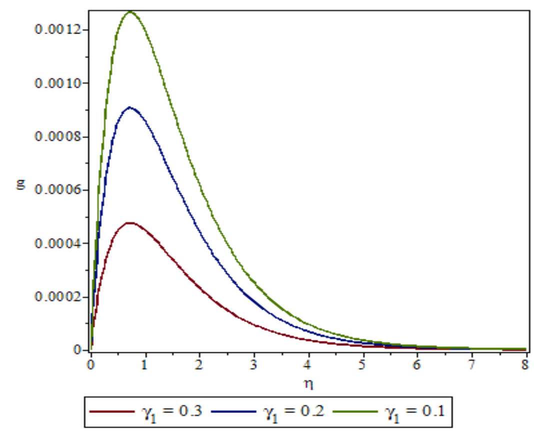

Figure 6: Variation of γ_1 on g

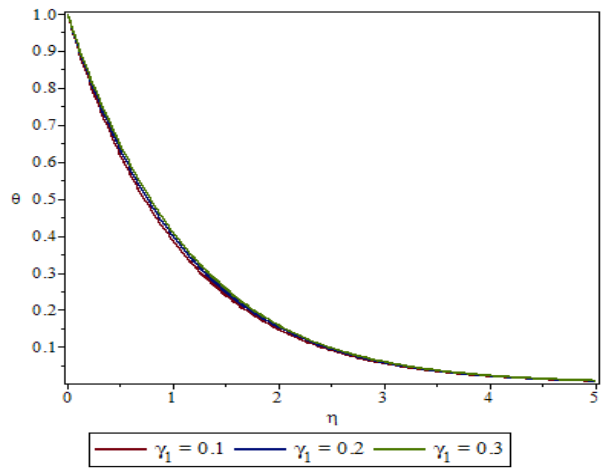

Figure 7: Variation of γ_1 on θ

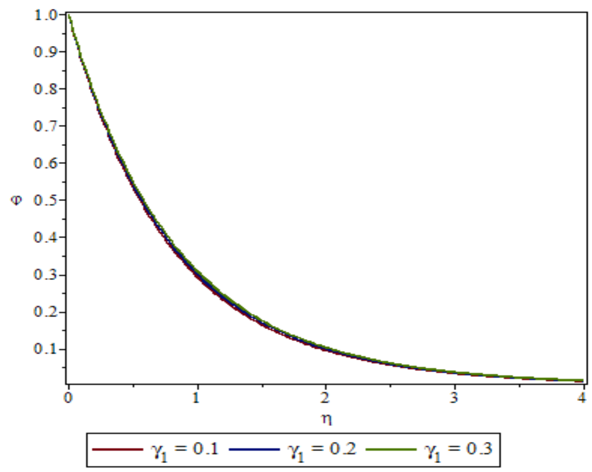

Figure 8: Variation of γ_1 on φ

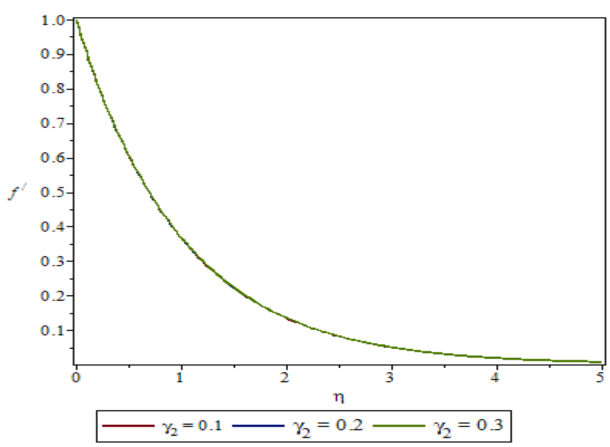

Figure 9: Variation of γ_2 on f'

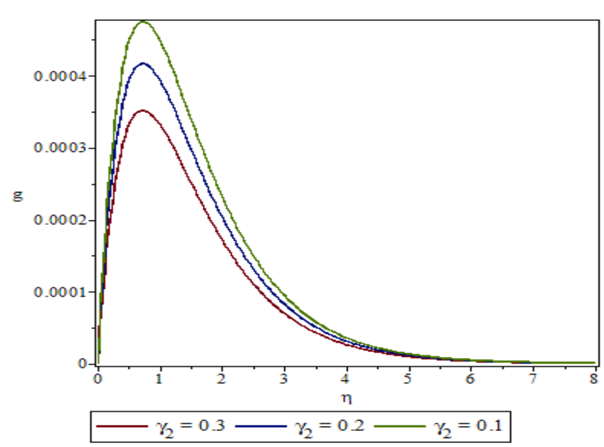

Figure 10: Variation of γ_2 on g

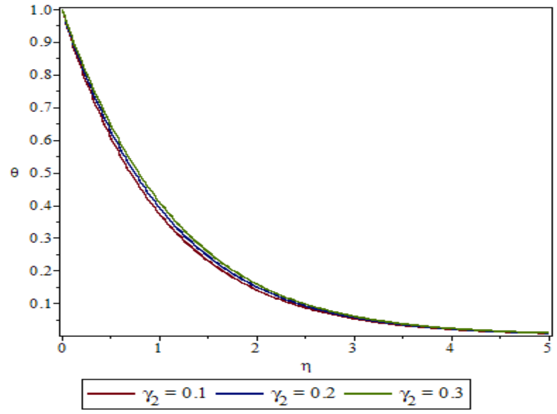

Figure 11: Variation of γ_2 on θ

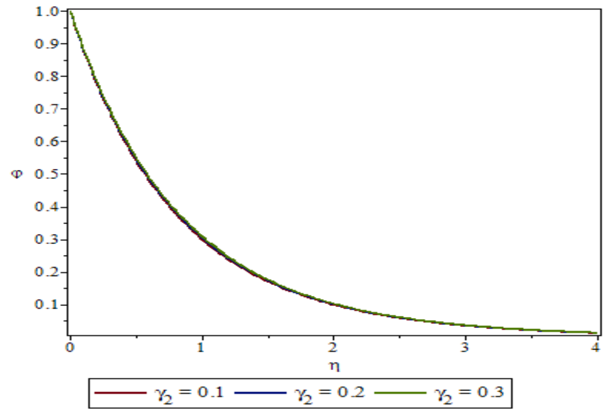

Figure 12: Variation of γ_2 on φ

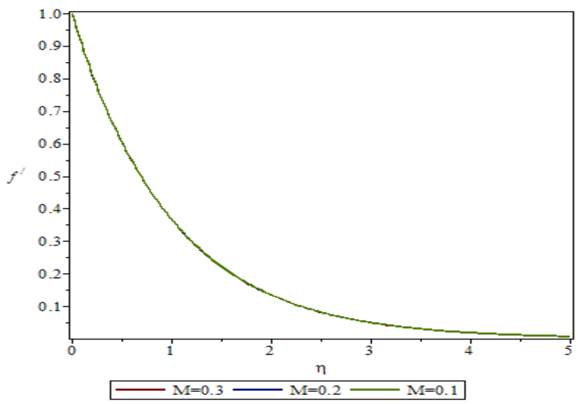

Figure 13: Variation of M on $f^{/}$

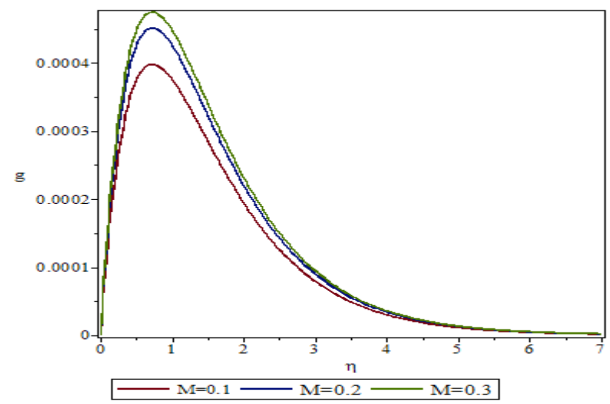

Figure 14: Variation of M on g

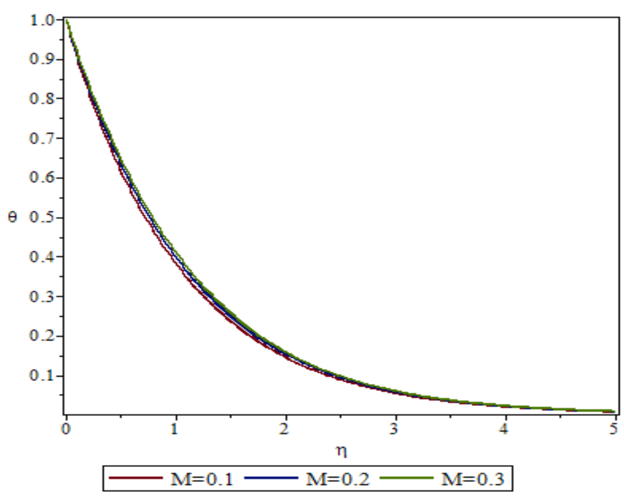

Figure 15: Variation of M on θ

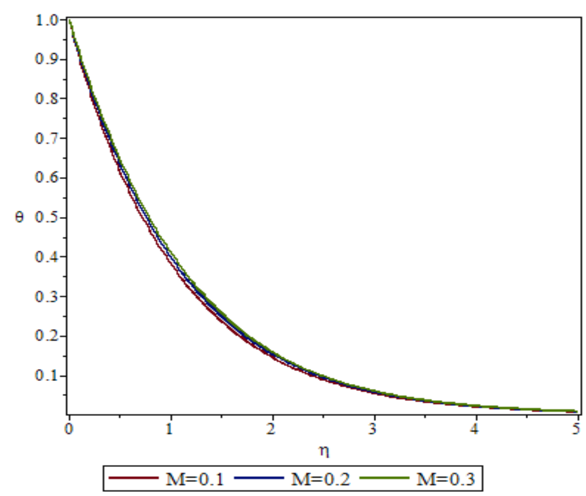

Figure 16: Variation of M on φ

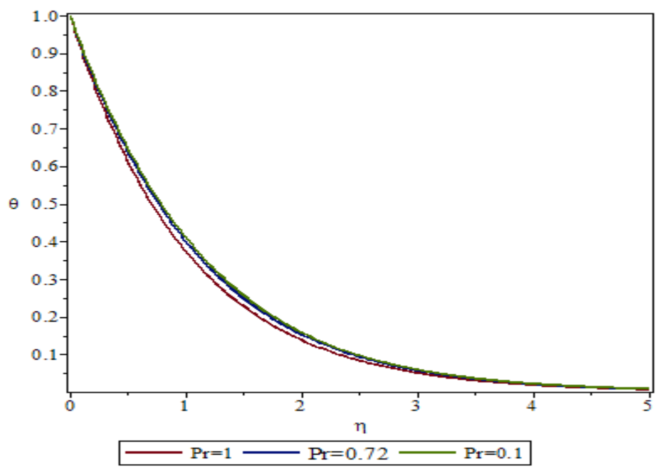

Figure 17: Variation of $\Pr$ on θ

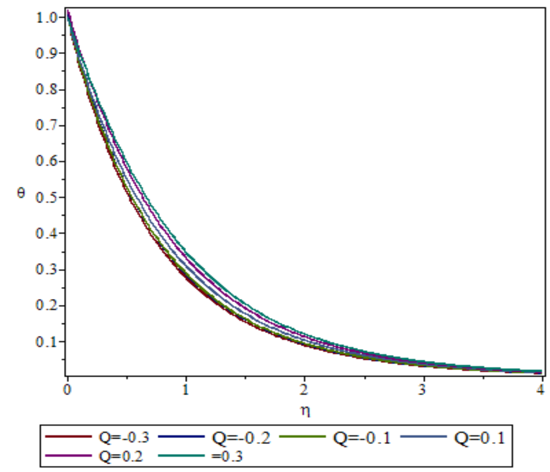

Figure 18: Variation of Q on θ

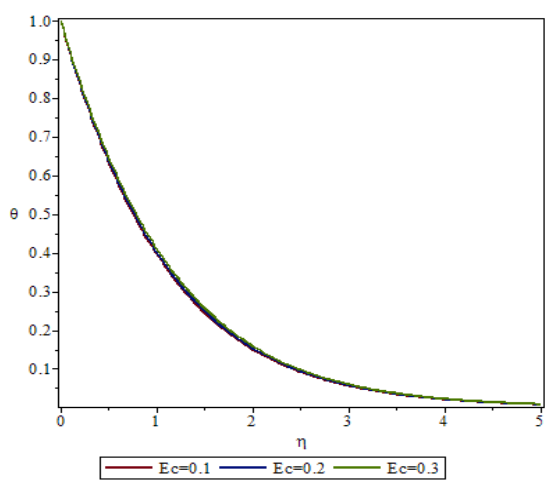

Figure 19: Variation of Ec on θ

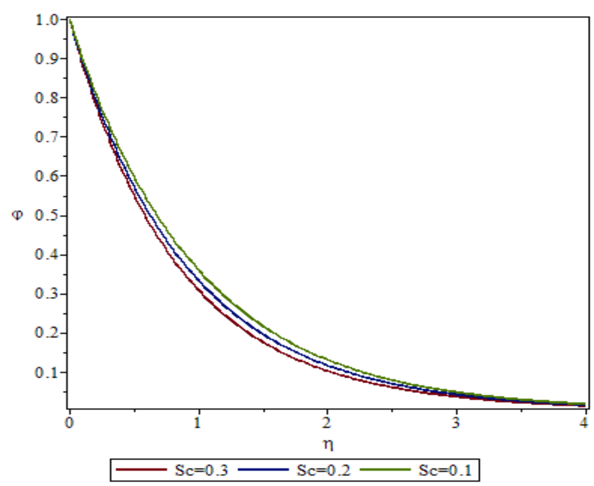

Figure 20: Variation of Sc on φ

CONCLUSION

This present work extended the work of Dawar *et al.* (2021) by introducing the heat generation and neglecting the slip parameter. The formulated problems were solved using decomposition method and the results obtained were compared with literature as presented in Table 1. These results show an agreement between the present work and the literature. The work is hereby concluded with the following observations:

1. The maximum velocity on the sheet surface remains constant due to the fact that the slip condition was neglected.
2. The micro rotation is zero on the sheet surface irrespective of the parameter vary.
3. The graphs presented in this work all satisfy both the initial and the boundary conditions.
4. The fluid velocity, micro rotation, temperature, and mass concentration all decayed to zero at free stream.

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