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Fractional order of pneumococcal pneumonia infection model with Caputo Fabrizio operator

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ABSTRACT

In this study, we present the Pneumococcal Pneumonia infection model using fractional order derivatives in the Caputo-Fabrizio sense. We use fixed-point theory to prove the existence of the solution and investigate the uniqueness of the model variables. The fractional Adams-Bashforth method is used to compute an iterative solution to the model. Finally, using the model parameter values to explain the importance of the arbitrary fractional order derivative, the numerical results are presented.

Introduction

Pneumonia is an acute breathing infection affecting the lungs. The lungs are composed of little bags called alveoli which, when a healthy person breathes, fill with air [1–3]. The greatest single infectious cause of death in children worldwide is pneumonia. In 2017, 808694 children under 5 years of age were infected by pneumonia, accounting for 15 percent of all child deaths under 5. Children and families everywhere are affected by pneumonia, but it is most common in South Asia and sub-Saharan Africa. It can be avoided with easy procedures, and managed with low-cost, low-tech treatment and care [4]. Children can be protected from pneumonia. There are a number of infectious agents, including viruses, bacteria and fungi, that cause pneumonia. In a variety of ways, pneumonia can be transmitted. The viruses and bacteria that are usually found in the nose or throat of a child will, if inhaled, infect the lungs. They can also spread from a cough or sneeze through airborne droplets. Moreover, pneumonia, particularly during and shortly after birth, can spread through the blood. The prevention of pneumonia in children is an important component of an infant mortality mitigation plan. Hib, pneumococcus, measles and whooping cough (pertussis) immunizations are the most effective way to stop pneumonia [5,6]. Many researchers have suggested models for understanding the dynamics of infectious diseases and for quantitative forecasts of various preventive measures and their efficacy. [6–12]. In the last decade, very few significant studies have been performed on the transmission dynamics of pneumonia[13–17]. A deterministic and stochastic mathematical model of pneumonia transmission dynamics was developed in all the above studies, but none of them considered the fractional order aspect of the model.However, up till now, there is no work which has been designed to analyze Fractional Order of Pneumococcal Pneumonia Infection Model with Caputo Fabrizio Operator. This, therefore, motivated us to undertake this study to fulfill this gap. The emerging field of mathematical modelling with fractional order derivatives has been used as a powerful method to investigate the complex dynamics of different real phenomena in various areas of science and engineering [18–27]. Several fractional operators of Order $\varepsilon \in [0,1]$ are presented in the literature. The definition of the most common derivative of Caputo fractional order and relevant concepts has been established [28–31]. The Caputo fractional order derivative has been used to express a variety of problems that can be found in different fields [32–37]. The remaining part of this paper is organized as follows: the Section “Formulation of the model” deals with the formulation of the model, Section “Fractional model” deals with the analysis of the fractional order model, in Section “Numerical scheme and simulations”, A numerical scheme and simulations

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Table 1
Details of parameters [38].

Parameter	Initial Value	Description
μ	0.0002/day	Natural death rate
Λ	10.00/day	Recruitment rate
θ	0.34/day	Proportion of vunerable people that join the carrier group
σ	0.33/day	Disease induced death rate
β	0.0.011/day	Rate of recovery for carrier individuals
α	0.03/day	Force of infection
τ	0.07/day	Rate of recovery rate for infected people
π	0.01/day	Symptom rate for carrier individuals
η	0.0241/day	Rate of loosing immunity for treated individuals
γ	0.06/day	Vaccination rate for susceptible people
ϕ	9.4/day	Vaccination rate for treated people
ω	0.00112/day	Transmission coefficient for asymptomatic people
δ	7.6/day	Rate of transmission
k	1–10/day	Rate of contact
p	0.0–1/days	Rate of infection per contact

based on Adams-Bashforth approach is presented, Section "Discussion of results" is the discussion of results, finally, we give a brief conclusion in the last section.

Formulation of the model

The model consists of four compartments that categorize individuals with respect to the disease based on their status. $S(t)$ represents susceptible individuals at risk of contracting infection with pneumonia at time t. $C(t)$ represents carrier individuals that bear the bacteria of pneumonia and are able to spread the infection at t. $I(t)$ represents contagious persons capable of transmitting the infection to persons at risk at time t and $R(t)$ is the recovered individuals treated with pneumonia at the time of t. The following system of ordinary equations is obtained based on assumptions and definitions of variables and parameters described in Table 1.

$$\left.\begin{array}{l} S' = \Lambda - (\alpha+\mu)S + \eta R, \\ C' = \alpha\theta S - (\mu+\beta+\pi)C, \\ I' = \alpha(1-\theta)S + \pi C - (\tau+\mu\sigma)I, \\ R' = \beta C + \tau I(\mu+\eta)R, \end{array}\right\} \tag{1}$$

where $\alpha = \delta(\frac{1+wC}{N})$, $\delta = kp$

Definition 1. The function $y \in H'(m,n), n > m, \varepsilon \in [0,1]$ for the fractional derivatives in the Caputo sense [39] is defined as

$$D_t^\varepsilon(y(t)) = \frac{A(\varepsilon)}{1-\varepsilon}\int_m^t Y'(y)\exp\left[-\varepsilon\frac{t-y}{1-\varepsilon}\right]dy \tag{2}$$

$A(\varepsilon)$ represents the normalized function satisfying $A(0) = A(1) = 1$ [16]. The Caputo derivatives for the case $y \in H'(m,n)$ can be express as

$$D_t^\varepsilon(y(t)) = \frac{\varepsilon A(\varepsilon)}{1-\varepsilon}\int_m^t (Y(t) - Y(y)\exp\left[-\varepsilon\frac{t-y}{1-\varepsilon}\right]dy. \tag{3}$$

Remark 2. If $\beta = \frac{1-\varepsilon}{\varepsilon} \in [0,\infty], \varepsilon = \frac{1}{1+\beta} \in [0,1]$, *then* (3) can be written as

$$D_t^\beta(y(t)) = \frac{B(\beta)}{\varepsilon}\int_m^t (Y'(y)\exp\left[-\frac{t-y}{\beta}\right]dy; \quad B(0) = B(\infty) = 1. \tag{4}$$

that is

$$\lim_{\beta\to 0}\frac{1}{\beta}\exp\left[-\frac{t-y}{\beta}\right] = \delta(y-t). \tag{5}$$

Definition 3. Let $0 < \varepsilon < 1$, and the fractional derivative is expressed as

$$D_t^\varepsilon(Y(t)) = h(t), \tag{6}$$

then the corresponding integral of fractional order ε is defined as

$$F_t^\varepsilon(Y(t)) = \frac{2(1-\varepsilon)}{(2-\varepsilon)A(\varepsilon)}h(t) + \frac{2\varepsilon}{(2-\varepsilon)A(\varepsilon)}\int_0^t h(s)ds, \ t \geqslant 0. \tag{7}$$

Remark 4. By using the result

$$\frac{2}{2A(\varepsilon) - \varepsilon(A)\varepsilon} = 1, \tag{8}$$

this implies that $A(\varepsilon) = \frac{2}{2-\varepsilon}, 0 < \varepsilon < 1$, from [36] the new CF fractional derivative of order $0 < \varepsilon < 1$ is expressed as

$$D_t^\varepsilon(Y(t)) = \frac{1}{1-\varepsilon}\int_0^t Y'(y)\exp\left[-\varepsilon\frac{t-y}{1-\varepsilon}\right]dy \tag{9}$$

Fractional model

The fractional pneumonia model in the Caputo sense is given below.

$$\left.\begin{array}{l} {}^{CF}D_t^\varepsilon S = \Lambda^\varepsilon - (\alpha^\varepsilon + \mu^\varepsilon)S + \eta^\varepsilon R, \\ {}^{CF}D_t^\varepsilon C = \alpha^\varepsilon\theta^\varepsilon S - (\mu^\varepsilon + \beta^\varepsilon + \pi^\varepsilon)C, \\ {}^{CF}D_t^\varepsilon I = \alpha^\varepsilon(1-\theta^\varepsilon)S + \pi^\varepsilon C - (\tau^\varepsilon + \mu^\varepsilon\sigma^\varepsilon)I, \\ {}^{CF}D_t^\varepsilon R = \beta^\varepsilon C + \tau^\varepsilon I(\mu^\varepsilon + \eta^\varepsilon)R, \end{array}\right\} \tag{10}$$

with the initial conditions

$$S(0) = a_1, C(0) = a_2, I(0) = a_3, R(0) = a_4. \tag{11}$$

Existence and uniqueness

In this section, we apply the fixed point results to show the existence and the uniqueness of the fractional model in (10).

System (10) can be expressed in the equivalent form as,

$$\left.\begin{array}{l} {}^{CF}D_t^\varepsilon[S(t)] = P_1(t,S), \\ {}^{CF}D_t^\varepsilon[C(t)] = P_2(t,C), \\ {}^{CF}D_t^\varepsilon[I(t)] = P_3(t,I), \\ {}^{CF}D_t^\varepsilon[R(t)] = P_4(t,R). \end{array}\right\} \tag{12}$$

Following the definition of Caputo fractional integral operator defined in [36], (12) can be written in the following integral equation and with Caputo fractional interval order as $0 < \varepsilon < 1$,

$$\left.\begin{array}{l} S(t) - S(0) = 2\frac{1-\varepsilon}{(2-\varepsilon)N(\varepsilon)}P_1(t,S) + 2\frac{\varepsilon}{(2-\varepsilon)N(\varepsilon)}\int_0^t P_1(\phi,S)d\phi, \\ C(t) - C(0) = 2\frac{1-\varepsilon}{(2-\varepsilon)N(\varepsilon)}P_2(t,C) + 2\frac{\varepsilon}{(2-\varepsilon)N(\varepsilon)}\int_0^t P_2(\phi,C)d\phi, \\ I(t) - I(0) = 2\frac{1-\varepsilon}{(2-\varepsilon)N(\varepsilon)}P_3(t,I) + 2\frac{\varepsilon}{(2-\varepsilon)N(\varepsilon)}\int_0^t P_3(\phi,I)d\phi, \\ R(t) - R(0) = 2\frac{1-\varepsilon}{(2-\varepsilon)N(\varepsilon)}P_4(t,R) + 2\frac{\varepsilon}{(2-\varepsilon)N(\varepsilon)}\int_0^t P_4(\phi,R)d\phi \end{array}\right\} \tag{13}$$

Theorem 5. *The kernel P_1 is said to satisfied the Lipchitz and contraction if the inequality below holds.*

Proof. Let S and S_1 be the two function for P_1, then

$$||P_1(t,S)-P_1(t,S_1)||=||-\delta(S(t)-S(t_1))+\delta w(t)(S(t)-S(t_1))-\mu(S(t)-S(t_1)||$$

by applying property of norm,

$$||P_1(t,S)-P_1(t,S_1)||\leqslant\{\delta wq+\mu+\delta\}||\{S(t)-S(t_1)\}|| \tag{14}$$

$$\leqslant\tau_1||\{S(t)-S(t_1)\}|| \tag{15}$$

taking $\tau_1=\{\delta wq+\mu+\delta\}$, where $||C(t)||\leqslant\delta w$ is bounded function, therefore,

$$||P_1(t,S)-P_1(t,S_1)||\leqslant\tau_1||\{S(t)-S(t_1)\}||. \tag{16}$$

Therefore, for P_1 the Lipschitz condition is obtained and if an additionally $0\leqslant(\delta wq+\mu+\delta)<1$ which gives a contraction. We can therefore verify the Lipschitz condition for other equations.

$$\left.\begin{array}{l}||P_2(t,C)-P_2(t,C_1)||\leqslant\tau_2||C(t)-C(t_1)||\\||P_3(t,I)-P_3(t,I_1)||\leqslant\tau_3||I(t)-I(t_1)||\\||P_4(t,R)-P_4(t,R_1)||\leqslant\tau_4||R(t)-R(t_1)||\end{array}\right\} \tag{17}$$

□ Next, we write the difference between the successive term in (11) in a recursive form, this can be expressed as

$$\left.\begin{array}{l}\theta_{1n}(t)=S_n(t)-S_{n-1}(t)=\dfrac{2(1-\varepsilon)}{(2-\varepsilon)N(\varepsilon)}(P_1(t,S_{n-1}))-P_1(t,S_{n-2})+2\dfrac{\varepsilon}{(2-\varepsilon)N(\varepsilon)}\displaystyle\int_0^t(P_1(\rho,S_{n-1}-P_1(\rho,S_{n-2})))d\rho\\\theta_{2n}(t)=C_n(t)-C_{n-1}(t)=\dfrac{2(1-\varepsilon)}{(2-\varepsilon)N(\varepsilon)}(P_2(t,C_{n-1}))-P_2(t,C_{n-2})+2\dfrac{\varepsilon}{(2-\varepsilon)N(\varepsilon)}\displaystyle\int_0^t(P_2(\rho,C_{n-1}-P_2(\rho,C_{n-2})))d\rho\\\theta_{3n}(t)=I_n(t)-I_{n-1}(t)=\dfrac{2(1-\varepsilon)}{(2-\varepsilon)N(\varepsilon)}(P_3(t,I_{n-1}))-P_3(t,I_{n-2})+2\dfrac{\varepsilon}{(2-\varepsilon)N(\varepsilon)}\displaystyle\int_0^t(P_3(\rho,I_{n-1}-P_3(\rho,I_{n-2})))d\rho\\\theta_{4n}(t)=R_n(t)-R_{n-1}(t)=\dfrac{2(1-\varepsilon)}{(2-\varepsilon)N(\varepsilon)}(P_4(t,R_{n-1}))-P_4(t,R_{n-2})+2\dfrac{\varepsilon}{(2-\varepsilon)N(\varepsilon)}\displaystyle\int_0^t(P_4(\rho,R_{n-1}-P_4(\rho,R_{n-2})))d\rho\end{array}\right\} \tag{18}$$

subject to the following initial conditions $S_0(t)=S(0),C(t)=C(0),I_0(t)=I(0)$ and $R_0(t)=R(0)$.

Applying norm on (18),

$$\begin{aligned}||\theta_1n(t)||=&\ ||S_n(t)-S_{n-1}(t)||\\=&\ ||2\frac{(1-\varepsilon)}{(2-\varepsilon)N(\varepsilon)}(P_1(t,S_{n-1}))-P_1(t,S_{n-2})\\&+2\frac{\varepsilon}{(2-\varepsilon)N(\varepsilon)}\int_0^t(P_1(\rho,S_{n-1})-P_1(\rho,S_{n-2})d\rho||\end{aligned} \tag{19}$$

By applying the triangular inequality, we expressed (19) as.

$||S_n(t)-S_{n-1}(t)||\leqslant2\frac{(1-\varepsilon)}{(2-\varepsilon)N(\varepsilon)}||(P_1(t,S_{n-1}))-P_1(t,S_{n-2})||+2\frac{\varepsilon}{(2-\varepsilon)N(\varepsilon)}||\int_0^t(P_1(\rho,S_{n-1})-P_1(\rho,S_{n-2})d\rho||$

By applying the Lipschitz condition in (16),

$||S_n(t)-S_{n-1}(t)||\leqslant2\frac{(1-\varepsilon)}{(2-\varepsilon)N(\varepsilon)}\tau_1||S_{n-1}-S_{n-2}||+2\frac{\varepsilon}{(2-\varepsilon)N(\varepsilon)}\tau_1\times\int_0^t||S_{n-1}-S_{n-2}||d\rho$

Therefore

$$||\phi_{1n}(t)||\leqslant2\frac{(1-\varepsilon)}{(2-\varepsilon)N(\varepsilon)}\tau_1||\theta_{n-1}(t)||+2\frac{\varepsilon}{(2-\varepsilon)N(\varepsilon)}\tau_1\int_0^t||\theta_{(n-1)}(\rho)||d\rho \tag{20}$$

In a similar way, from (18), we obtain

$$\left.\begin{array}{l}||\phi_{2n}(t)||\leqslant2\dfrac{(1-\varepsilon)}{(2-\varepsilon)N(\varepsilon)}\tau_2||\theta_{2(n-1)}(t)||+2\dfrac{\varepsilon}{(2-\varepsilon)N(\varepsilon)}\tau_2\displaystyle\int_0^t||\theta_{2(n-1)}(\rho)||d\rho\\||\phi_{3n}(t)||\leqslant2\dfrac{(1-\varepsilon)}{(2-\varepsilon)N(\varepsilon)}\tau_3||\theta_{3(n-1)}(t)||+2\dfrac{\varepsilon}{(2-\varepsilon)N(\varepsilon)}\tau_3\displaystyle\int_0^t||\theta_{3(n-1)}(\rho)||d\rho\\||\phi_{4n}(t)||\leqslant2\dfrac{(1-\varepsilon)}{(2-\varepsilon)N(\varepsilon)}\tau_4||\theta_{4(n-1)}(t)||+2\dfrac{\varepsilon}{(2-\varepsilon)N(\varepsilon)}\tau_4\displaystyle\int_0^t||\theta_{4(n-1)}(\rho)||d\rho\end{array}\right\} \tag{21}$$

From (21), we write that

$$\left.\begin{array}{l}S_n(t)=\displaystyle\sum_{i=1}^n\theta_{1i}(t),\\C_n(t)=\displaystyle\sum_{i=1}^n\theta_{2i}(t),\\I_n(t)=\displaystyle\sum_{i=1}^n\theta_{3i}(t),\\R_n(t)=\displaystyle\sum_{i=1}^n\theta_{4i}(t),\end{array}\right\} \tag{22}$$

Theorem 6. *The system of solution of fractional pneumococcal pneumonia infection Caputo-Farizio model exist if an only if the inequality below hold and one can find* t_1

$$2\frac{(1-\varepsilon)}{(2-\varepsilon)N(\varepsilon)}\tau_1+2\frac{\varepsilon t_1}{(2-\varepsilon)N(\varepsilon)}\tau_i<1,\ \textit{for}\ i=1,\ldots,4.$$

Proof. By considering Eq. (21), and applying the recursive technique, the following succeeding results are obtained.

$$\left.\begin{array}{l}||\theta_{1n}(t)||\leqslant||S_n(0)||\left[\left(\dfrac{2(1-\varepsilon)}{(2-\varepsilon)N(\varepsilon)}\tau_1\right)+\left(\dfrac{2\varepsilon}{(2-\varepsilon)N(\varepsilon)}\tau_1t\right)\right]^n\\||\theta_{2n}(t)||\leqslant||C_n(0)||\left[\left(\dfrac{2(1-\varepsilon)}{(2-\varepsilon)N(\varepsilon)}\tau_2\right)+\left(\dfrac{2\varepsilon}{(2-\varepsilon)N(\varepsilon)}\tau_2t\right)\right]^n\\||\theta_{3n}(t)||\leqslant||I_n(0)||\left[\left(\dfrac{2(1-\varepsilon)}{(2-\varepsilon)N(\varepsilon)}\tau_3\right)+\left(\dfrac{2\varepsilon}{(2-\varepsilon)N(\varepsilon)}\tau_3t\right)\right]^n\\||\theta_{4n}(t)||\leqslant||R_n(0)||\left[\left(\dfrac{2(1-\varepsilon)}{(2-\varepsilon)N(\varepsilon)}\tau_4\right)+\left(\dfrac{2\varepsilon}{(2-\varepsilon)N(\varepsilon)}\tau_4t\right)\right]^n\end{array}\right\} \tag{23}$$

Hence, the solution of the system exist and continuous. Furthermore, we consider the uniqueness of solution. Suppose there exists another solution of the model, say, $S_1(t)$, $C_1(t)$, $I_1(t)$, and $R_1(t)$, then

$$\begin{aligned}S(t)-S_1(t)=&\ 2\frac{(1-\varepsilon)}{(2-\varepsilon)N(\varepsilon)}(P_1(t,S)-P_1(t,S_1))\\&\times\int_0^t(P_1(\rho,S)-P_1(\rho,S_1))d\rho\end{aligned} \tag{24}$$

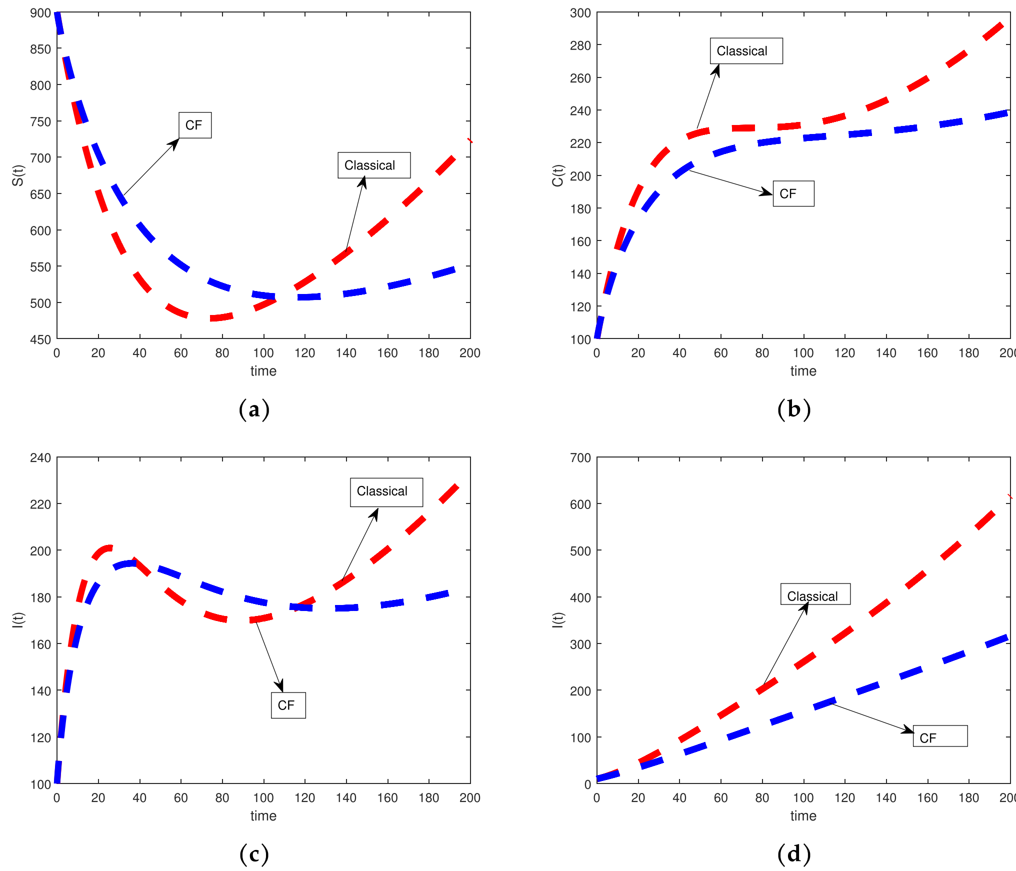

Fig. 1. Comparison of each state variables in classical and CF sense for the values of the fractional order $\varepsilon = 0.8890$.

Applying norm to (24),

$$||S(t) - S_1(t)|| \leq 2\frac{(1-\varepsilon)}{(2-\varepsilon)N(\varepsilon)}||P_1(t,S) - P_1(t,S_1)|| + 2\frac{\varepsilon}{(2-\varepsilon)N(\varepsilon)} \times \int_o^t ||P_1(\rho,S) - P_1(\rho,S_1(t))||d\rho. \tag{25}$$

Using the Lipschiz condition in (16),

$$||S(t) - S_1(t)|| \leq 2\frac{(1-\varepsilon)}{(2-\varepsilon)N(\varepsilon)}\tau_1||s(t) - S_1(t)|| + 2\frac{\varepsilon}{(2-\varepsilon)N(\varepsilon)} \times \tau_1 t||S(t) - S_1(t))||. \tag{26}$$

This simplifies to

$$||S(t) - S_1(t)||\left(1 - \frac{2(1-\varepsilon)}{(2-\varepsilon)N(\varepsilon)}\tau_1 - \frac{2\varepsilon}{(2-\varepsilon)N(\varepsilon)}\tau_1 t\right) \leqslant 0 \tag{27}$$

□

Theorem 7. *If the condition below is valid,*

$$\left(1 - \frac{2(1-\varepsilon)}{(2-\varepsilon)N(\varepsilon)}\tau_1 - \frac{2\varepsilon}{(2-\varepsilon)N(\varepsilon)}\tau_1 t\right)\rangle 0$$

then the solution of the fractional order model will be unique.

Proof. Assuming the condition in (27), holds, then

$$||S(t) - S_1(t)||\left(1 - \frac{2(1-\varepsilon)}{(2-\varepsilon)}\tau_1 - \frac{2\varepsilon}{(2-\varepsilon)N(\varepsilon)}\tau_1 t\right) \leq 0 \tag{28}$$

therefore

$$||S(t) - S_1(t)|| \leqslant 0 \tag{29}$$

$\Rightarrow$

$$S(t) = S_1(t) \tag{30}$$

Following the same approach, we can obtain a similar equality for the remaining equations. Hence, the solution of the fractional model is unique. □

Numerical scheme and simulations

We present the approximate solution of the fractional order pneumococcal model using two-step fractional Adam-Bashforth approach for the Caputo-Fabrizio fractional derivatives [25]. First, we write the system of equations in the form of fractional volterra using the elementary theorem of integration. We consider the first equation of the model in order to arrive at the desired iterative scheme. From the first equation in (11), we obtain.

$$S(t) - S(0) = \frac{(1-\varepsilon)}{N(\varepsilon)}P_1(t,S) + \frac{\varepsilon}{N(\varepsilon)}\int_0^t P_1(\rho,S)d \quad \rho t = t_{n+1}, \text{ suchthat, } n = 0,1,2,\ldots \tag{31}$$

we have

$$S(t_{n+1}) - S_n = \frac{1-\varepsilon}{N(\varepsilon)}\{P_1(t_n,S_n) - P_1(t_{n-1},S_{n-1})\} + \frac{\varepsilon}{N(\varepsilon)}\int_{t_n}^{t_{n+1}} P_1(t,S)dt \tag{32}$$

Approximating the function $P_1(t,S)$ by interpolation polynomial in the interval $[t_k, t_{(k+1)}]$, we obtain

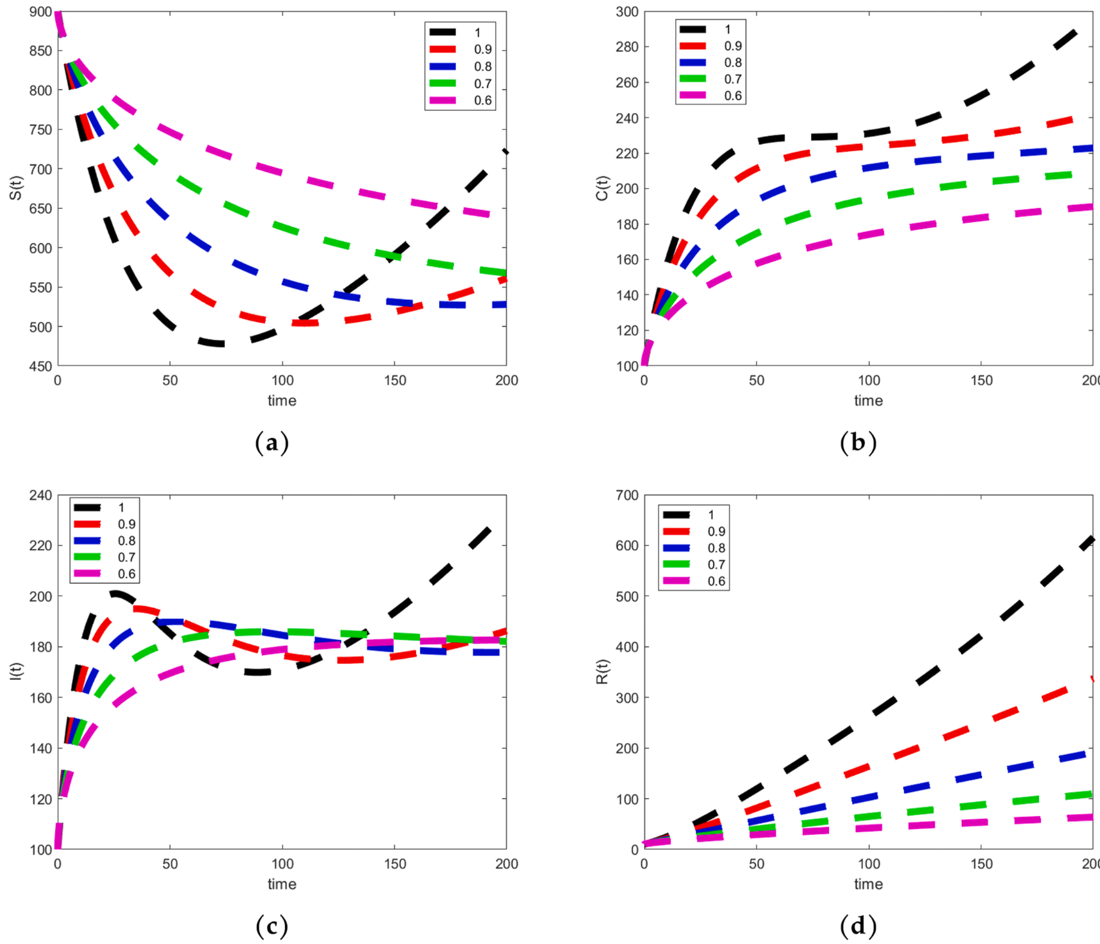

Fig. 2. Profile of each state variables for different values of the fractional order $\varepsilon = 1, 0.9, 0.8, 0.7, 0.6$.

$$Q_k(t) \cong \frac{f(t_k, x_k)}{j}(t - t_{k-1}) - \frac{f(t_{k-1}, x_{k-1})}{hj}(t - t_k) \quad \text{where } t_n - t_{n-1}. \tag{33}$$

by applying the polynomial in 33 to 32 and calculating the interest in (31), we obtain

$$\int_{t_n}^{t_{n+1}} P_1(t, S)dt = \int_{t_n}^{t_{n+1}} \frac{P_1(t_n, S_n)}{j}(t - t_{n-1}) - \frac{P_1(t_{n-1}), S_{n-1}}{j}(t - t_n)dt \tag{34}$$

$$= \frac{3j}{2}P_1(t_n, S_n) - \frac{j}{2}P_1(t_{n-1}, S_{n-1}). \tag{35}$$

substituting (35) into (31) and simplifying we obtain

$$S_{n+1} = S_n + \left(\frac{1-\varepsilon}{N(\varepsilon)} + \frac{3j}{2N(\varepsilon)}\right)P_1(t_n, S_n). \tag{36}$$

Using the same approach, the rest of system of Eq. (12) recursive formula can be obtained as

$$C_{n+1} = C_0 + \left(\frac{1-\varepsilon}{N(\varepsilon)} + \frac{3j}{2N(\varepsilon)}\right)P_2(t_n, C_n) - \left(\frac{1-\varepsilon}{N(\varepsilon)} + \frac{\varepsilon j}{2N(\varepsilon)}\right)P_2(t_{n-1}, C_{n-1}),$$

$$I_{n+1} = I_0 + \left(\frac{1-\varepsilon}{N(\varepsilon)} + \frac{3j}{2N(\varepsilon)}\right)P_3(t_n, I_n) - \left(\frac{1-\varepsilon}{N(\varepsilon)} + \frac{\varepsilon j}{2N(\varepsilon)}\right)P_3(t_{n-1}, I_{n-1})$$

$$R_{n+1} = R_0 + \left(\frac{1-\varepsilon}{N(\varepsilon)} + \frac{3j}{2N(\varepsilon)}\right)P_4(t_n, R_n) - \left(\frac{1-\varepsilon}{N(\varepsilon)} + \frac{\varepsilon j}{2N(\varepsilon)}\right)P_4(t_{n-1}, R_{n-1})$$

Discussion of results

This is the role in which we gain a deep insight into the model's dynamic behavior. The current section provides numerical simulations of the model while using the biological parameters as described earlier. As it was explained in the introduction, for more than 2 million children under 5 years of age, pneumonia is the leading cause of respiratory morbidity, mainly in low-income countries. It is a lung infection caused by bacteria, fungi, viruses, or other pathogens. Streptococcus pneumonia, also known as pneumococcus, is the most common cause of bacterial pneumonia. It is mainly marked by inflammation in the lungs' air sacs filled with fluid or pus, making it difficult to breathe. Rather alarming is the subtle existence of this virus. In the analysis of the dynamic of the virus, serious attention is needed. As a result, we intend to provide in-depth insights into the complex behavior of the Pneumococcal Pneumonia infection model in a fractional sense using Caputo Fabrizio operator. Fractional derivatives have been shown to be very successful in modeling real-world problems. In light of this, we employ the initial conditions as $S(0) = 900, C(0) = 100, I(0) = 100$ and $R(0) = 10$, and also the parameter values $\mu = 0.0002, \Lambda = 10.00, \theta = 0.34, \sigma = 0.33, \beta = 0.011, \alpha = 0.03, \tau = 0.07, \pi = 0.01$ and $\eta = 0.0241$. The propagation dynamics of an infectious ailment can be thoroughly comprehended under different computational simulations of the proposed fractional version of the model for state variables of interest. To understand the complex behavior of the model and to see the effect of the fractional order, we have carried out numerous simulations. In Fig. 1, a comparison between classical variant $(\upsilon = 1)$ and CF variant $(\upsilon = 0.8890)$ of the Pneumococcal Pneumonia infection model has been depicted. It can be observed that, the CF has given a better approximate

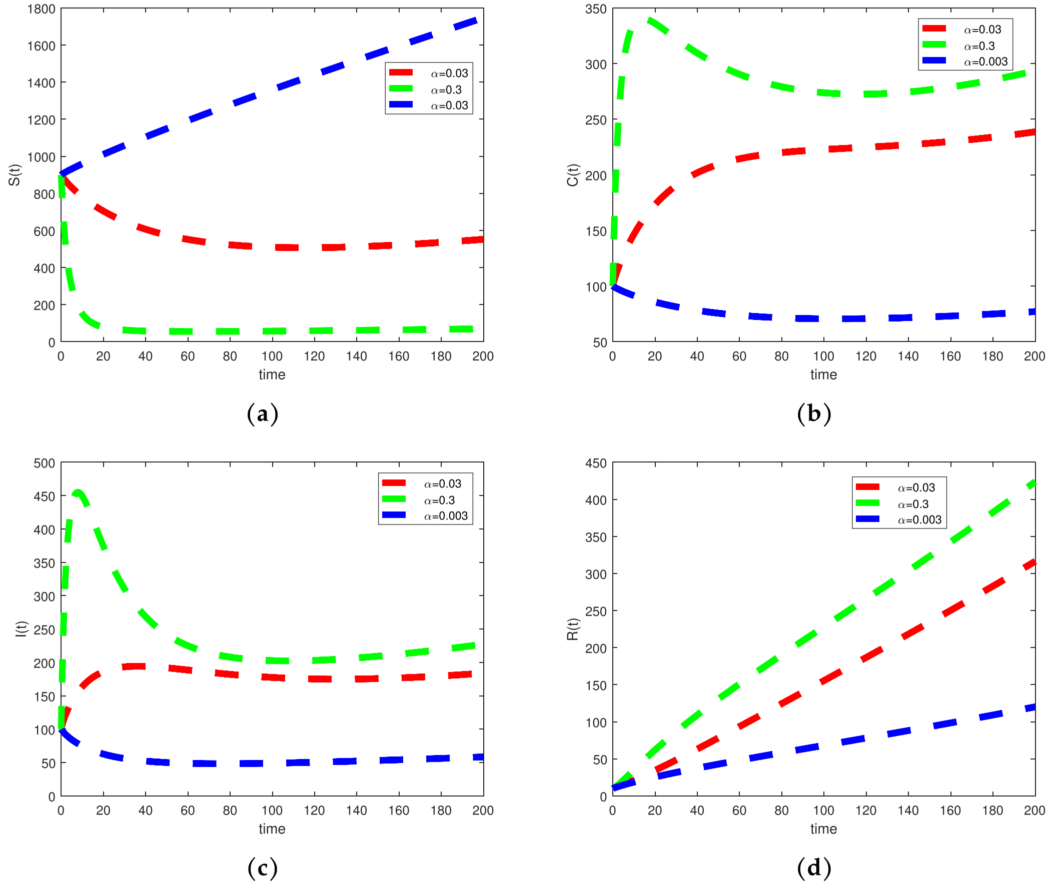

Fig. 3. Profile of each state variables with increasing and decreasing values of the force of infection $\alpha = 0.003, 0.03, 0.3$.

solution than the classical variant. Moreover, the value of the fractional order υ has been varied for $\upsilon = 1, 0.9, 0.8, 0.7, 0.6$ in Fig. 2, in order to analyze the impact of the fractional order υ on the model's dynamic behavior for all the four compartments. When the $S(t)$ decreases, the $C(t)$ as well as the $I(t)$ get increased. However, at some point both the $C(t)$ and $I(t)$ start to decrease. An important point should be noted in Fig. 2, and Fig. 2(c), for $\upsilon = 1$, after increasing, the curves start to decrease and consequently get increased again. Whereas, for the fractional variant the curves continue to decrease. And this gets $R(t)$ increased as seen in Fig. 2 (c). This is one of the advantages of fractional calculus over classical calculus. Furthermore, one of the most important and effective parameter called the force of infection α which is the rate at which susceptible individuals acquire an infectious disease has been varied for increasing and decreasing values in order to see further the dynamical characteristics of the model. The results depicted in Fig. 3, with the decreasing/increasing values of α, it can clearly be observed how effective is the force of infection in the behavior of the model.

Conclusion

The Pneumococcal Pneumonia infection model was modelled in the current study by one of the robust non-local fractional operators called the Caputo Fabrizio. For more than 2 million children under 5 years of age, pneumonia is the leading cause of respiratory morbidity, especially in low income countries. A more critical study of the dynamics of this subtle virus is of vehement importance. In the literature for researching the transmission dynamics of a disease, the fractional operator employed has been shown to be ideally suited. The fractionalized order is υ and the dimensional consistency between the rest of the parameters was considered. As a consequence, many significant features of the proposed fractional version of the model, such as the formation of the model, the nature and uniqueness of the solution via the fixed point theorem, have been reported. It should be noted that the model of the fractional type disease under investigation understands the disease's actions more correctly than the integer order version. In addition, by means of an effective numerical scheme, various numerical simulations were carried out in order to shed more light on the model's characteristics. It will be in our best interests if we can have access to country-specific real data to validate our model

Author contribution statement

O. J. Peter, conceived the idea, O. J. Peter, A. Yusuf, and K. Oshinubi, A. A. Ibrahim, K. Oshinubi, A.I. Abioye, T.A. Ayoola and J.O. Lawal did the simulation, writing-original draft, review and editing, O. J. Peter, Ayoola and F.A.Oguntolu formulated the model, O. J. Peter, and A. Yusuf wrote the coding aspect.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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