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A Mathematical Model of a Yellow Fever Dynamics with Vaccination

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Abstract

In this paper, a mathematical model describing the dynamics of yellow fever epidemics, which involves the interactions of two principal communities of Hosts (Humans) and vectors (mosquitoes) is considered .The existence and uniqueness of solutions of the model were examined by actual solution. We conduct local stability analysis for the model. The results show that it is stable under certain conditions. The system of equations describing the phenomenon was solved analytically using parameter-expanding method coupled with direct integration. The results are presented graphically and discussed. It is discovered that improvement in Vaccination strategies will eradicate the epidemics.

Keywords: Yellow fever, model, vaccination, epidemics

1.0 Introduction

Yellow fever (YF), a hemorrhagic fever caused by a *Flavivirus*, family *Flaviviridae* [1, 2], is characterized by fever, chills, losses of appetite, nausea, muscle pains particularly in the back, and headaches [3]. There are more than 200,000 infections and 30,000 deaths every year [3]. About 90% of YF cases occur in Africa [4], and a billion people live in an area of the world where the disease is common [3]. It also affects tropical areas of South America, but not Asia [3, 5, 6]. The number of cases of yellow fever has been increasing in the last 30 years [3, 7], probably due to fewer people being immune, more people living in cities, people moving frequently, and changing climate [3]. The origin of the disease is Africa, from where it spread in South America through the slave trade in the 17th century [8, 9]. The yellow fever virus was the first human virus discovered [10], and its family comprises approximately 70 viruses [2], most of which are transmitted by arthropod insects (hence the name arthropod borne viruses or arboviruses). A safe and effective vaccine against yellow fever exists and some countries require vaccinations for travelers [3]. In rare cases (less than one in 200,000 to 300,000 doses), the vaccination can cause yellow fever vaccine-associated viscerotropic disease (YEL-AVD), which is fatal in 60% of cases, probably due to the genetic morphology of the immune system. Another possible side effect is an infection of the nervous system, which occurs in one in 200,000 to 300,000 cases, causing yellow fever vaccine-associated neurotropic disease (YEL-AND), which can lead to meningoencephalitis, fatal in less than 5% of cases [6]. In some rare circumstances, however, the fatality rate of vaccine induced diseases can reach alarming proportions, as observed recently by Mascheretti et al. [11], who found 1 death per million doses applied in a Southeastern Brazilian region.

This present study investigates the criteria under which the effectiveness of vaccination could lead to the stability of the equilibrium point. We establish the conditions for existence and uniqueness of the solution of models, conducted local stability analysis of the models and provide an analytical solution via parameter-expanding method.

2.0 Model Formulation

Following [12], the equations describing yellow fever epidemics are:

$$\frac{dS_h}{dt} = \beta_1 N - (\mu_1 + \delta)S_h - \alpha_1 I_m S_h \tag{1}$$

$$\frac{dI_h}{dt} = -(\mu_1 + \alpha + \rho)I_h + \alpha_1 I_m S_h \tag{2}$$

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$$\frac{dR_h}{dt} = -\mu_1 R_h + \delta S_h + \alpha I_h \tag{3}$$

$$\frac{dS_m}{dt} = \beta_2(S_m + (1-\theta)I_m) - \mu_2 S_m - \alpha_2 S_m I_h \tag{4}$$

$$\frac{dI_m}{dt} = \theta\beta_2 I_m - \mu_2 I_m + \alpha_2 S_m I_h \tag{5}$$

$$N_h(t) = S_h(t) + I_h(t) + R_h(t) \tag{6}$$

As initial condition based on our assumptions, we choose

$$S_h(0) = S_{h0}, \quad I_{h0}(0) = I_{h0}, \quad R_h(0) = R_{h0}, \quad S_m(0) = S_{m0}, \quad I_m(0) = I_{m0} \tag{7}$$

Where

Variables	Parameters
$S_h(t)$ - the density of susceptible humans	β_1 - the natural birthrate for hosts
$I_h(t)$ - the density of infected humans	β_2 - the natural birthrate for vectors
$R_h(t)$ - the density of recovered humans	μ_1 - the natural mortality rate for hosts
$S_m(t)$ - the density of susceptible mosquitoes	μ_2 - the natural mortality rate for vectors
$I_m(t)$ - the density of infected mosquitoes	α - the recovery rate
	α_1 - the effective bitting interaction rate between $S_h(t)$ *and* $I_m(t)$ compartments
	α_2 - the effective bitting interaction rate between $S_m(t)$ *and* $I_h(t)$ compartments
	ρ - the death rate from infection
	θ - the proportion of the offsprings of $I_m(t)$ that is infected vertically
	δ - the vaccination rate

3.0 Method of Solution

3.1 Existence and Uniqueness of Solution

Theorem 1: Let $\rho = 0$ .Then the equations (1) – (5) with initial conditions (7) has a unique Solution for all $t \geq 0$.

Proof: Let $\rho = 0$, $\phi(t) = S_h(t) + I_h(t) + R_h(t)$ and $\varphi(t) = S_m(t) + I_m(t)$ .We obtain

$$\frac{d\phi}{dt} = (\beta_1 - \mu_1)\phi, \qquad \phi(0) = (S_{h0} + I_{h0} + R_{h0}) = \phi_0 \tag{8}$$

$$\frac{d\varphi}{dt} = (\beta_2 - \mu_2)\varphi, \qquad \varphi(0) = (S_{m0} + I_{m0}) = \varphi_0 \tag{9}$$

By direct integration, we obtain the solution of problem (8) and (9) as

$$\phi(t) = \phi_0 e^{(\beta_1 - \mu_1)t} \quad \textit{and} \quad \varphi(t) = \varphi_0 e^{(\beta_2 - \mu_2)t} \tag{10}$$

Then, we obtain

$$S_h(t) = \phi_o e^{(\beta_1 - \mu_1)t} - (I_h(t) + R_h(t)) \tag{11}$$

$$I_h(t) = \phi_o e^{(\beta_1 - \mu_1)t} - (S_h(t) + R_h(t)) \tag{12}$$

$$R_h(t) = \phi_o e^{(\beta_1 - \mu_1)t} - (S_h(t) + I_h(t)) \tag{13}$$

$$S_m(t) = \varphi_0 e^{(\beta_2-\mu_2)t} - I_m(t) \quad (14)$$

$$I_m(t) = \varphi_0 e^{(\beta_2-\mu_2)t} - S_m(t) \quad (15)$$

Hence, there exists a unique solution of problem (1) – (5). This completes the proof.

3.2 Stability Analysis

Since the equation (3) depends on equation (1) and (2), it will not be consider in the analysis.
Our system of equations (1) – (5) has a trivial steady state:

$$\overline{S_h} = \frac{\beta_1 N_h}{\mu_1+\delta}, \quad \overline{I_h} = 0, \quad \overline{S_m} = 0, \quad \overline{I_m} = 0 \quad (16)$$

And a non-trivial steady state:

$$\left.\begin{aligned} \widehat{S_h} &= \frac{\beta_1 N_h - \Phi_2(\mu_1+\alpha+\rho)}{\mu_1+\delta} \\ \widehat{I_h} &= \frac{(\theta\beta_2-\mu_2)(\beta_2-\mu_2)}{\alpha_2(\theta\beta_2-\mu_2)-\alpha_2\beta_2(1-\theta)} \\ \widehat{S_m} &= \frac{\Phi_1\alpha_1(\theta\beta_2-\mu_2)}{\Phi_2\Phi_2\alpha_2(\mu_1+\alpha+\rho)} \\ \widehat{I_m} &= \frac{\Phi_2(\mu_1+\alpha+\rho)}{\Phi_1\alpha_1} \end{aligned}\right\} \quad (17)$$

corresponding to clearance of infection and active disease respectively,

where $\Phi_1 = \dfrac{\beta_1 N_h - k_2(\mu_1+\alpha+\rho)}{\mu_1+\delta}$, $\Phi_2 = \dfrac{(\theta\beta_2-\mu_2)(\beta_2-\mu_2)}{\alpha_2(\theta\beta_2-\mu_2)-\alpha_2\beta_2(1-\theta)}$

Theorem 2: If $\dfrac{\beta_1 N_h}{\mu_1+\delta} \neq \dfrac{\beta_1 N_h - k_2(\mu_1+\alpha+\rho)}{\mu_1+\delta}$ there exist two equilibria.

Proof: The infection-free equilibrium is given by $p_1 = \left(\dfrac{\beta_1 N_h}{\mu_1+\delta}, 0, 0, 0\right)$

If $I_h \neq 0$, $S_m \neq 0$, $I_m \neq 0$, *then* $S_h = \dfrac{\beta_1 N_h - k_2(\mu_1+\alpha+\rho)}{\mu_1+\delta}$

Hence, the other equilibrium is

$$p_2 = \left(\frac{\beta_1 N_h - \Phi_2(\mu_1+\alpha+\rho)}{\mu_1+\delta}, \frac{(\theta\beta_2-\mu_2)(\beta_2-\mu_2)}{\alpha_2(\theta\beta_2-\mu_2)-\alpha_2\beta_2(1-\theta)}, \frac{\Phi_1\alpha_1(\theta\beta_2-\mu_2)}{\Phi_2\Phi_2\alpha_2(\mu_1+\alpha+\rho)}, \frac{\Phi_2(\mu_1+\alpha+\rho)}{\Phi_1\alpha_1}\right)$$

$$p_2 = (\Phi_1, \Phi_2, \Phi_3, \Phi_4)$$

This completes the proof.

Next, we shall conduct stability analysis of the critical points.

Then, the jacobian matrix of our system of equations (1) – (5) is

$$Df(S_h, I_h, S_m, I_m) = \begin{pmatrix} -(\mu_1+\delta)-\alpha_1 I_m & 0 & 0 & -\alpha_1 S_h \\ \alpha_1 I_m & -(\mu_1+\alpha+\rho) & 0 & \alpha_1 S_h \\ 0 & -\alpha_2 S_m & (\beta_2-\mu_2-\alpha_2 I_h) & \beta_2(1-\theta) \\ 0 & \alpha_2 S_m & \alpha_2 I_h & (\theta\beta_2-\mu_2) \end{pmatrix} \quad (18)$$

The linearization of (18) at $p_1 = \left(\dfrac{\beta_1 N_h}{\mu_1+\delta}, 0, 0, 0\right)$ is

$$Df\left(\frac{\beta_1 N_h}{\mu_1+\delta},0,0,0\right)=\begin{pmatrix} -(\mu_1+\delta) & 0 & 0 & \frac{-\alpha_1\beta_1 N_h}{\mu_1+\delta} \\ 0 & -(\mu_1+\alpha+\rho) & 0 & \frac{\alpha_1\beta_1 N_h}{\mu_1+\delta} \\ 0 & 0 & (\beta_2-\mu_2) & \beta_2(1-\theta) \\ 0 & 0 & 0 & (\theta\beta_2-\mu_2) \end{pmatrix} \quad (19)$$

With eigenvalues:

$\lambda_1=-(\mu_1+\delta)<0$, $\lambda_2=-(\mu_1+\alpha+\rho)<0$, $\lambda_3=-(\mu_2-\beta_2)<0$ *iff* $\mu_2>\beta_2$

$\lambda_4=-(\mu_2-\theta\beta_2)<0$ *iff* $\mu_2>\theta\beta_2$

Here, we have four eigenvalues λ_i, $i=1,2,3,4$, with distinct non-zero and negative roots. Then, the equilibrium point $\left(\frac{\beta_1 N_h}{\mu_1+\delta},0,0,0\right)$ is locally asymptotically stable if all the conditions above hold.

Now, let us denote the endemic equilibrium (EE) points $(\Phi_1,\Phi_2,\Phi_3,\Phi_4)$ where each component corresponds to an earlier specified value.

The linearization of (18) at $p_2=(\Phi_1,\Phi_2,\Phi_3,\Phi_4)$ is

$$Df(\Phi_1,\Phi_2,\Phi_3,\Phi_4)=\begin{pmatrix} -b_1 & 0 & 0 & -b_2 \\ b_3 & -b_4 & 0 & b_2 \\ 0 & -b_5 & -b_6 & b_7 \\ 0 & b_5 & b_8 & -b_9 \end{pmatrix} \quad (20)$$

where,

$b_1=(\mu_1+\delta)+b_3$, $b_2=\alpha_1\Phi_1$, $b_3=\alpha_1\Phi_4$, $b_4=\mu_1+\alpha+\rho$, $b_5=\alpha_2\Phi_2$, $b_6=\mu_2-\beta_2+b_8$

$b_7=\beta_2-\beta_2\theta$, $b_8=\alpha_2\Phi_2$, $b_9=\mu_2-\theta\beta_2$

To evaluate the jacobian at the endemic equilibrium to determine the stability of the system.

Using elementary row transformation [13] , the equation (20) becomes

$$Df(\Phi_1,\Phi_2,\Phi_3,\Phi_4)=\begin{pmatrix} -b_1 & 0 & 0 & -b_2 \\ 0 & -b_1b_4 & 0 & (b_1b_2-b_2b_3 \\ 0 & 0 & -(b_6-b_8) & (b_7-b_9) \\ 0 & 0 & 0 & -b_{10} \end{pmatrix} \quad (21)$$

where, $b_{10}=(b_8-b_6)(b_1b_2b_5-b_2b_3b_5-b_1b_4b_9)-b_1b_4b_8(b_7-b_9)$

Hence, the eigenvalues are:

$\lambda_1=-b_1=-(\mu_1+\delta+\alpha\Phi_4)<0$

$\lambda_2=-b_1b_4=-(\mu_1+\delta+\alpha\Phi_4)(\mu_1+\delta+\rho)<0$

$\lambda_3=-(\mu_2-\beta_2)<0$ *iff* $\mu_2>\beta_2$

$\lambda_4=-b_{10}<0$ *iff* $(b_8-b_6)(b_1b_2b_5-b_2b_3b_5-b_1b_4b_9)>(b_1b_4b_8)(b_7-b_9)$

Therefore, the endemic equilibrium (EE) is locally asymptotically stable if and only the the above conditions hold.

3.3 Solution by Parameter-Expanding Method

Parameter-expanding method proposed by He was successfully applied to various engineering problems [14]. We applied Parameter-expanding method to equations (1) – (5), where details can be found in [14]. Suppose the solutions $S_h(t)$, $I_h(t)$, $R_h(t)$, $S_m(t)$ *and* $I_m(t)$ in (1) – (5) can be expressed as

$$\left.\begin{aligned} S_h(t) &= S_{h0}(t) + \alpha_1 S_{h1}(t) + \alpha_1^2 S_{h2}(t) + h.o.t \\ I_h(t) &= I_{h0}(t) + \alpha_1 I_{h1}(t) + \alpha_1^2 I_{h2}(t) + h..o.t \\ R_h(t) &= R_{h0}(t) + \alpha_1 R_{h1}(t) + \alpha_1^2 R_{h2} + h.o.t \\ S_m(t) &= S_{m0}(t) + \alpha_1 S_{m1}(t) + \alpha_1^2 S_{m2}(t) + h.o.t \\ I_m(t) &= I_{m0}(t) + \alpha_1 I_{m1}(t) + \alpha_1^2 I_{m2}(t) + h.o.t \end{aligned}\right\} \quad (22)$$

Let $\alpha_2 = a\alpha_1$

where h.o.t read "higher order terms in α_1" and in our analysis, we assume α_1 is small, so we are interested only in the first two terms.

Substituting (22) into (1) – (5) and processings, we obtain:

$$\frac{dS_{h0}}{dt} = \beta_1 N_h - k_1 S_{h0}, \qquad S_{h0}(0) = S_{h0} \quad (23)$$

$$\frac{dI_{h0}}{dt} = -k_2 I_{h0}, \qquad I_{h0}(0) = I_{h0} \quad (24)$$

$$\frac{dR_{h0}}{dt} = -\mu_1 R_{h0} + \delta S_{h0} + \alpha I_{h0}, \qquad R_{h0}(0) = R_{h0} \quad (25)$$

$$\frac{dS_{m0}}{dt} = \beta_2 S_{m0} + \beta_2(1-\theta) I_{m0} - \mu_2 S_{m0}, \qquad S_{mo}(0) = S_{m0} \quad (26)$$

$$\frac{dI_{m0}}{dt} = \theta\beta_2 I_{m0} - \mu_2 I_{m0}, \qquad I_{m0}(0) = I_{m0} \quad (27)$$

$$\frac{dS_{h1}}{dt} = -k_1 S_{h1} - I_{m0} S_{h0}, \qquad S_{h1}(0) = 0 \quad (28)$$

$$\frac{dI_{h1}}{dt} = -k_2 I_{h1} + I_{m0} S_{h0}, \qquad I_{h1}(0) = 0 \quad (29)$$

$$\frac{dR_{h1}}{dt} = -\mu_1 R_{h1} + \delta S_{h1} + \alpha I_{h1}, \qquad R_{h1}(0) = 0 \quad (30)$$

$$\frac{dS_{m1}}{dt} = \beta_2 S_{m1} + \beta_2(1-\theta) I_{m1} - \mu_2 S_{m1} - a S_{m0} I_{h0}, \qquad S_{m1}(0) = 0 \quad (31)$$

$$\frac{dI_{m1}}{dt} = \theta\beta_2 I_{m1} - \mu_2 I_{m1} + a S_{m0} I_{h0}, \qquad I_{m0}(0) = I_{m0} \quad (32)$$

Solving equations (23) – (32) by direct integration, we obtain

$$S_{h0}(t) = a_1 + a_2 e^{-k_1 t} \quad (33)$$

$$I_{h0}(t) = I_{h0} e^{-k_2 t} \quad (34)$$

$$R_{h0}(t) = a_3 e^{-\mu_1 t} + a_4 e^{-k_1 t} + a_5 e^{-k_2 t} + a_6 \quad (35)$$

$$S_{m0}(t) = a_7 e^{(\beta_2 - \mu_2)t} - I_{m0} e^{(\theta\beta_2 - \mu_2)t} \quad (36)$$

$$I_{m0}(t) = I_{m0} e^{(\theta\beta_2 - \mu_2)t} \quad (37)$$

$$S_{h1}(t) = a_8 e^{-k_1 t} + a_9 e^{2(\theta\beta_2 - \mu_2)t} - a_{10} e^{(\theta\beta_2 + \beta_2 - 2\mu_2)t} \quad (38)$$

$$I_{h1}(t) = a_{11} e^{-k_2 t} + a_{12} e^{(\theta\beta_2 + \beta_2 - 2\mu_2)t} - a_{13} e^{2(\theta\beta_2 - \mu_2)t} \quad (39)$$

$$R_{h1}(t) = a_{14} e^{-\mu_1 t} + a_{15} e^{2(\theta\beta_2 - \mu_2)t} + a_{16} e^{(\theta\beta_2 - \beta_2 - 2\mu_2)t} + a_{17} e^{-k_1 t} + a_{18} e^{-k_2 t} \quad (40)$$

$$S_{m1}(t) = a_{22} e^{(\beta_2 - \mu_2)t} + a_{23} e^{-(\mu_2 - \beta_2 + k_2)t} + a_{24} e^{(\theta\beta_2 - \mu_2 - k_2)t} - a_{25} e^{(\theta\beta_2 - \mu_2)t} \quad (41)$$

$I_{m1}(t) = a_{19}e^{(\beta_2 - \mu_2 - k_2)t} + a_{20}e^{(\theta\beta_2 - \mu_2 - k_2)t} - a_{21}e^{(\theta\beta_2 - \mu_2)t}$

where

$$k_1 = \mu_1 + \delta \quad , \quad k_2 = \mu_1 + \alpha + \rho, \quad a_1 = \frac{\beta_1 N_h}{k_1}, \quad a_2 = S_{h0} - a_1, \quad a_3 = R_{h0} - \frac{a_1\delta}{\mu_1} - \frac{a_2\delta}{\mu_1 - k_1} - \frac{\alpha I_{h0}}{\mu_1 - k_2} \tag{42}$$

$$a_4 = \frac{a_2\delta}{\mu_1 - k_1}, \quad a_5 = \frac{\alpha I_{h0}}{\mu_1 - k_2}, \quad a_6 = \frac{a_1\delta}{\mu_1}, \quad a_7 = I_{m0} + S_{m0}, \quad a_8 = \frac{a_7 I_{m0}}{\theta\beta_2 + \beta_2 - 2\mu_2 + k_1} - \frac{(I_{m0})^2}{2\theta\beta_2 - 2\mu_2 + k_1}$$

$$a_9 = \frac{(I_{m0})^2}{2\theta\beta_2 - 2\mu_2 + k_1}, \quad a_{10} = \frac{a_7 I_{m0}}{\theta\beta_2 + \beta_2 - 2\mu_2 + k_1}, \quad a_{11} = \frac{(I_{m0})^2}{2\theta\beta_2 - 2\mu_2 + k_2} - \frac{a_7 I_{m0}}{\theta\beta_2 + \beta_2 - 2\mu_2 + k_2}$$

$$a_{12} = \frac{a_7 I_{m0}}{\theta\beta_2 + \beta_2 - 2\mu_2 + k_2}, \quad a_{13} = \frac{(I_{m0})^2}{2\theta\beta_2 - 2\mu_2 + k_2}, \quad a_{14} = \frac{\alpha a_{13}}{2\theta\beta_2 - 2\mu_2 + \mu_1} - \frac{\alpha a_{12}}{\theta\beta_2 + \beta_2 - 2\mu_2 + \mu}$$

$$\frac{\alpha a_{11}}{\mu_1 - k_2} + \frac{a_{10}\delta}{\theta\beta_2 + \beta_2 - 2\mu_2 + \mu_1} - \frac{a_9\delta}{2\theta\beta_2 - 2\mu_2 + \mu_1} - \frac{a_8\delta}{\mu_1 - k_1}$$

$$a_{15} = \frac{a_9\delta}{2\theta\beta_2 - 2\mu_2 + \mu_1} - \frac{\alpha a_{13}}{2\theta\beta_2 - 2\mu_2 + \mu_1}, \quad a_{16} = \frac{\alpha a_{12}}{\theta\beta_2 + \beta_2 - 2\mu_2 + \mu_1} - \frac{a_{10}\delta}{\theta\beta_2 + \beta_2 - 2\mu_2 + \mu_1}$$

$$a_{17} = \frac{a_8\delta}{\mu_1 - k_1}, a_{18} = \frac{\alpha a_{11}}{\mu_1 - k_2}, a_{19} = \frac{a_0 a_7 I_{h0}}{\beta_2 - \theta\beta_2 - k_2}, a_{20} = \frac{a_0 I_{h0} I_{m0}}{k_2}, a_{21} = a_{19} + a_{20}$$

$$a_{22} = \frac{a_{19}\beta_2(1-\theta)}{k_2} - \frac{\beta_2 a_{20}(1-\theta)}{\theta\beta_2 - \beta_2 - k_2} + \frac{a_{21}\beta_2(1-\theta)}{\theta\beta_2 - \beta_2} - \frac{a_0 a_7 I_{h0}}{k_2} - \frac{a_0 I_{h0} I_{m0}}{\theta\beta_2 - \beta_2 - k_2}$$

$$a_{23} = \frac{a_0 a_7 I_{h0}}{k_2} - \frac{a_{19}\beta_2(1-\theta)}{k_2}, \quad a_{24} = \frac{a_{20}\beta_2(1-\theta)}{\theta\beta_2 - \beta_2 - k_2} + \frac{a_0 I_{h0} I_{mo}}{\theta\beta_2 - \beta_2 - k_2}, \quad a_{25} = \frac{a_{21}\beta_2(1-\theta)}{\theta\beta_2 - \beta_2}$$

The computations were done using computer symbolic algebraic package MAPLE.

4.0 Results and Discussion

Here the existence and uniqueness of solution of our system of equations (1) – (5) is proved by actual solutions. Also, u certain conditions, we have conducted local analysis of the disease-free and endemic equilibrium states. The results show both the disease-free and endemic equilibrium states are stable. Analytical solutions of equations (1) – (5) are achieved Parameter-expanding method and computed for the values of

$\beta_1 = 0.000095, \beta_2 = 0.001, \mu_1 = 0.000035, \mu_2 = 0.09, \alpha = 0.143, \alpha_1 = 0.01, \alpha_2 = 0.1, \rho = 0.02,$

$\theta = 0.01, \quad \delta = 0.5, S_{h0} = 100, I_{h0} = 50, R_{h0} = 0, S_{m0} = 50, I_{m0} = 30, N = 150$

The population of Susceptible ,Infected and Recovered individuals are depicted graphically in Figures 1 – 6.

From **Figure 1,**we observe that as the vaccination rate δ increases , the susceptible individuals reduces with time.

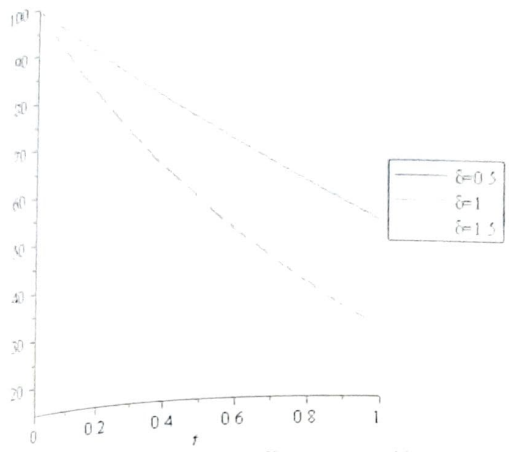

Figure 1: Plots of $S_h(t)$ against t for different values of δ and $\beta_1 = 0.000095$, $\beta_2 = 0.001$, $\mu_1 = 0.000035$, μ_2, $\alpha = 0.143$, $\alpha_1 = 0.01$, $\alpha_2 = 0.1$, $\rho = 0.02$, $\theta = 0.01$, $N = 150$, $S_{h0} = 100$, $I_{h0} = 50$, $R_{h0} = 0$, $S_{m0} = 50$, $I_{m0} = 30$

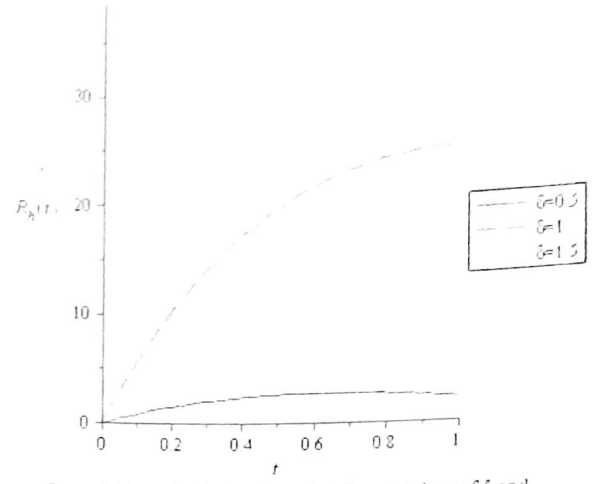

Figure 2: Plots of $R_h(t)$ against t for different values of δ and $\beta_1 = 0.000095$, $\beta_2 = 0.001$, $\mu_1 = 0.000035$, μ_2, $\alpha = 0.143$, $\alpha_1 = 0.01$, $\alpha_2 = 0.1$, $\rho = 0.02$, $\theta = 0.01$, $N = 150$, $S_{h0} = 100$, $I_{h0} = 50$, $R_{h0} = 0$, $S_{m0} = 50$, $I_{m0} = 30$

From Figure 2, we can conclude that with the increase in vaccination rate δ ,the recover individuals increases with time.

From Figure 3, we can conclude that with the increases in effective bitting interaction rate α_1 between $S_h(t)$ *and* $I_m(t)$ the susceptible individuals reduces to move to infected individuals due to increases in contact rate.

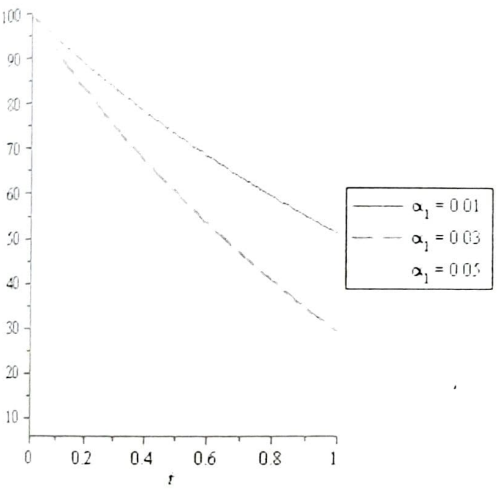

Figure 3: Plots of $S_h(t)$ against t for different values of α_1 and $\beta_1 = 0.000095$, $\beta_2 = 0.001$, $\mu_1 = 0.000035$, μ_2, $\alpha = 0.143$, $\delta = 0.5$, $\alpha_2 = 0.1$, $\rho = 0.02$, $\theta = 0.01$, $N = 150$, $S_{h0} = 100$, $I_{h0} = 50$, $R_{h0} = 0$, $S_{m0} = 50$, $I_{m0} = 30$

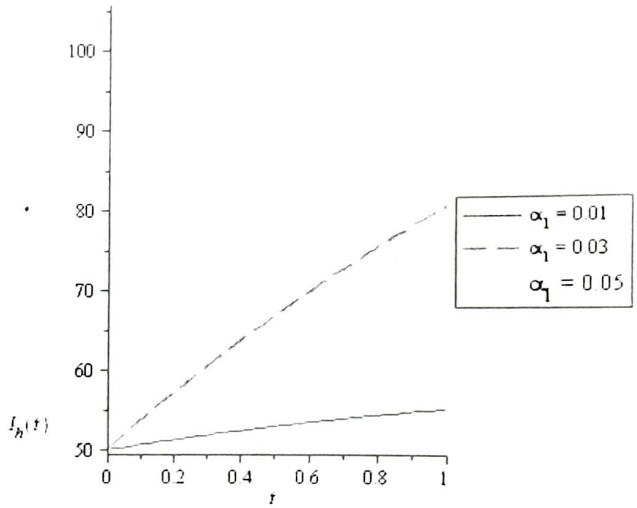

Figure 4: Plots of $I_h(t)$ against t for different values of α_1 and $\beta_1 = 0.000095$, $\beta_2 = 0.001$, $\mu_1 = 0.000035$, μ_2, $\alpha = 0.143$, $\delta = 0.5$, $\alpha_2 = 0.1$, $\rho = 0.02$, $\theta = 0.01$, $N = 150$, $S_{h0} = 100$, $I_{h0} = 50$, $R_{h0} = 0$, $S_{m0} = 50$, $I_{m0} = 30$

From Figure 4, we can conclude that with the increases in effective bitting interaction rate α_1 between $S_h(t)$ *and* $I_m(t)$ the infected individuals increases with time.

From Figure 5 and 6, we can conclude that with the increases in effective bitting interaction rate α_2 between $S_h(t)$ *and* $I_h(t)$, the susceptible individuals reduces and the infected individuals increases with time.

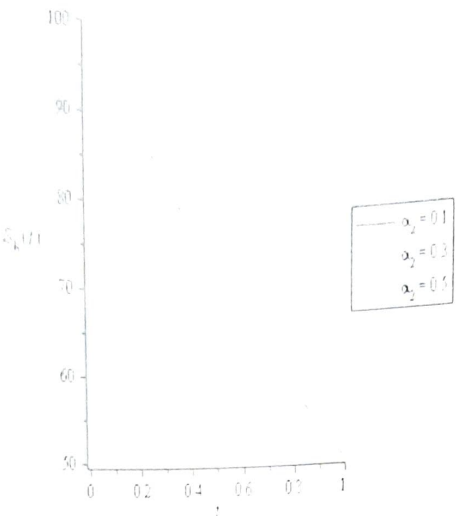

Figure 5 Plots of $S_h(t)$ against t for different values of α_2 and $\beta_1 = 0.000095$, $\beta_2 = 0.001$, $\mu_1 = 0.000035$, μ_2, $\alpha = 0.143$, $\delta = 0.5$, $\alpha_1 = 0.01$, $\rho = 0.02$, $\theta = 0.01$, $N = 150$, $S_{h0} = 100$, $I_{h0} = 50$, $R_{h0} = 0$, $S_{m0} = 50$, $I_{m0} = 30$

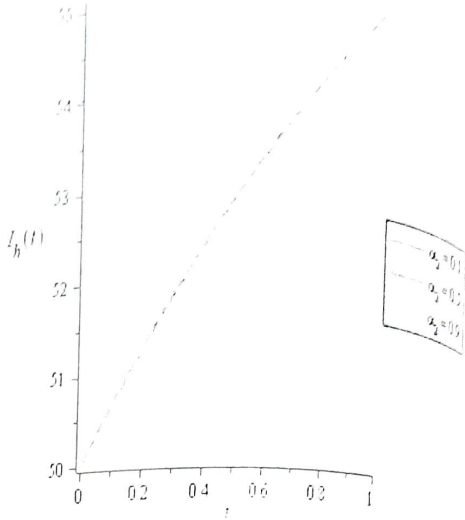

Figure 6 Plots of $I_h(t)$ against t for different values of α_2 and $\beta_1 = 0.000095$, $\beta_2 = 0.001$, $\mu_1 = 0.000035$, μ_2, $\alpha = 0.143$, $\delta = 0.5$, $\alpha_1 = 0.01$, $\rho = 0.02$, $\theta = 0.01$, $N = 150$, $S_{h0} = 100$, $I_{h0} = 50$, $R_{h0} = 0$, $S_{m0} = 50$, $I_{m0} = 30$

5.0 Conclusion

A non-linear mathematical model has been proposed and analyzed to study the effect of vaccination on the transmi... Yellow fever infection in a population. The disease free and endemic equilibria were obtained and their sta... investigated. The model showed that both the disease free equilibrium and endemic equilibrium are locally stable ... certain conditions. From the studies made on this paper we conclude that the most effective way to reduce the transmis... Yellow fever epidemic infection is to increase the effective vaccination rate and the outbreak of this epidemics will s... eradicated from our society.

6.0 References

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