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THE EFFECTS OF SORET AND DUFOUR ON THE TRANSIENT SOLUTIONS OF SECOND GRADE FLUIDS FLOW AND HEAT TRANSFER PAST A VERTICAL INFINITE PLATE

***Y.M. Aiyesimi*[1], * *B.N. Akintewe*[2,3], *M. Jiya*[2], *R.O. Olayiwola*[2], *G.A. Bolarin*[2] *and A. Yusuf*[2]**

[1]Department of Computer Sciences, Edo University Iyamho, Edo State, Nigeria.
[2]Department of Mathematical Sciences, Federal University of Technology, Minna, Niger State, Nigeria.
[3]Department of Mathematics, Adeyemi College of Education, Ondo, Ondo State, Nigeria

Abstract

This study discusses the transient state solutions to a system of a coupled equations; which address the continuity, momentum, energy and specie concentration equations for a second grade fluid. The Adomian Decomposition method was used to solve the problem for the flow geometry of a vertical infinite plate. The Soret and Dufour effects were investigated in terms of velocity, temperature and specie concentration profiles. The findings show that higher values of the Prandtl number, Soret and the suction parameter impede flow velocity, heat transfer temperature and concentration profiles while the Dufour number reinforces fluid flow velocity and fluid temperature fields.

***Keywords*:** Second grade fluids, Soret effect, Dufour effect.

1.0 Introduction

Second grade fluids form the simplest subclass of the differential type of the non-Newtonian fluids whose stress tensor sum up all the tensors formed from velocity field with up to two derivatives. Second grade fluids can model dilute polymer solutions, slurry flows, and industrial oils amongst others. The wide applicability and use of these fluids in several areas such as food processing, movement of biological fluids, plastic manufacture and performance of lubricants to mention but a few have elicited a lot of researches over the years.

The magnetic influence on the unsteady free convection flow of a second grade fluid near an infinite vertical plate with ramped wall temperature embedded in a porous material was studied in [1]. Velocity and skin friction for ramped temperature were found to be far less than the isothermal temperature; other assumptions as well as the effect of other parameters were equally investigated. Studies undertaken in [2] proffered closed form solutions for unsteady free convection flows of a second grade fluid near an isothermal vertical plate oscillating in its plane using the Laplace transform. Expressions were obtained for velocity and temperature and graphs were displayed for different dimensionless numbers, visco-elastic parameter, phase angle and time. Their work is an extension of known solutions in literature and has among other findings established the fact that the skin friction increases with time and phase angle. A steady case, two dimensional non-Newtonian second grade fluid under the influence of temperature dependent viscosity and thermal conductivity; other influences such as radiative heat, viscous dissipation and heat source/sink were also considered in [3].

The system of equations were then transformed and solved using the Runge Kutta shooting technique. Their findings showed that the visco-elastic parameter decreases with temperature distribution within the flow region which is affected by adjusting Prandtl number and the surface temperature parameter. Stagnation point flow of a second grade fluid over an unsteady stretching surface in the presence of variable free stream was discussed in [4]. Flow analysis that addressed dimensionless velocity and temperature were considered profiles using the Homotopy Analysis Method; chief among their findings was the inverse relationship existing between temperature and Prandtl number. In another research, [5] analyzed the unsteady mixed convection slip flow for casson fluid towards a non-linearly stretching sheet with slip and convective boundary conditions. They also investigated the effect of Soret, Dufour, viscous dissipation and heat generation absorption via the numerical method of solution and were able to show that fluid velocity

Corresponding Author: Akintewe B.N., Email: akintewe_bridget@yahoo.com, Tel: +2348034655108

the local unsteadiness and casson parameters while the influence of the Dufour number on temperature is more pronounced when compared with specie concentration. It was discovered that the temperature increases for the case of non linear thermal radiation. Another research in [6] worked on the effect of Soret and Dufour parameters as well as unsteadiness mass flux, thermophoresis and Brownian motion parameters on heat transfer characteristics for unsteady boundary layer using the numerical method. Dual solutions were obtained for reduced skin friction coefficient, reduced Nusselt number as well as the velocity, temperature and Nanoparticle volume fraction profiles. The Soret and Dufour parameters were found to influence the rate of heat transfer at the surface.
This study seeks to investigate the Soret and Dufour effects on the heat transfer parameters such as velocity, temperature and specie concentration. The method of Adomian Decomposition will be applied to the system of coupled equations. A graphical display of the findings will also be made available

2.0 MATHEMATICAL FORMULATION OF THE PROBLEM

The constitutive equation for second-grade fluid is;

$$T = -pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 \qquad (1)$$

where T is the Cauchy stress, $-pI$ is the spherical part of the stress due to constraint of incompressibility, p is the scalar pressure, I is the identity tensor, μ, α_1 and α_2 are measurable material constants. They denote, respectively, the viscosity, elasticity and cross-viscosity. A_1 and A_2 are Rivlin and Ericksen kinematical tensors and they denote, respectively, the rate of strain and acceleration. The Rivlin and Ericksen kinematical tensors, are described in [7] as;

$$A_1 = (grad\ V) + (grad\ V)^T \qquad (2)$$

$$A_2 = \frac{d}{dt} A_1 + A_1 (grad\ V)^T + A_1 (grad\ V) \qquad (3)$$

Theoretical investigations have indicated that for an exact model, satisfying the Clausius Duhem inequality and the assumption that the specific Helmholtz free energy be a minimum in equilibrium, the following conditions must hold:

$$\mu \geq 0 \quad \alpha_1 > 0 \quad \alpha_1 + \alpha_2 = 0 \qquad (4)$$

Consider an unsteady two-dimensional mixed convective boundary layer flow of an electrical conducting non-Newtonian second grade fluid through an infinite vertical plate heated in the presence of thermal and concentration buoyancy effects. The problem is being treated under boundary layer and Boussinesq approximation. The effect of viscous dissipation and Joule heating are also taken into account. The x-axis is taken in the upward direction of the plate and y-axis is normal to it. A constant magnetic field of strength B_0 is applied perpendicular to the plate and the effect of the induced magnetic field is being neglected. All the other fluid properties are then assumed to be isotropic and constant.

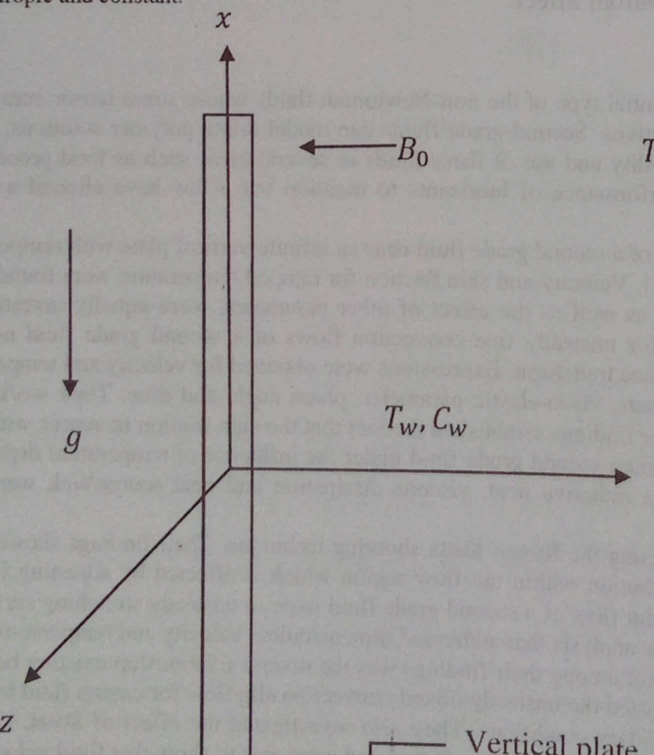


Fig. 1: Geometry of the problem

Since the plate is infinite and motion is steady, all the flow variables depend only on y- coordinate. Thus the conservation of mass, momentum, energy and species concentration equations lead to the following coupled partial differential equations related by [8] with the cross diffusion effects also added to their model gives:

$$\frac{\partial v}{\partial y}=0 \tag{5}$$

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\upsilon\left(\frac{\partial^2 u}{\partial y^2}\right)+\frac{\alpha_1}{\rho}\left(\left(\frac{\partial^3 u}{\partial t\partial y^2}\right)+v\left(\frac{\partial^3 u}{\partial y^3}\right)\right)+g\beta_T(T-T_\infty)+g\beta_c(C-C_\infty)-\frac{\sigma B_0^2 u}{\rho} \tag{6}$$

$$\rho c_p\left(\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2+\sigma B_0^2u^2+\frac{\rho Dk_T}{c_s}\left(\frac{\partial^2 C}{\partial y^2}\right) \tag{7}$$

$$\frac{\partial C}{\partial t}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}+\frac{Dk_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right) \tag{8}$$

Symbols u and v denote the fluid velocity in the x and y directions. T and C represent the temperature and concentration fields respectively. ρ is the density, μ is the coefficient of viscosity. D is the mass diffusivity. g is the acceleration due to gravity. β_T is the coefficient of thermal expansion. β_c is the coefficient of volumetric expansion, σ is the electrical conductivity. c_p is the specific heat at constant pressure. k_T is thermal diffusion, and k is the thermal conductivity. It is clear from equation (5) is identically satisfied such that v is a constant or function of time only, but according to [9];

Where. V_0 is a non-zero positive constant referred to as suction parameter, while the negative sign indicates that the suction is towards the vertical plate. The boundary conditions for the equations (5) - (8) are as follows:

$$u(y,0)=u_0,\quad T(y,0)=T_w,\quad C(y,0)=C_w \quad at\ t=0$$

$$\left.\begin{array}{l} u(0,t)=u_0,\quad T(0,t)=T_w,\quad C(0,t)=C_w \quad at\ y=0 \\ u\to 0,\quad \frac{\partial u}{\partial y}\to 0,\quad T\to T_\infty,\quad C\to C_\infty \quad as\ y\to\infty \end{array}\right] t>0 \tag{10}$$

These equations are subject to the following similarity variables:

$$u=u_0f(\eta),\ \eta=\frac{y}{2\sqrt{\upsilon t}},\ \theta=\frac{T-T_\infty}{T_w-T_\infty},\ \phi=\frac{C-C_\infty}{C_w-C_\infty},\ \lambda_1=\frac{2\rho\upsilon t-\alpha_1}{v_0\alpha_1},\ \beta=\frac{4\rho\upsilon t}{\alpha_1},$$

$$\lambda_2=\frac{2\rho\upsilon t-\alpha_1}{v_0^2\alpha_1},\ S_c=\frac{\upsilon}{D},\ M=\frac{8\upsilon\sigma B_0^2t^2}{v_0\alpha_1},\ G_r=\frac{8\rho\upsilon gB_Tt^2(T_w-T_\infty)}{u_0v_0\alpha_1},\ S_r=\frac{DK_T(T_w-T_\infty)}{\upsilon T_\infty(C_w-C_\infty)} \tag{11}$$

$$G_m=\frac{8\rho\upsilon gB_Ct^2(C_w-C_\infty)}{u_0v_0\alpha_1},\ P_r=\frac{\rho\upsilon c_p}{k},\ E_c=\frac{u_0^2}{c_p(T_w-T_\infty)},\ D_u=\frac{DK_T(C_w-C_\infty)}{c_sc_p\upsilon(T_w-T_\infty)}$$

Where, $\upsilon=\dfrac{\mu}{\rho}$ is the kinematic viscosity. The subscripts w and ∞ refer to the condition at the wall and far away from the plate respectively. Introducing these similarity variables into the transient state coupled nonlinear dimensionless partial differential equations under the electromagnetic Boussinesq approximation yields;

$$f'''-\lambda_1f''+\lambda_2f''-\beta f'+Mf-M\eta f-G_r\theta+\frac{G_r\eta}{v_0}-G_m\phi+\frac{G_m\eta}{v_0}=0 \tag{12}$$

$$\theta''+2P_r(\eta+v_0)\theta'+P_rE_c(f')^2+P_rE_cMf+D_u\phi''=0 \tag{13}$$

$$\phi''+2S_c\eta\phi'+2S_cv_0\phi'+S_rS_c\theta''=0 \tag{14}$$

The corresponding boundary conditions for are;

$$\begin{array}{llll} f=1, & \theta=1, & \phi=1 & at\ \ \eta=0 \\ f=0, & f'=0, & \theta=0, & \phi=0\ \ as\ \ \eta\to\infty \end{array} \tag{15}$$

The unbounded domain of the independent variable $\eta\in[0,\infty)$ is changed into a bounded one using a new variable $x\in[0,1)$ by applying the transformation $x=1-e^{-\eta}$ in [10] and its derivatives are:

$$\eta=-ln(1-x) \tag{16}$$

$$\frac{df}{d\eta}=(1-x)\frac{df}{dx};\quad \frac{d\theta}{d\eta}=(1-x)\frac{d\theta}{dx};\quad \frac{d\phi}{d\eta}=(1-x)\frac{d\phi}{dx} \tag{17}$$

$$\frac{d^2f}{d\eta^2}=(1-x)^2\frac{d^2f}{dx^2}-(1-x)\frac{df}{dx};\quad \frac{d^3f}{d\eta^3}=(1-x)^3\frac{d^3f}{dx^3}-3(1-x)^2\frac{d^2f}{dx^2}+(1-x)\frac{df}{dx}$$

$$\frac{d^2\theta}{d\eta^2}=(1-x)^2\frac{d^2\theta}{dx^2}-(1-x)\frac{d\theta}{dx};\quad \frac{d^3\theta}{d\eta^3}=(1-x)^3\frac{d^3\theta}{dx^3}-3(1-x)^2\frac{d^2\theta}{dx^2}+(1-x)\frac{d\theta}{dx} \tag{18}$$

$$\frac{d^2\phi}{d\eta^2}=(1-x)^2\frac{d^2\phi}{dx^2}-(1-x)\frac{d\phi}{dx};\quad \frac{d^3\phi}{d\eta^3}=(1-x)^3\frac{d^3\phi}{dx^3}-3(1-x)^2\frac{d^2\phi}{dx^2}+(1-x)\frac{d\phi}{dx}$$

The relations (17) to (18) are obtained using the chain rule in Differential Calculus. To tackle the challenge of the singular point, the function $\frac{1}{(1-x)}$ is approximated with the series form $\sum_{n=0}^{\infty} x^n$, where $x \in [0,1)$. With these in place the systems (12) – (15) become;

$$f''' + (1+x+x^2)(\lambda_2 - \lambda_1 - \beta)f'' + (1+2x+x^2)(\lambda_1 - \lambda_2)f' + (1+3x+3x^2)(x+\frac{x^2}{2}-1)Mf$$
$$-(1+3x+3x^2)(Gr\theta + G_m\phi) + (x+\frac{x^2}{2})(1+3x+3x^2)\frac{(Gr\theta + G_m\phi)}{v_0} = 0 \quad (19)$$

$$\theta'' + (1+x+x^2)(2P_r v_0 - 2P_r(-x-\frac{x^2}{2}) - 1)\theta' + P_r E_c (f')^2 + (1+2x+x^2)P_r E_c Mf \quad (20)$$
$$+D_u\phi'' - D_u(1+x+x^2)\phi' = 0 \quad (21)$$
$$\phi'' + 2S_c(v_0 + (x+\frac{x^2}{2}) - 1)(1+x+x^2)\phi' + S_r S_c \theta'' - S_r S_c(1+x+x^2)\theta' = 0$$

Subject to the boundary conditions:

$$f=1, \quad \theta=1, \quad \phi=1 \quad \text{at} \quad x=0 \quad (22)$$
$$f=0, \quad f'=0, \quad \theta=0, \quad \phi=0 \quad \text{as} \quad x=1$$

3.0 METHOD OF SOLUTION

Briefly discussed here are the basic principles of the ADM using an initial value problem for a nonlinear ordinary differential equation in the form; (23)

$$Lu + Ru + Nu = g$$

Where: g is the systems input and u is the systems output, L is the linear operator to be inverted; usually the highest order of the differential operator, R is the linear remainder operator while N is assumed to be the analytic nonlinear operator. It is important to note that the choice of L and its inverse L^{-1} depends on the kind of equation to be solved. Generally, $L = \frac{d^p}{dx^p}(W$ for the P^{th} order differential equation so that it's inverse L^{-1} follows the p-fold definite integration operator from x_0 to x. Clearly, $L^{-1}Lu = u - \phi$ where ϕ covers the initial values as, $\phi = \sum_{v=0}^{p-1} \beta_v \frac{(x-x_0)^v}{v!}$. Applying the inverse linear operator L^{-1} to both sides of equation (23) gives

$$u = \gamma(x) - L^{-1}(Ru + Nu) \quad (24)$$

Where: $\gamma(x) = \phi + L^{-1}g$. The ADM, then decomposes the solution into a series,

$$u = \sum_{n=0}^{\infty} u_n \quad (25)$$

and also decomposes the nonlinear term Nu into a series,

$$Nu = \sum_{n=0}^{\infty} A_n \quad (26)$$

where the A_n which depends on,

$u_0, u_1, u_2, ..., u_n$ are called the Adomian polynomials obtained for the non linear term $Nu = f(u)$ by the definitional equation (23)

$$A_n = \frac{1}{n!}\frac{\partial^n}{\partial\lambda^n}\left[f\left(\sum_{k=0}^{\infty} u_k \lambda^k\right)\right]_{\lambda=0}, n = 0,1,2,..., \quad (27)$$

Where:

λ is the grouping parameter of convenience on substituting the Adomian decomposition series for the solution u(x) and series of the Adomian polynomials for the nonlinearity Nu from equations (25) and (26) into equation (24) yields

$$\sum_{n=0}^{\infty} u_n = \gamma(x) - L^{-1}\left[R\sum_{n=0}^{\infty} u_n + \sum_{n=0}^{\infty} A_n\right] \quad (28)$$

The solution components $u_n(x)$ can now be estimated by any good recursion scheme depending on the choice of the initial solution component $u_0(x)$ starting with the classic Adomian recursion scheme

$$u_0(x) = \gamma(x)$$

$$u_{n+1}(x) = -L^{-1}\left[Ru_n + A_n\right], n \geq 0, \quad (29)$$

Where the initial solution component chosen for Adomian is,
$u_0(x) = \gamma(x)$, the n-term approximation of the solution is given by;

$$\varphi_n(x) = \sum_{k=0}^{n} u_k(x) \quad (30)$$

Various recursion schemes can be designed and used in solving the problem. The transient solutions for velocity, temperature and concentration profiles are considered under different thermo-physical properties of valuable interest in Engineering and other fields relevant to this study.

4.0 RESULTS AND DISCUSSIONS

In this section is displayed the graphical results obtained for the unsteady state solution of the system of coupled equations using the semi analytic method of the Adomian Decomposition. The influence of physical parameters and dimensionless numbers on the thermo-physical quantities, dimensionless velocities, temperature and concentration are also discussed. These factors play keys roles in handling second grade fluids whether at the industrial, manufacturing or Engineering point.

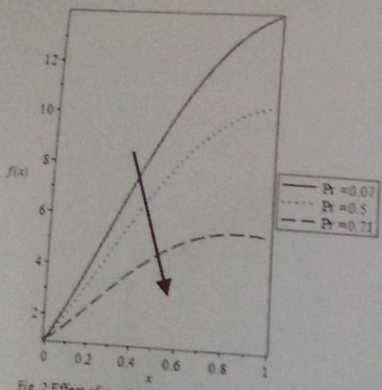
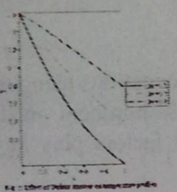
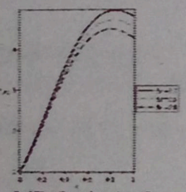
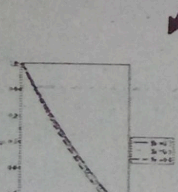
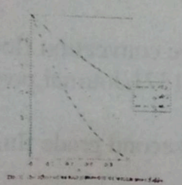


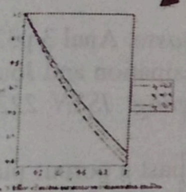
Fig. 2 Effect of varying Prandtl number with velocity

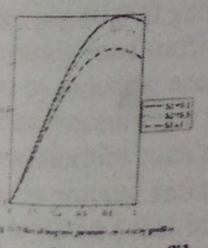












The velocity profiles tend to increase for reduced values of Prandtl and Soret numbers as seen in figures 2 and 6. This implies that fluid flow velocity can be significantly controlled by adjusting the values of these parameters. In figs. 3 and 8, an inverse relationship is also observed for tempeature profiles with prandtl number and suction parameter. A similar trend is also seen for concentration fields, see figs. 7 and 9. The Dufour number on the other hand shows a positive correlation with both velocity and temperature fields as shown on figs 4 and 5. Higher values of the magnetic parameter have an inverse effect on the velocity field. till a certtain peak value is attained after which there is a decline in the same order. Asimilar pattern of behavior is observed for different values of the soret number on the velocity field.

CONCLUSION

Key factors to consider in altering fluid flow velocity include the Prandtl number, Soret and the Dufour numbers. Altering tempature fields will require adjusting prandtl number, suction and dufour parameters. Obviously a mildly inverse accord is witnessed for Soret effect with the concentration fields. Prandtl measures the relative impotance of heat conduction and the fluids viscosity and is usually large for small thermal conductivities and high viscosities. It invariably showcases the relative importance of viscosity to thermal dissipation. In multicomponent fluid mixtures it is important to take into account the "cross effects" namely the mass flows brought about by temperature gradient also refered to as thermal diffusion or the Soret effect and the energy flows due to density gradient also known as diffusion thermal effect or the Dufour effect.

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