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On the Use of Big M Method in Modeling Production Process: A Case Study of NARICT, Zaria

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Abstract

In the National Research Institute for chemical Technology (NARICT), optimal control of production and distribution has a fundamental role to play. One inevitable area where production planning has proven useful is in the allocation of scarce resources to meet certain production demand, also in the distribution of finished goods to different demand location at a minimum cost. In this study, the particular scenario presented concerns NARICT with potentials for production of multiple items under different facilities was considered. The items produced are distributed to number locations whose demands are known. Three integer programming models were analyzed to address the production planning problem, while the modified version of simplex method was used as the numerical tool for the computation of the optimal solution. The optimal solutions were verified using the Windows based Quantitative System for Business (WINQSB) package. The results obtained showed that not all items should be produced if a minimum cost is to be achieved (i.e. some item should have zero production units).we see that if demand is known at a starting point our decisions are not exposed to uncertainty, since the demand determines the actual distribution of quantities and consequently the minimum distribution cost.Adequate production planning is essential to ensure less operational cost and increased profits.

Keywords: Optimization, Production Planning, Linear Programming.

1.0 Introduction

Optimization is a vital instrument for decision making process based on past experiencewhich can affect the present and the future. Most decision making is part of the unending history of actions. Earlier choices affect the present, while current decisions may influence the future and so on.One of the important tools of Optimization is "Linear Programming" (L.P.). A Linear Programming Problem (LPP) is specified by a linear, multi-variable function which is to be optimized (maximized or minimized) subject to a set of stated restrictions, or linear constraints. The function which is to be optimized is called the objective function. The Simplex method, also called Simple Technique or Simplex Algorithm was developed in 1947 to solve problems of this type in [1] an American Mathematician. It is the basic workhouse for solving Linear programming problems till date. Though there have been modifications to the method, especially to take advantage of computer implementations, the essential elements are still the same as they were when the method was invented [2,3].

Production and distributioncompany are often faced with decisions relating to the use oflimited resources.These resources may include men, materials and money. In other sector, thereare insufficient resources available to do as many things as management would wish. Theproblem is based on how to decide on which resources would be allocated to obtain the bestresult, which may relate to profit or cost or both. Linear Programming is heavily used in micro-economics and company management such as planning, production,transportation, technology and other issues. Although the modern management issues are error changing, most companies would like to maximize profits or minimize cost with limited resources. Therefore, many issues can be characterized as Linear Programming Problems [4].

In this paper, we formulate production processes as integer linear programming and present relevant solution approaches for the models formulated. The modified version of simplex method was adopted and tested on the production models, and the results obtained showed that the method is highly reliable and efficient. The remainder of this paper is organized as follows;

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Details of the simplex method and its modification are discussed in section 2. In section 3, 4 we present the model formulations of the production process and effectively apply the modified version simplex method to solving them. Finally the concluding remarks are given in section 5.

2.0 Materials and Method

Simplex Method and its Implementation

Step 1: Find the PIVOT element. The pivot element is that number which is at the intersection of working column (WC) and row (WR). The working column is the column with the most negative number in the last row (if no negative number exists, we multiply all the elements in that row by -1), excluding the last column. To identify the WR, consider the strictly positive elements in the WC and form ratios with the corresponding elements in the last column, excluding the element in the last row and last column. The row that provides the least ration is the WR.

Step 2: Remove the basic variable (BV) in the WR and replace it with non-basic variable in the WC.

Step 3: Convert the pivot element to one by dividing all the elements in the WR with the pivot element, excluding that element in the first column. Then, reduce all other elements in the WC to zero by use of row operations

Step 4: Repeat steps 1 – 3, until all the elements in the last row are greater than or equal to zero.

Step 5: The problem has no solution working if there is no strictly positive element in the WC or if artificial variable is part of the final set of basic variables.

2.1 Modified Version of the Simplex Method (The Big M method)

If a constraint of an Integer Linear programming is strictly equal (=), we write the program in standard form by adding artificial variable (AV) only. But if it is an inequality with greater than or equal restriction (≥), then make it standard by subtracting surplus variable (SV) and at the same time adding artificial variable. The coefficient of the surplus variable in the objective function is zero, while that of the artificial variable is M, which is assumed to be very high and it is considered as a penalty cost.

2.2 Implementation of the Big M Method

If the problem is written in standard form then the last row of the initial tableau is expressed in term of the big M, which is then broken into two rows; the first row contains terms independent of M and the second one involves the coefficients of M. Hence we apply simplex method as usual;

(i) If an artificial variable leaves the set of basic variables, its entire column is deleted for further consideration.

(ii) If the last row (for coefficient of M) contains all positive elements then consider second to the last row, for identifying the Working column. Iterations are terminated if all the elements in the last rows are zero or positive.

2.3 Formulation of Integer Linear Programming Problems

The IP optimizes a linear objective function subject to a set of linear equalities or inequalities. The general production planning maximization models is;

Model (P1)

$$\left.\begin{array}{l} Optimize\ Z : c_1x_1 + c_2x_2 + \ldots + c_nx_n \\ Subject\ to: \\ \qquad b_{11}x_1 + b_{12}x_2 + \ldots + b_{1n}x_n \le R_1 \\ \qquad b_{21}x_1 + b_{22}x_2 + \ldots + b_{2n}x_n \le R_2 \\ \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \qquad b_{m1}x_1 + b_{m2}x_2 + \ldots + b_{mn}x_n \le R_m \\ \qquad x_j \ge 0 \\ \qquad j = 1,2,3\ldots n; \quad i = 1,2,3,\ldots,m \end{array}\right\} \qquad (1)$$

Where,

Z = Objective function that maximized selling profits.

x_j = Choice variable (production item) for which the problem solved.

c_j = Coefficient measuring the contribution of the j^{th} choice variable to the objective functions.

R_i = Constraint or restrictions placed upon the problem.

b_{ij} = Coefficient measuring the effect of the i^{th} constraint on the j^{th} choice variable.

3.0 Model Formulation

Let x_{ij} denote the quantity to be distributed from location of a certain warehouse i to another demand point j; the optimization problem will be formulated as model P2

Model (P2)

$$\left.\begin{aligned} &\underset{x_{ij}}{minimize}\ f = \sum_{i,j} c_{ij}x_{ij} \\ &Subject\ to: \quad \sum_{j} x_{ij} = w_i, \quad i = 1,\dots,m \\ &\qquad\qquad\quad \sum_{i} x_{ij} = d_j, \quad i = 1,..,n \\ &\qquad\qquad\quad x_{ij} \geq 0 \end{aligned}\right\} \quad (2)$$

Where,

c_{ij} = denote the distribution cost from warehouse *(i)* to distribution point *(j)*

d_j = demand at distribution point j

w_i = Capacity of each warehouse or (supply point).

3.1 Model of Production Planning with Lost Demand and Inventory

We consider a case where a factory is planning for the production of three items, this item requires three available resources and then we formulate a model for the factory so as to maximize profit as in Model P3, i.e.

3.2 Model P3

$$\begin{aligned} Max\ Z:\ & r_1(d_1 - u_1) + r_2(d_2 - u_2) + r_3(d_3 - u_3) - \\ & cp_1(p_1) - cp_2(p_2) - cp_3(p_3) - cq_1(q_1) - \\ & cq_2(q_2) - cq_3(q_3) - cu_1(u_1) + cu_2(u_2) + cu_3(u_3) \end{aligned}$$

$$\begin{aligned} subject\ to: \quad & a_{11}p_1 + a_{12}p_2 + a_{13}p_3 \leq b_1 \\ & a_{21}p_1 + a_{22}p_2 + a_{23}p_3 \leq b_2 \\ & a_{31}p_1 + a_{32}p_2 + a_{33}p_3 \leq b_3 \\ & p_1 - q_1 + u_1 = d_1 \\ & p_2 - q_2 + u_2 = d_2 \\ & p_3 - q_3 + u_3 = d_3 \end{aligned}$$

Where,

r_i = profit for item i

cp_i = cost of producing item i

cq_i = unit inventory holding cost of item i

a_{ik} = type of resources k required to produce item i

b_k = amount of resource k available.

d_i = amount of demand for item i.

p_i = amount of output of item i.

q_i = inventory of item i.

u_i = amount of unmet demand of item i.

Where, the values for the variables r_i, cp_i, cq_i, a_{ik} $d_i and$ b_i for (i = 1, 2, 3) are known.
Thus, the model developed above will be used and from the available data; parameters would be substituted to get the optimal solution.

4.0 Illustrative Problems

4.1 Cost Minimization and Optimal Control of Inputs

National Research Institute for Chemical Technology (NARICT), Zaria produces four types of goods namely; belts, sandals, boots and bags, the manufacture of these items is constrained by a budget of N100, 000. To engage in the production of these four items NARICT uses three resources; raw materials, labour and over-time. NARICT needs 5,000kg of raw materials, 180 personnel and 250hrs over-time to produce 1000 units each of the four items. The resource requirements for each item and the cost are given by Table 1.

4.2 Profit Maximization and Optimal Control for Three Inputs

NARICT is planning for production of three items namely; industrial boots, belts, and bags. The manufacture of each item requires three resources. These raw materials are number of workers, overtime and varying stock. The amount of the three raw materials required is illustrated in Table 3.the amount of available resources and the demand for each item are represented in Table 4 and 5.
The cost of production, the unit stock holding cost, and the profit for each item in each time state is illustrated in Table 6

4.3 Solution of the Problems

Data Presentation

We present a summary of data required for the problem in 4.1 and using model P1 we formulate an appropriate integer linear programming and further solve it using modified version of the simplex method (Big M Method).

Table 1: Data of the Cost Minimization and Optimal control of inputs

Items	Raw Material (Kg)	Labour	Over Time (hrs)	Cost (N)
Belts	500	15	40	5000
Sandals	1200	22	18	6000
Boots	1800	18	20	4500
Bags	1300	30	20	5000

The problem involves determining the optimal production combination of these four items that will generate a minimum cost.
Now, we formulate the mathematical programming and optimization problem as follows;

$$\begin{aligned} &\text{minimize: } 5000x_1 + 6000x_2 + 4500x_3 + 5000x_4 \\ &\text{Subject to: } 500x_1 + 1200x_2 + 1800x_3 + 1300x_4 \geq 5000 \\ &15x_1 + 22x_2 + 18x_3 + 30x_4 \geq 180 \\ &40x_1 + 18x_2 + 20x_3 + 20x_4 \geq 250 \\ &5000x_1 + 6000x_2 + 4500x_3 + 5000x_4 \leq 100000 \\ &x_1, x_2, x_3, x_4 \geq 0 \end{aligned} \tag{3}$$

We apply the Modified version of Simplex method (Big M Method) to the problem above and thus have a reformulation as follows;

$$\left.\begin{aligned} &\text{minimize: } 5000x_1 + 6000x_2 + 4500x_3 + 5000x_4 + (0)y_1 + (0)y_2 + (0)y_3 + MA_1 + \\ &\quad MA_2 + MA_3 \\ &\text{Subject to: } 5000x_1 + 6000x_2 + 4500x_3 + 5000x_4 + S_1 = 100000 \\ &500x_1 + 1200x_2 + 1800x_3 + 1300x_4 - y_1 + A_1 = 5000 \\ &15x_1 + 22x_2 + 18x_3 + 30x_4 - y_2 + A_2 = 180 \\ &40x_1 + 18x_2 + 20x_3 + 20x_4 - y_3 + A_3 = 250 \\ &x_1, x_2, x_3, x_4, y_1, y_2, y_3, S_1, A_1, A_2, A_3 \geq 0 \end{aligned}\right\} \tag{4}$$

Where,

y_1, y_2, y_3 are surplus variables

S_1 is a slack variable

A_1, A_2, A_3 are artificial variables.

Table 2: Summary of Results of Cost Minimization and Optimal Control of Inputs

	Basic Variables	Entering Basic Variable	Leaving Basic Variable	RHS Value	Min. Ratio	Obj. Function Value
Iteration 1	A_1	x_3	A_1	5000	2.7	5430
	A_2			180		
	A_3			250		
	S_1			100000		
Iteration 2	x_3	x_1	A_3	$\frac{25}{9}$	$\frac{175}{31}$	$\frac{2920}{9}$
	A_2			130		
	A_3			$\frac{1750}{9}$		
	S_1			87500		
Iteration 3	x_3	x_4	x_3	$\frac{75}{62}$	$\frac{25}{14}$	$\frac{2280}{31}$
	A_2			$\frac{2280}{31}$		
	x_1			$\frac{75}{31}$		
	S_1			66330.6		
Iteration 4	x_4	y_3	x_4	$\frac{25}{14}$	150	$\frac{645}{14}$
	A_2			$\frac{645}{14}$		
	x_1			$\frac{75}{14}$		
	S_1			64285.7		
Iteration 5	y_3	y_1	A_2	150	1000	30
	A_2			30		
	x_1			10		
	S_1			50000		
Iteration 6	y_3	x_4	y_3	230	3.83	0
	y_1			1000		
	x_1			12		
	S_1			40000		
Iteration 7	x_4^*			$\frac{23^*}{6}$		**40833.3**
	y_1			2150		
	x_1^*			$\frac{13^*}{3}$		
	S_1			59166.7		

Data for Profit Maximization and Optimal Control of three inputs

Table 3: Required resources of production

	Production output requirement		
Total required Resources a_{ik}	**Industrial Boots**	**Belts**	**Bags**
No of Labour	10	7	5
Overtime	2	3	2
Varying stock	10	20	30

Table 4: Amount of available Resources

	Production Output Requirement
Total required resources $[b_k]$	
No of Labour	60
Overtime	250
Varying Stock	60

Table 5: Amount of Demand Resources

Production requirement	**Demand**
No of Labour	500
Overtime	600
Varying Stock	500

Table 6: Cost of Production, Cost of Unmet demand,Unit Stock Holding Costs and Profit

Production Requirement	p_i	cu_i	q_i	r_i
Industrial Boots	30000	4000	1000	60000
Belts	20000	3000	2000	40000
Bags	30000	6000	3000	60000

Now, applying Model P3 and from the data on the Table 3 to 5 we formulate our maximization problem as follows;

$$\left.\begin{aligned} &Max\ Z: 60000(d_1-u_1)+40000(d_2-u_2)+60000(d_3-u_3)-\\ &\quad 30000p_1-20000p_2-30000p_3-1000q_1-2000q_2-\\ &\quad 3000q_3-4000u_1-3000u_2-6000u_3\\ &subject\ to:\\ &\quad 10p_1+7p_2+5p_3\le b_1\\ &\quad 2p_1+3p_2+2p_3\le b_2\\ &\quad 10p_1+20p_2+30p_3\le b_3\\ &\quad p_1-q_1+u_1=d_1\\ &\quad p_2-q_2+u_2=d_2\\ &\quad p_3-q_3+u_3=d_3 \end{aligned}\right\} \quad (5)$$

Where,

p_1 = Amount of output of industrial boots.

p_2 = Amount of output of belts.

p_3 = Amount of output of bags

q_1 = Amount of stock of industrial boots

q_2 = Amount of stock of belts.

q_3 = Amount of stock of bags

d_1 = Demand for industrial boots.

d_2 = Demand for belts.

d_3 = Demand for bags.

b_1 = Amount of labour available

b_2 = amount of Overtime available

b_3 = Amount of varying stock

$$
\left.\begin{aligned}
\max z : & \, 84000000 - 30000p_1 - 20000p_2 - 30000p_3 - 1000q_1 - \\
& 2000q_2 - 3000q_3 - 64000u_1 - 43000u_2 - 66000u_3 \\
s.t \quad & 10p_1 + 7p_2 + 5p_3 \leq 60 \\
& 2p_1 + 3p_2 + 2p_3 \leq 250 \\
\text{i.e.,} \quad & 10p_1 + 20p_2 + 30p_3 \leq 60 \\
& p_1 - q_1 + u_1 = 500 \\
& p_2 - q + u_2 = 600 \\
& p_3 - q_3 + u_3 = 500
\end{aligned}\right\} \quad (6)
$$

$$p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2, u_3 \geq 0$$

Now, using the modified version of simplex method, we reformulate the program as;

$$
\begin{aligned}
\max z : & \, 84000000 - 30000p_1 - 20000p_2 - 30000p_3 - 1000q_1 - \\
& 2000q_2 - 3000q_3 - 64000u_1 - 43000u_2 - 66000u_3 + 0(s_1) + \\
& 0(s_2) + 0(s_3) + MA_1 + MA_2 + MA_3 \\
s.t \quad & 10p_1 + 7p_2 + 5p_3 + s_1 = 60 \\
& 2p_1 + 3p_2 + 2p_3 + s_2 = 250 \\
& 10p_1 + 20p_2 + 30p_3 + s_3 = 60 \\
& p_1 - q_1 + u_1 + A_1 = 500 \\
& p_2 - q + u_2 + A_2 = 600 \\
& p_3 - q_3 + u_3 + A_3 = 500 \\
& p_1, p_2, p_3, q_1, q_2, q_3, u_1, u_2, u_3, s_1, s_2, s_3, A_1, A_2, A_3 \geq 0
\end{aligned} \quad (7)
$$

Where,

s_1, s_2, s_3 are slack variables added to the constraints with $\leq$ sign.

A_1, A_2, A_3 are artificial variables added to the constraint with the equality sign.

Table 7: Summary of results of Profit maximization and optimal control of three inputs

	Basic variables	Entering basic Variable	Leaving basic variable	RHS Value	Min. Ratio	Obj. Funtion Value
Iteration 1	S_1	x_2	S_3	60		0
	S_2			250		
	S_3			60		
	A_1			500		
	A_2			600		
	A_3			500		
Iteration 2	S_1	x_8	A_2	39	597	-60000
	S_2			241		
	x_2			3		
	A_1			500		
	A_2			597		
	A_3			500		
Iteration 3	x_1	x_1	S_1	39	6	--25731000
	S_2			241		
	x_2			3		
	A_1			500		
	x_8			597		
	A_3			500		
Iteration 4	x_1	x_3	x_2	6	347	-25980000
	S_2			238		
	x_2			0		
	A_1			294		
	x_8			600		
	A_3			500		
Iteration 5	x_1	x_7	A_1	6	294	-25980000
	S_2			238		
	x_3			0		
	A_1			294		
	x_8			600		
	A_3			500		

Iteration 6	x_1			6	500	-44796000
	S_2			238		
	x_3			0		
	x_7			294		
	x_8			600		
	A_3			500		
Iteration 7	x_1			6		-77796000
	S_2			238		
	x_3			0		
	x_7			294		
	x_8			600		
	x_9			500		

4.4 Discussion of Results

From the results obtained in problem 4.1 any short of supply of raw materials the optimal production will be significantly affected. Therefore a raw material combined in a proportion that gives optimal profit is essential.
Similarly from the second problem 4.2 there is a holding cost for goods, the optimal production and profits is dependent on the holding cost of the goods.

From the optimal solution of the problem 4.1 given by $\left\{\frac{23}{6},0,0,\frac{13}{3}\right\}$, we observe that this is not of an integer form, therefore we approximate to the nearest integer values. The solution becomes $\{4,0,0,4\}$, this is because the program is of the integer nature.

To ensure that the production, expenses and profits remains at optimum, the following should be considered;

(i) 4 x 1000 units of belts and bags should be produced.
(ii) No units of sandals and boots should be produced.

If production of other items must be made (i.e. boots and sandals), the most sensitive inputs which are raw materials and available budget must be increased. The amount of output of industrial boots should be kept at 6000 units to maximize the profits of the factory while other items should be kept constant at zero.

5.0 Conclusion

This paper has succeeded in using the concept of integer linear programming to model various production and distribution problems which arises in everyday living. We also demonstrated how the modified version of simplex method approach (Big M method) could be used as a very reliable and effective tool for obtaining the optimal solution to diverse integer linear programming problems.

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