Boubaker Polynomials Expansion Scheme Solution to the Heat Transfer Equation Inside Laser Heated Biological Tissues

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In this study, an analytical solution to the heat transfer equation in biological tissues during laser heating is presented. The results were compared to recently published numerical simulations. [DOI: 10.1115/1.4005744]

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1 Introduction

Laser interaction with biological tissues has different aspects, which involve photothermal, acoustical, and photochemical [1–3]. The most interesting applications are laser thermokeratoplasty (LTK) [4], laser-induced interstitial thermotherapy (LITT) [5], laser-induced hyperthermia (HT) [6], and interstitial laser photocoagulation therapy (ILP) [7]. In recent years, modeling human tissue response to external solicitations has become a realistic method in many research and medical fields [8–12] for the prediction of damage, in-situ clinical medical treatments, destroying tumor tissue, and pathogen cell elimination.

In performing laser heating treatments, tissue temperature rise has always been the most quoted parameter for determining response amplitude. Besides research interests, it is desirable to have complete knowledge of the temperature distribution during medical laser applications which are based on heating. Nevertheless, the complexity of the problem lies in the fact that this temperature rise depends conjointly on factors intrinsic to the body, such as the thermal conductivity and diffusivity of the tissue and the energy dissipated inside it by the source.

In this study, we propose an analytical expression to the heat equation in a model of living tissue using the Boubaker Polynomials Expansion Scheme (BPES).

2 Theoretical Investigations

Laser tissue heating is reflected in temperature changes within the tissue by interaction with penetrating waves. The heat response of the human body depends wholly on the thermal parameters of the targeted zone (Fig. 1).

The thermal parameters of the different zones are gathered in Table 1. It is expected that the temperature changes only in one direction (Fig. 1), then we can write

$$\begin{cases} \dfrac{\partial^2 T_1(x,t)}{\partial x^2} = \dfrac{1}{D_1}\dfrac{\partial T_1(x,t)}{\partial t} - \dfrac{1}{\kappa_1}\dfrac{P_1}{V_1}; & 0 < x < L_1 \\ \dfrac{\partial^2 T_2(x,t)}{\partial x^2} = \dfrac{1}{D_2}\dfrac{\partial T_2(x,t)}{\partial t} - \dfrac{1}{\kappa_2}\dfrac{P_2}{V_2}; & L_1 < x < L_1 + L_2 \\ \dfrac{\partial^2 T_3(x,t)}{\partial x^2} = \dfrac{1}{D_3}\dfrac{\partial T_3(x,t)}{\partial t} - \dfrac{1}{\kappa_3}\dfrac{P_3}{V_3}; & L_1 + L_2 < x < L_1 + L_2 + L_3 \end{cases} \quad (1)$$

where $T_i|_{i=1..3}$ is the temperature rise in the zone Z_i, $D_i|_{i=1..3}$ $\kappa_i|_{i=1..3}$, and $T_i|_{i=1..3}$ are tissue thermal diffusivity ($m^2\ s^{-1}$), thermal conductivity ($Wm^{-1}\ K^{-1}$), and volume of zone Z_i, respectively. The total heat power deposited in the volume V_i is represented by $P_i|_{i=1..3}$.

The physical parameters are linked by the relation

$$\left.\frac{D_i \rho_i c_i}{\kappa_i}\right|_{i\in[1,2,3]} = 1 \quad (2)$$

In region Z_2, the specific absorption rate ϖ_2, which represents the rate of energy deposited in the region body per unit mass, describes the distribution and amount of absorbed power per kilogram sample weight. Hence, we may rewrite the related equation as follows:

$$\frac{\kappa_2}{\rho_2}\frac{\partial^2 T_2(x,t)}{\partial x^2} = c_2 \frac{\partial T_2(x,t)}{\partial t} - \varpi_2 \quad (3)$$

or

$$\frac{\partial^2 T_2(x,t)}{\partial x^2} = \frac{1}{D_2}\frac{\partial T_2(x,t)}{\partial t} - \frac{\rho_2}{\kappa_2}\varpi_2 \quad (4)$$

Consequently, and provided that the zone does not contain two tissue types (skin, fat, muscle, core), ϖ_2, ρ_2, and κ_2 are all

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