A solution to Laser-induced Heat Equation inside a Two-layer Tissue Model Using Boubaker Polynomials Expansion Scheme

M.A. Aweda[a], M. Agida[b], M. Dada[b], O. B. Awojoyogbe[b], K. Isah[b], O.P. Faromika[b], K. Boubaker [c*], K. De[b] and O.S. Ojambati[b]

a Department of Radiation Biology and Radiotherapy, College of Medicine of the University of Lagos. Idi-Araba. P. M. B. 12003, Lagos State, Nigeria

b Department of Physics, Federal University of Technology, Minna, Niger-State, Nigeria

c Unité de Physique de Dispositifs à Semiconducteurs -UPDS- Faculté des Sciences de Tunis Campus Universitaire 2092 Tunis, Tunisia.

[*] Corresponding author, email: mmbb11112000@yahoo.fr;
Additionnal contacts: boubaker_karem@yahoo.com , das_niger@yahoo.co.uk

In this paper, an analytical solution to the heat transfer equation in a particual two-layer model of *in-vivo* tissues during laser heating is proposed. The application of tissues continuous or pulsed lasers heating is not yet a routine technique which can soon be widely implemented into practice. A two layer model of living tissue is used in order to study temperature rise during laser heating. The resolution of the heat transfer between the two regions is carried out using a polynomial expansion scheme.

DOI:10.2961/jlmn.2011.02.0002

Keywords: Laser ; modeled tissues; Heat equation; *In-vivo.*

PACS 2010: 87.85.Lf ; 82.56.Pp; 87.61.-c

1. Introduction

In recent years, laser heating treatment soldering and ablation have become some of the most investigated methods in nano-engineering, microbiology and medicine sciences [1-4]. Understanding of the underlying physics and interrelation of the processes taking place in tissues irradiated by continuous or pulsed laser allows optimization of relevant parameters in therapeutic and medical applications. In this context, Stanowski *et al.* [2] achieved a quantitative description of the IR laser-QWI process leading to an accurate knowledge of temporal and spatial temperature profiles induced by the laser solving a 3-dimensional heat diffusion. Mrochen *et al.* [5] investigated in the same context the influence of temporal and spatial spot sequences on an *in-vivo* surface temperature increase during laser surgery with a high-repetition-rate excimer laser.

The most interesting measurements in this field are hence temperature profiles. In fact, in performing laser heating treatments, tissues temperature rise has always been considered as the key for determining response amplitude [5-12].

Besides research interests, it is desirable to have complete knowledge of the temperature distribution during medical laser applications based on heating. Nevertheless, the complexity of the problem lies in the fact that this temperature rise depends conjointly on factors intrinsic to the body such as the thermal conductivity and diffusivity of the tissue and the energy dissipated inside it by the source.

In this study we propose an analytical expression to the heat equation in a modelled living tissue using the Boubaker Polynomials Expansion Scheme.

2. Theoretical investigations

Laser tissue heating by is reflected in temperature changes within the tissue during interaction with penetrating waves. The heat response of the human body depends wholly on the thermal parameters of the targeted zone (Fig. 1). The thermal equivalents of the different zones are gathered in Table 1.

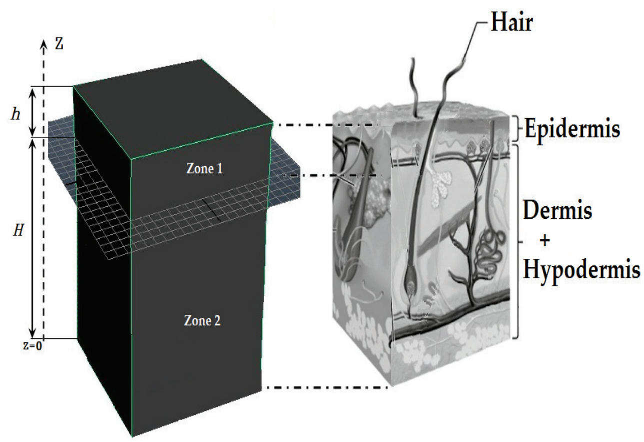

Fig. 1. Geometrical equivalent scheme of the targeted tissue

It is expected that the temperature changes only in one direction (Fig. 1), then we can write:

$$\begin{cases} \dfrac{\partial^2 T_1(z,t)}{\partial z^2} = \dfrac{1}{D_1}\dfrac{\partial T_1(z,t)}{\partial t} - \dfrac{1}{\lambda_1}\dfrac{P_1}{V_1}; & \mathrm{H} < z < h + H \\ \dfrac{\partial^2 T_2(z,t)}{\partial z^2} = \dfrac{1}{D_2}\dfrac{\partial T_2(z,t)}{\partial t} - \dfrac{1}{\lambda_2}\dfrac{P_2}{V_2}; & 0 < z < H \end{cases} \quad (1)$$

under the boundary condition:

$$\begin{cases} T_1(z,t)\big|_{z=H} = T_2(z,t)\big|_{z=H} \\ \lambda_1 \dfrac{\partial T_1(z,t)}{\partial z}\bigg|_{z=H} = \lambda_2 \dfrac{\partial T_2(z,t)}{\partial z}\bigg|_{z=H} \end{cases} \quad (2)$$

with :

T_1 : Temperature rise in the zone Z_1
D_1 : Thermal conductivity in the zone $Z_1\,(m^2 s^{-1})$
λ_1 : Thermal conductivity in the zone $Z_1\,(W m^{-1} K^{-1})$
P_1 : Total heat power deposited in the volume V_1.
T_2 : Temperature rise in the zone Z_2
D_2 : Thermal conductivity in the zone $Z_2\,(m^2 s^{-1})$
λ_2 : Thermal conductivity in the zone $Z_2\,(W m^{-1} K^{-1})$
P_2 : Total heat power deposited in the volume V_2.

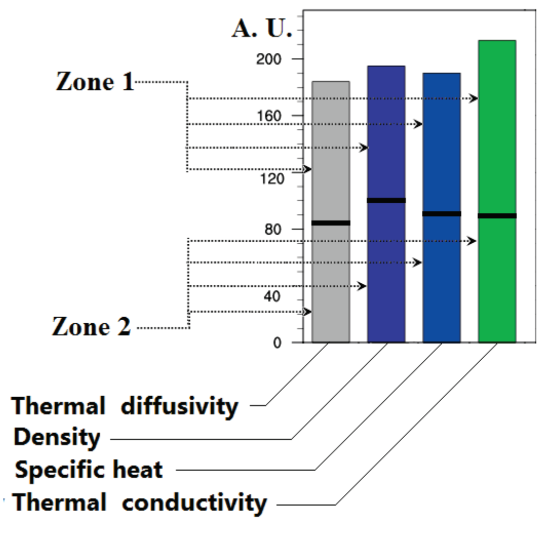

Figure 2. Relative distribution of some physical parameters between two tissue zones

By denoting $P_0(t)$ the specific absorption rate, which represents the rate of energy deposited in the region Z_2 in the body per unit mass, and which describes the distribution and amount of absorbed power per kilogram sample weight, we have :

$$\frac{\partial^2 T_2(z,t)}{\partial z^2} = \frac{1}{D_2}\frac{\partial T_2(z,t)}{\partial t} - \frac{\rho_2}{\lambda_2} P_0(t) \quad (3)$$

By setting:

$$T_2(z,t) = \hat{T}_2(z,t) + \varpi_2(t), \quad (4)$$

and by substituting this expression in equation (1,3), it follows that :

$$\frac{\partial^2 \hat{T}_2(z,t)}{\partial z^2} = \frac{1}{D_2}\left(\frac{\partial \hat{T}_2(z,t)}{\partial t} + \varpi_2'\right) - \frac{\rho_2}{\lambda_2} P_0(t) \quad (5)$$

Then, if we set the system:

$$\begin{cases} \dfrac{\partial^2 \hat{T}_2(z,t)}{\partial z^2} = \dfrac{1}{D_2}\dfrac{\partial \hat{T}_2(z,t)}{\partial t} \\ \varpi_2' = D_2 \dfrac{\rho_2}{\lambda_2} P_0(t) \end{cases} \quad (6)$$

it follows that, and using the method of separation of variables: $\hat{T}_2(z,t) = U(z)W(t)$:

$$\frac{1}{U}\frac{d^2U(z)}{dz^2} = \frac{1}{D_2 W}\frac{dW(t)}{dt} \quad (7)$$

The above equation must be equal to a constant $-\Phi^2$

$$\frac{1}{U}\frac{d^2U(z)}{dz^2} = \frac{1}{D_2 W}\frac{dW(x)}{dt} = -\Phi^2 \quad (8)$$

So that we now have :

$$\begin{cases} \dfrac{d^2U(z)}{dz^2} = -\Phi^2 U(z) \\ \dfrac{dW(t)}{dt} = -D_2 \Phi^2 W(t) \end{cases} \quad (9)$$

Solutions to equation (9) are given as follows

$$\begin{cases} U(x) = \mu_1 \cos\Phi z + \mu_2 \sin\Phi z \\ W(t) = \mu_3 e^{-D_2\Phi^2 t} \end{cases} \quad (10)$$

Then :

$$\hat{T}_2(z,t) = \left(\hat{\mu}_1 \cos\Phi z + \hat{\mu}_2 \sin\Phi z\right) e^{-D_2\Phi^2 t} \quad (11)$$

where $\hat{\mu}_1 = \mu_1\mu_3$ and $\hat{\mu}_2 = \mu_2\mu_3$.

3. Results and discussion

Using equation (6) and (11), we have:

$$\begin{cases} T_2(z,t) = \hat{T}_2(z,t) + \varpi_2(t); \\ \hat{T}_2(z,t) = \left(\hat{\mu}_1 \cos\Phi z + \hat{\mu}_2 \sin\Phi z\right)e^{-D_2\Phi^2 t} \\ \varpi_2(t) = \int_0^{\Delta t} D_2 \frac{\rho_2}{\lambda_2} P_0(t)dt \end{cases} \quad (12)$$

where Δt is the duration of laser beam application.

The second term in Eq. (12) is evaluated using the Boubaker polynomials [13-18] Expansion Scheme *BPES*, namely by proposing the expansions:

$$P_0(t) = \frac{1}{N}\sum_{q=1}^{N} \mu_q B_{4q}\left(\frac{\beta_q}{\Delta t}t\right) \quad (13)$$

where β_q is 4q-Boubaker polynomial minimal positive root, N is a prefixed integer, $\mu_q\big|_{q=1..N}$ are unknown real coefficients related to the laser beam.

This scheme *BPES* has been proposed for resolution of several applied physics problems in the field of biophysics [13], thermodynamics [14] heat transfer [15,16] semiconductors and materials sciences [17-19]. Advantages of this scheme, when applied to many boundary-conditioned physical problems, mainly those involving Alembert (hyperbolic), consist of embedding the boundary condition thanks to 4q-Boubaker polynomials properties.

When applied to Eq. (12)-(13), these conditions give a solution $\mu_q^{sol.}\big|_{q=1..N}$ which minimizes the function:

$$\int_0^{\Delta t}\left(P_0(t) - \frac{1}{N}\sum_{q=1}^{N} \mu_q^{sol.} B_{4q}\left(\frac{\beta_q}{\Delta t}t\right)\right)^2 dt \quad (14)$$

The final temperature expression is hence:

$$\begin{cases} T_2(z,t) = \hat{T}_2(z,t) + \varpi_2(t); \\ \hat{T}_2(z,t) = \left(\sqrt{\hat{\mu}_1^{\,2} + \hat{\mu}_2^{\,2}}\cos(\Phi z + \phi)\right)e^{-D_2\Phi^2 t} \\ \varpi_2(t) = D_2 \frac{\rho_2}{N\lambda_2} \int_0^{\Delta t} \sum_{q=1}^{N} \mu_q^{sol.} B_{4q}\left(\frac{\beta_q}{\Delta t}t\right)dt \end{cases} \quad (15)$$

Plots of the solution, for the values gathered in Table 1, along with results of Tung *et al.* [20] are represented in Fig. 2 in terms of dimensionless temperature $\widetilde{T}$:

$$\widetilde{T} = \frac{T_2(z,t) - T_{\min}}{T_{\max} - T_{\min}}$$

Table 1: Experimental values

Exposure duration Δt	Initial temp. T_0	Incident Laser power
2 s	35° C	5× 10^4 Wcm^{-2}

The obtained profile is in good agreement with that recorded by Stanowski *et al.* [2]. Solution patterns are also close to those of the Laplace transforms-related analytical solution of proposed by Mrochen *et al.* [5]. Time-dependent evolution is a theoretical and numerical supply to the recent studied on Laser-based surgery and medicine models [5-12,20-24]

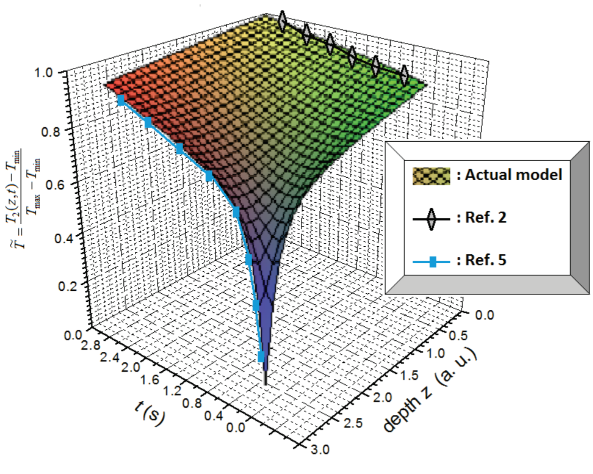

Figure 2. Solution 3D plot

4. Conclusion

In this paper, a two-layer model of targeted tissue has been used in order to study temperature rise during laser heating. The considerations were based on the solution of two-dimensional, stationary heat conduction equation obtained by separation-of-variables approach. Aanalytical expressions were provided. The obtained analytical profile presents a supply to recent investigation on tissue alteration during treatments. The results have been found in good agreement with some recently proposed solutions. The BPES protocol, introduced in this study, seems to be a versatile tool with applications far beyond the scope of the model problem considered here.

Extension of the current method to multi-later modeled tissues the subject of ongoing research.

References

[1] T. Furumoto, T.Ueda, T. Tsukamoto, N. Kobayashi, A. Hosokawa, S. Abe, Study on Laser Consolidation of Metal Powder with Yb:fiber laser - Temperature Measurement of Laser Irradiation Area -, J. Laser Micro/Nanoeng. 4 (2009) pp. 22-27.

[2] R. Stanowski, O. Voznyy, J. J. Dubowski, Finite element model calculations of temperature profiles in Nd:YAG laser annealed GaAs/AlGaAs quantum well microstructures, J. Laser Micro/Nanoeng. 1 (2006) pp.17-22.

[3] N. M. Bulgakova, I. M. Burakov, Y. P. Meshcheryakov, R. Stoian, A. Rosenfeld, I. V. Hertel, Theoretical Models and Qualitative Interpretations of Fs Laser Material Processing, J. Laser Micro/Nanoeng. 2 (2007) pp.76-86.

[4] M. Strassl, V. Wieger, D. Brodoceanu, F. Beer, A. Moritz, E. Wintner, Ultra- short pulse laser ablation of biological hard tissue and biocompatibles, J. Laser Micro/Nanoeng. 3 (2008) pp. 30-40.

[5] M. Mrochen, U. Schelling, C. Wuellner, C. Donitzky, Effect of time sequences in scanning algorithms on the surface temperature during corneal laser surgery with high-repetition-rate excimer laser, J Cataract Refract Surg. 35 (2009) pp. 738–746

[6] K.H. Kim, Z. Guo, Multi-time-scale heat transfer modeling of turbid tissuesexposed to short-pulsed irradiations, Comput. Meth. Prog. Biomed. 86 (2007) pp.112-123.

[7] C. Das, A. Trivedi, K. Mitra, T. Vo-Dinh, Experimental and numerical analysis of short pulse laser interaction with tissue phantoms containing inhomogeneities, Appl. Opt. 42 (2003) pp. 5173-5180.

[8] P.L. Djorev, E. Borisova, L. Avramov, Interaction of the IR laser radiation with human skin – Monte-Carlo simulation, Proc. SPIE (2003) pp. 403-407.

[9] A. Kozlowska, M. Latoszek, J.W. Tomm, F. Weik, T. Elsaesser, M.Zbroszczyk, M. Bugajski, B. Spellenberg, M. Bassler, Analysis of thermal images from diode lasers: temperature profiling and reliability screening, Appl. Phys. Lett 86 (2005) 203503-3.

[10] B. Laikhtman, A. Gourevitch, D. Donetsky, D. Westerfeld, G. Belenky, Current spread and overheating of high power laser bars, J. Appl. Phys. 95 (2004) pp/ 3880-3889.

[11] B. Laikhtman, A. Gourevitch, D. Westerfeld, D. Donetsky, G.Belenky, Thermal resistance and optimal fill factor of a high power diode laser bar, Semicond. Sci. Technol. 20 (2005) 1087-1095.

[12] M. Ben-David, R. Cantor, N. Balbul, M. Yehuda, I. Gannot, Measuring tissue heat penetration by scattered light measurements, Lasers in Surgery and Medicine 40 (2008) pp. 494-499.

[13] O. B. Awojoyogbe, K. Boubaker, A solution to Bloch NMR flow equations for the analysis of hemodynamic functions of blood flow system using m-Boubaker polynomials, Curr. App. Phys. 9 (2009) pp. 278-288.

[14] Zhang, D. H., On the higher order derivatives of the 4q-Boubaker polynomials, Eur. Phys. J. Appl. Physics 50 (1) (2010) pp. 11201-11212

[15] S. Slama, J. Bessrour, K. Boubaker, M. Bouhafs, A dynamical model for investigation of A3 point maximal spatial evolution during resistance spot welding using Boubaker polynomials, Eur. Phys. J. Appl. Phys. 44, (2008) pp. 317-322.

[16] S. Slama, M. Bouhafs, K. B. Ben Mahmoud, A Boubaker polynomials solution to heat equation for monitoring A3 point evolution during resistance spot welding, Int. J. of Heat and Technology, 26(2) (2008) pp. 141-146.

[17] T. Ghrib, K. Boubaker, M. Bouhafs, Investigation of thermal diffusivity-microhardness correlation extended to surface-nitrured steel using Boubaker polynomials, Modern Physics Letters B, 22 (2008) pp. 2893-2907.

[18] S. Fridjine, M. Amlouk, A new parameter: An ABACUS for optimizing func- tional materials using the Boubaker polynomials expansion scheme, Modern Phys. Lett. B. 23 (2009) 2179-2181

[19] M. Dada, O.B. Awojoyogbe, O. F. Moses, O.S. Ojambati, D.K. De, K. Boubaker, A mathematical analysis of stenosis geometry, NMR magnetization and signals based on the Bloch NMR flow equations, and Bessel and Boubaker polynomials, J. of Biological Physics and Chemistry 9 pp. (2009) pp. 101-106.

[20] D. Arora, M. Skliar, R.B. Roemer, Minimum-time thermal dose control of thermal therapies, IEEE Trans. Biomed. Eng. 52 (2005) pp.191-200.

[21] M. A. Childers, W. Franco, J. S. Nelson , G. Aguilar, Laser surgery of port wine stains using local vacuum pressure: changes in skin morphology and optical properties (Part I), Lasers in Surgery and Medicine 39 (2007) pp. 108-117.

[22] D.S. Robinson, J.M. Parel, D.B. Denham, X. Gonzalez-Cirre, F. Manns, P.J. Milne, R.D. Schachner, A.J. Herron, J. Comander, G. Hauptmann, Interstitial laserhyperthermia model development for minimally invasive therapy of breastcarcinoma, J. Am. Coll. Surg. 186 (1998) pp. 284-292.

[23] S. Zangos, M.G. Mark, J.O. Balzer, B.K. Engelmann, R. Straub, K. Eichler, C.Hergoz, T. Lehnert, O. Sollner, M. Heller, A. Thalhammer, T.J. Vogl, Neoadjuvant transarterial chemoembolization (TACE) before percutaneous MR-guided laser-induced thermotherapy (LITT): results in large-size primary and secondary liver tumors, Med. Laser Appl. 19 (2004) pp. 98-108.

[24] M. Pech, G. Wieners, F. Fischbach, S. Hengst, A. Beck, G. Warschewske, E.L.Hanninen, P. Wust, J. Ricke, Synchronous CT-guided brachytherapy in patients at risk for incomplete interstitial laser ablation of liver malignancies, Med. Laser Appl. 19 (2004) pp.73-82.

(Received: January 11, 2011, Accepted: March 22, 2011)