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Article

Investigation of Nano-keyhole Cooling Evolution using Boubaker Polynomials Expansion Scheme (BPES)

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Abstract

This study proposes an analytical expression for temperature evolution inside a nano-keyhole modeled device. Several assumptions have been taken into account. The validity of the model has been tested through compatibility with experiment and Newtonian cooling laws.

Keywords: Keyhole, Heat equation, Newtonian cooling, Boubaker Polynomials.

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1. Introduction:

In recent years, numerical modeling has become a realistic method in many applied physics fields [1-5] as for the prediction of weld geometries and time dependent evolution. A schematic illustration of the geometrical features of the keyhole weld is provided in Fig. (1). In this model tree main assumption are taken into account:

- ✓ The keyhole wall temperature corresponds to the metal boiling point.
- ✓ The exciting beam thermal and optical profiles are coherent..
- ✓ The absorption coefficient is constant on keyhole wall.

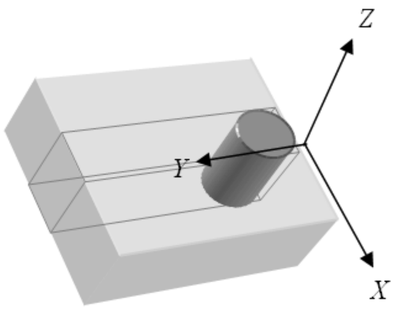

Figure 1 Nano-keyhole model geometrical features

As shown in Fig. 1, the y-direction is the direction of motion of the laser beam (welding direction) and since according to the nano-keyhole approximation model, the nano-keyhole vertical edges temperature is equal to the boiling point of the material, it would be expected that heat transfer is in the x-direction. Under these assumptions, the one-dimensional diffusion, or heat equation for this experimental setup would be derived as follows.

2 Temperature dimensionless expression derivation

Let $T_l(x,t)$ represent the dimensionless temperature of a metal bar at a point *x* at time *t*. The first step is the derivation of a *continuity equation* for the heat flow in the bar shown above. If the bar has a cross sectional area *A*, so that the infinitesimal volume of the bar between x and $x+\Delta x$ is $A\Delta x$. The quantity of heat contained in this volume is $\rho c_p T_l A\Delta x$, with c_p the specific heat, ρ the mass per unit volume; their product has dimensions $[\rho c_p] = ML^{-1}T^{-2}\theta^{-1}$. In a time interval *dt* this heat changes by an amount $\rho c_p\left(\frac{\partial T_l}{\partial t}\right)A\Delta x dt$ due to the change in temperature. This change in the heat must come from laser beam extraordinary excitation of the atoms of the bar, and is the result of a flux of heat *q*(*x*,*t*) (released by the atoms that have been excited by the laser beam) through the area *A* (*q* is the heat flowing through a unit area per unit time).

Into the left side of the volume an amount of heat *qAdt* flows in a time *dt*; on the right hand side of the volume a quantity: $A\left[q+\left(\frac{\partial q}{\partial x}\right)\Delta x\right]dt$, schematic illustration of the geometrical features of the nano-keyhole weld flows out in a time *dt*, so that the net accumulation of heat in the volume is $-A\left(\frac{\partial q}{\partial x}\right)\Delta x dt$. Equating the two expressions for the rate of change of the heat in the volume $A\Delta x$, we find:

$$\rho c_p \frac{\partial T_l}{\partial t} = -\frac{\partial q}{\partial x} \qquad (1)$$

which is the equation of continuity [6]. It is a mathematical expression of the conservation of heat in the infinitesimal volume $A\Delta x$. We supplement this with a phenomenological law of heat conduction, known as *Fourier's law*: the heat flux is proportional to the negative of the local temperature gradient (heat flows from a hot region to a cold region):

$$q = -\kappa \frac{\partial T_l}{\partial t} \qquad (2)$$

with κ the *thermal conductivity* of the metal bar. The thermal conductivity is usually measured in units of $Wm^{-1}K^{-1}$, and has dimensions $[\kappa] = MLT^{-3}\theta^{-1}$. Combining Eq. (1) and Eq. (2), we obtain the *diffusion equation* (often called the *heat equation*)

$$\frac{\partial T_l}{\partial t} = D\frac{\partial^2 T_l}{\partial x^2} \qquad (3)$$

where $D = \frac{\kappa}{\rho c_p}$ is the *thermal diffusivity* of the bar; it has dimensions $[D] = \frac{[\kappa]}{[\rho c_p]} = L^2T^{-1}$, as it should. Eq. (3) is the diffusive equation for heat. It usually results from combining a continuity equation with an empirical law which expresses a current or flux in terms of some local gradient.

Suppose that the bar is very long, so that we can consider the idealized case of an infinite bar. At initial time t = 0, we add an amount of heat H (with dimensions $[H] = ML^2T^{-2}$) at some point of the slab (H is the heat energy equivalent of the laser beam power absorbed by the slab during welding; it is as a result of the excitation of the slab atoms), which we will arbitrarily call x = 0. The heat is conserved at all times, so that:

$$\rho c_p A\int_0^\infty T_l(x,t)dx = H \qquad (4)$$

However, the material to be welded is not infinite in length and provided that heat energy is still conserved within the finite spatial limit N, in consequence Eq. (4) alters to:

$$\rho c_p A\int_0^N T_l(x,t)dx = H \qquad (5)$$

The temperature T_l depends upon x, t, and the diffusivity D, From Eq. (4), it also depends upon the initial conditions through the combination $Q \equiv \frac{H}{\rho c_p A}$.

Dimensional analysis yields a solution to the diffusion equation of the form:

$$\alpha = \frac{x}{(Dt)^{1/2}} \qquad (6)$$

By using the chain rule to calculate various derivatives of T_l, we have:

$$\begin{cases} \frac{\partial T_l}{\partial x} = \frac{Q}{(Dt)^{1/2}}\frac{\partial \alpha}{\partial x}\frac{d\phi(\alpha)}{d\alpha} = \frac{Q}{Dt}\frac{d\phi(\alpha)}{d\alpha} \\ \frac{\partial^2 T_l}{\partial x^2} = \frac{Q}{(Dt)^{3/2}}\frac{d^2\phi(\alpha)}{d\alpha^2} \\ \frac{\partial T_l}{\partial t} = -\frac{1}{2}\frac{Q}{D^{1/2}t^{3/2}}\left[\phi(\alpha) + \alpha\frac{d\phi(\alpha)}{d\alpha}\right] \end{cases} \qquad (7)$$

Then, by substituting Eq. (7) into the diffusion equation Eq. (3), and canceling various factors, we obtain a differential equation for ϕ:

$$\frac{d^2\phi(\alpha)}{d\alpha^2} + \frac{\alpha}{2}\frac{d\phi(\alpha)}{d\alpha} + \frac{1}{2}\phi(\alpha) = 0 \qquad (8)$$

Dimensional analysis has reduced the problem from the solution of a partial differential equation in two variables to the solution of an ordinary differential equation in one variable. The normalization condition, Eq. (4), becomes in these variables:

$$\int_{-\infty}^{\infty}\phi(\alpha)d\alpha = 1 \qquad (9)$$

Eq. (8) is an exact differential, and it follows that

$$\frac{d}{d\alpha}\left[\frac{d\phi}{d\alpha}+\frac{\alpha}{2}\phi\right]=0 \qquad (10)$$

which we can integrate once to obtain

$$\frac{d\phi}{d\alpha}+\frac{\alpha}{2}\phi = Const \qquad (11)$$

However, since any physically reasonable solution would have both $\phi \to 0$ and $\frac{d\phi}{dx} \to 0$ as $x \to \infty$ (since there is no way the thermal vibration could get to such point and hence the temperature there would be constant with respect to the immediate environment), the integration constant must be zero. We now need to solve a first order differential equation, which we do by dividing Eq. (11) gives:

$$\phi(\alpha) = Ce^{-\alpha^2/4} \qquad (12)$$

where C a constant. To determine C, we use the normalization condition, Eq. (9):

$$C\int_{-\infty}^{\infty} e^{-\alpha^2/4}d\alpha = C(4\pi)^{1/2} = 1 \qquad (13)$$

where the integral (known as a *Gaussian integral*) can be found in integral tables. If we have a very big slab to weld such that the axis through which the laser beam would pass is quite small compared to the overall dimension of the slab, the expression of Eq. (13) would hold, at least approximately. Therefore:

$$T_l(x,t) = \frac{H/\rho c_p A}{(4\pi Dt)^{1/2}} e^{-x^2/4Dt} \qquad (14)$$

This is the complete solution for the temperature distribution in a within the one-dimensional bar due to a point source of heat (Fig. 2):

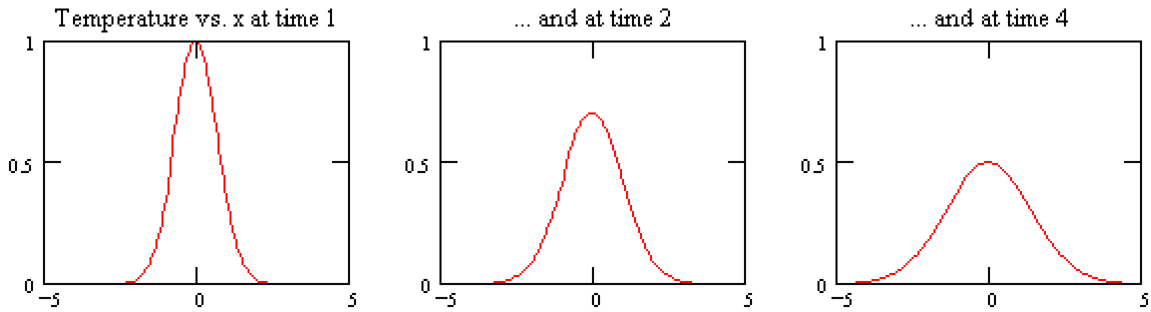

Figure 2 Solution for temperature distribution

3 Maximum Temperature rise T_0 expression derivation

3.1 Coulomb approximation

By analogy with Coulomb approximation [7], the maximum central temperature T_0 rise could be determined:

$$T_0 = \frac{P}{2\pi\kappa}\frac{1}{b} \qquad (15)$$

where b is the radius of the cylindrical targeted zone, receiving a uniformly distributed power P (Fig. 3), its surface constant temperature rise is expressed by (15).

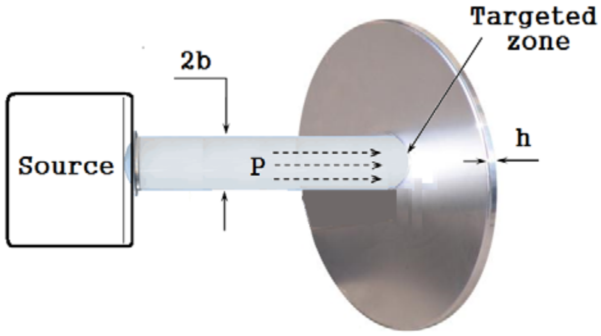

Figure 3: Coulomb approximation scheme

Now, provided that T_0 is the change (rise) in the surface temperature just immediately after the beam passes the nano-keyhole and since T_l is the temperature measured at any given point (we take T_l as our final temperature for this particular situation) we may write that:

$$T_0 = T_l - T_\infty$$

Where we have assumed that T_∞ is our initial temperature of the slab.

In the absence of work done, a change in internal energy per unit volume in the material, ΔH, is proportional to the surface temperature rise. That is:

$$\Delta H = \rho c_p \Delta T_l \qquad (16)$$

where c_p is the specific heat capacity, $T_0 = \Delta T_l = T_l - T_\infty$ and ρ is the mass density of the material.

Now since the rate of change of the heat energy H is the result of the power delivered per unit volume of the slab (the power is responsible for increasing the internal energy), we may then write that:

$$\frac{P}{V} = \rho c_p \frac{d(T_0 + T_\infty)}{dt}\frac{P}{V} = \rho c_p\left(\frac{dT_0}{dt} + \frac{dT_\infty}{dt}\right) = \frac{P}{V} = \rho c_p \frac{dT_0}{dt}$$

Since only T_0 depends on time.

Since the diffusivity is given as $D = \frac{k}{c_p \rho}$, and using Eq. (15), we have:

$$T_0 \frac{2D}{b} = \frac{dT_0}{dt} \qquad (17)$$

3.2 Boubaker polynomials expansion scheme BPES

Equation (17), it solved using the Boubaker polynomials expansion scheme BPES [8-18]. For this purpose, the t-dependent component of T_0 is expressed as:

$$T_0(t) = \lim_{N \to +\infty}\left[\frac{1}{2N}\sum_{n=1}^{N} \xi_n . \hat{B}_{4n}(\alpha_n t)\right] \qquad (18)$$

where α_n are the minimal positive roots of the 4n-Boubaker [9-17] polynomials $\hat{B}_{4n}$, N is an integer parameter, and ξ_n are coefficients to be found.

According to the BPES [12-16] principles, and the reformulation of Eq. (17), the coefficients ξ_n minimize the real function Γ :

$$\Gamma = \left|\frac{2D}{b}\lim_{N \to +\infty}\left[\frac{1}{2N}\sum_{n=1}^{N} \xi_n . \hat{B}_{4n}(\alpha_n t)\right] - \lim_{N \to +\infty}\left[\frac{1}{2N}\sum_{n=1}^{N} \xi_n \alpha_n . \frac{d\left[\hat{B}_{4n}(\alpha_n t)\right]}{dt}\right]\right| \qquad (19)$$

Thanks to the properties of the 4n-Boubaker polynomials Eq.(20), the main initial conditions are intrinsically respected.

$$\begin{cases} \lim\limits_{N \to +\infty} \left[\dfrac{1}{2N} \sum\limits_{n=1}^{N} \dfrac{\partial \left(\hat{B}_{4n}(X) \right)}{\partial X} \right]_{X=0} = 0 \\ \lim\limits_{N \to +\infty} \left[\dfrac{1}{2N} \sum\limits_{n=1}^{N} \hat{B}_{4n}(X) \right]_{X=0} = 1 \end{cases} \quad (20)$$

A plot of the obtained expression of T_0 against t for Carbon steel slab ($D = 1.172 \times 10^{-5} m^2 / s$), is given in Fig. 4.

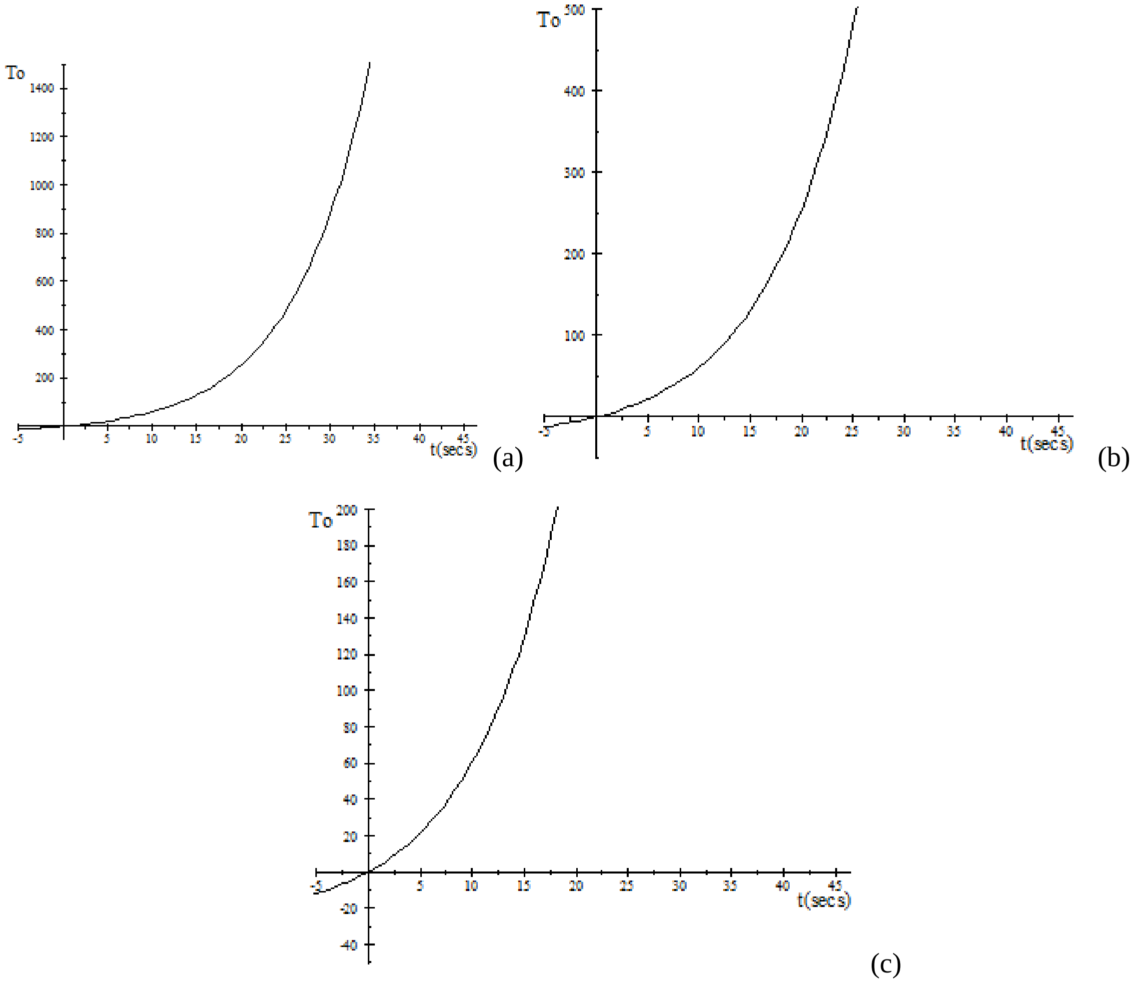

Fig. 4 Plots of T_0 against time for different ranges (in Degree Celsius)

Figure 5 show the dependence of the yielded solution versus the radius b.

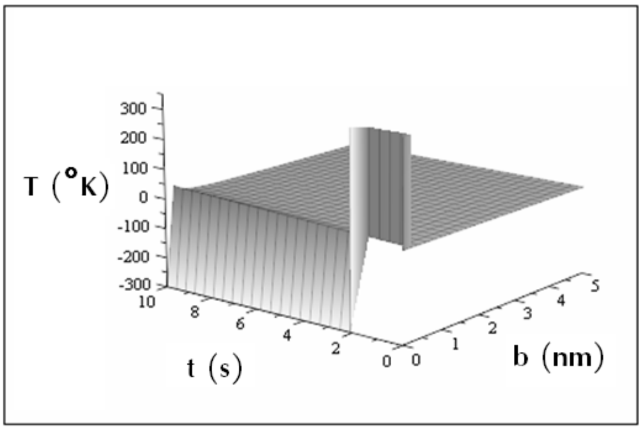

Fig. 5 A plot of T_0 against time and the nano-keyhole radius b.

4 Newtonian Cooling

A bigger challenge is actually to see if the Newton's law of cooling could be used to explain the cooling of the slab after the application of the laser beam.

Newton's law of cooling states that *the rate of heat loss of a body is proportional to the difference in temperatures between the body and its surroundings, or environment.* The law is:

$$\frac{dQ}{dt} = hA(T_l - T_\infty) \quad (21)$$

Q = Thermal energy transfer in joules, h = Heat transfer coefficient, A = Surface area of the slab, T_0 = Temperature of the object's surface, T_∞ = Temperature of the environment.

As this form of heat loss principle is sometimes not very precise; an accurate formulation may require analysis of heat flow, based on the (transient) heat transfer equation in a nonhomogeneous, or else poorly conductive, medium. The following simplification may be applied so long as it is permitted by the Biot number (which relates surface conductance to interior thermal conductivity in a body). If this ratio permits, it shows that the body has relatively high internal conductivity, such that (to good approximation) the entire body is at same uniform temperature as it is cooled from the outside, by the environment.

If has relatively high internal conductivity, then it is easy to derive from these conditions the behavior of exponential decay of temperature of a body. In such cases, the entire slab is treated as lumped capacitance heat reservoir, with total heat content which is proportional to simple total heat capacity $Q_h = mcT$, and the temperature of the body. If $T(t)$ is the temperature of such a body at time t, and T_∞ is the temperature of the environment around the body, then

$$\frac{dT_l(t)}{dt} = r(T_l - T_\infty) \quad (22)$$

Where r is a positive constant characteristic of the slab, which must be in units of 1/time, and is therefore sometimes expressed in terms of a time constant: $r = 1/t_0$.

The solution of this differential equation, by standard methods of integration and substitution of boundary conditions, gives:

$$T_l(t) = T_\infty + (T(0) - T_\infty)e^{-rt} \quad (23)$$

Here, $T_l(t)$ is the temperature at time t, and $T_l(0)$ is the initial temperature at zero time, or $t = 0$.

If: $\Delta T_l(t)$ is defined as: $T_l(t) - T_\infty$ where $\Delta T_l(0)$ is the initial temperature difference at time 0, then the Newtonian solution is written as:

$$\Delta T_l(t) = \Delta T_l(0)e^{-rt} \qquad (24)$$

Since the emphasis here is on cooling, we may assume that our investigation process starts immediately after the source of increased temperature has been removed. Hence, $T_l(0)$ is the highest temperature achieved after which surface temperature begin to drop (cooling). The behaviour of $T_l(t)$ is shown in Fig. (6).

($T_l(0)$ is taken to be 640°C while T_∞ is taken to be 27°C).

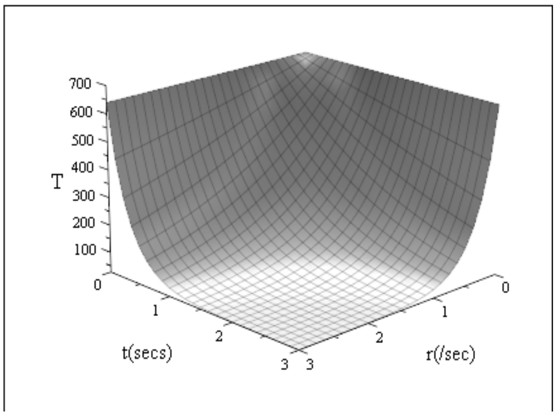

Fig. 6 A plot of T_l (or T) against time and the time constant

5 Conclusion

In this work, theoretical investigations have been performed in order to predict temperature distribution evolution in a particular device model: The nano-keyhole model [19-24]. The use of the Colombian approximation [7], Newtonian laws along with the already established Boubaker polynomials expansion scheme BPES [8-18], allowed monitoring temperature dynamical profiles. The yielded profiles could be compared successfully compared to those published by R. Rai *et al.* [25], H. Al-Kazzaz *et al.* [26], P. Solana *et al.* [27], and C. S. Wu *et al.* [28].

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