



A mathematical analysis of stenosis geometry, NMR magnetizations and signals based on the Bloch NMR flow equations, Bessel and Boubaker polynomials expansions

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Magnetic Resonance Imaging (MRI) has great potential in modern medical imaging, as it is non-invasive and provide comprehensive information about stenotic and nonstenotic plaque and myocardial viability. The aim of this paper is to model the variation of NMR magnetizations and signals over a stenosis under cylindrical geometry, based on the Bloch NMR flow equations. A cylindrical coordinate is constructed such that its maximum radius indicates a totally blocked blood vessel. A differential equation in terms of NMR transverse magnetization was solved for blood molecules which tunnel through the plaque and could be located at the centre of the plaque or any other point within the plaque. Such analysis can be very useful for assessing the relative importance of several flow parameters such as pressure loss due to friction and due to dynamic losses. Analytical expressions are proposed as guides to future investigations. Furthermore, new parameters were derived, which may allow one to accurately define the fluid resistance to shear or flow and to measure the adhesive/cohesive or frictional fluid property. These new parameters are directly or inversely dependent on the length or diameter of the stenosis and the dynamic viscosity of blood flow.

Keywords: bloch NMR flow equations, coronary heart disease, cylindrical geometry, dynamic viscosity, stenosis, vorticity

1. INTRODUCTION

Coronary artery disease, also called coronary heart disease, or simply, heart disease, is the primary cause of death in many parts of the world. Coronary artery disease (CAD) is a chronic disease in which blood flow is obstructed through the coronary arteries that supply the heart with oxygen-rich blood. This obstruction is caused by a disease known as atherosclerosis, which is sometimes called “hardening of the arteries”. Also referred to as coronary heart disease, CAD is the most common form of cardiovascular diseases.

While tests to accurately diagnose coronary artery disease [1–15] are currently the focus of intense clinical research, their incremental value is not yet proven. Accordingly, their use requires a careful clinical assessment of potential risks and benefits in individual patients. A new MRI processing technique called “black-blood” MRI (so called because it produces an image of an artery in which the blood appears black, and the wall of the artery appears white) seems to be able to distinguish effectively between normal and atherosclerotic

coronary arteries. While further refinements are necessary, such techniques are bringing us very close to the day in which MRI will be able to replace cardiac catheterization for diagnosing coronary artery disease [16].

However, MRI has the potential of detecting changes in the tiny blood vessels of the heart—the microvascular circulation—that are completely missed by cardiac catheterization. Detecting such changes seem to be useful in predicting the outcome of patients after a heart attack, and may prove to be useful in assessing patients with cardiac syndrome X, diabetes, and certain other conditions.

In this study, we consider the blood particle which either initially or in some average sense is in steady rotation. We apply a mathematical algorithm to describe in detail the dynamical state of the flowing blood particles starting from the NMR flow equations. We study the flow properties of the modified time independent Bloch NMR flow equations which describes the dynamics of fluid flow under the influence of a radio-frequency $B_1(x)$ magnetic field as given in previous studies [17–23] where it is convenient to use as dependent variable the departure of the stream function from its classical form [18, 22]

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$$\frac{d^2\psi}{dx^2} + \frac{\gamma^2 B_1^2}{V^2} \psi = 0 \quad (1a)$$

subject to the following two conditions: $e^{\lambda x} \neq 0$ and $f_o = \gamma B - \omega = 0$.

Eqn (1a) completely describes the wave properties of the Bloch NMR equations in terms of the mechanical wave function $\psi(x)$, the velocity V , radio-frequency $\omega_1 = \gamma B_1(x)$, T_1 and T_2 relaxation parameters. The uniqueness of eqn (1a) is apparent from the B_1 field which can be a constant or spatially vary. The relaxation parameters

$$T_g = \frac{1}{T_1 T_2} \quad \text{and} \quad \frac{1}{T_o} = \frac{1}{T_1} + \frac{1}{T_2}$$

are intrinsic properties of the NMR system and are constants. The wave function $\psi(x)$, expresses the mathematical form of particle wave properties associated with the fluid particles of the flow system.

2. SOLUTION OF THE TIME INDEPENDENT BLOCH NMR FLOW EQUATIONS BY THE METHOD OF SEPARATION OF VARIABLES

The flow of blood in straight blood vessels, like the flow of liquids in narrow rigid tubes, is normally laminar (streamline). Within the blood vessels, an infinitely thin layer of blood in contact with the wall of the vessel does not move. The next layer within the vessel has a low velocity, the next a higher velocity, and forth, velocity being greatest in the centre of the stream. Laminar flow occurs at velocities up to a certain critical velocity. At or above this velocity, flow is turbulent. The probability of turbulence is also related to the diameter of the vessel and the viscosity of the blood. In humans, the critical velocity is sometimes exceeded in the ascending aorta at the peak of systolic ejection, but usually exceeded only when an artery is constricted. Turbulence occurs more frequently in anaemia because the viscosity of blood is lower. This may be the explanation of the systolic murmurs that are common in anaemia.

In this study, we shall suppose the horizontal extent of the flow is small enough for the flow to be regarded as taking place in a plane layer with the coriolis parameter f uniform and equal to f_o say. The direction of the stenosis in the blood vessel is immaterial, and we choose it for convenience to be in the y -direction (Fig. 1). The stream approaching the stenosis will be assumed to have zero relative vorticity and uniform velocity with components (V, V_1) , with the coriolis force being balanced by a uniform pressure gradient. At a point over the stenosis where the layer thickness is h_1 , the relative vorticity \mathcal{G} is

$$\text{given by: } \frac{f_o + \mathcal{G}}{h_1} = \frac{f_o}{h}$$

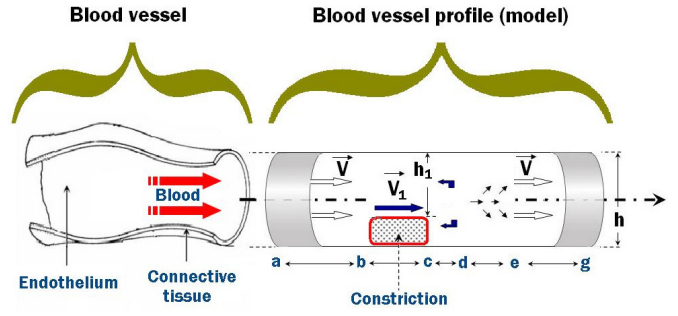


Figure 1. Effect of constriction on the velocity profile in a blood vessel: (ab) Laminar flow velocity V , (bc) High velocity V_1 , (de) Turbulent and (eg) Laminar flow. The diameter of the blood vessel is h .

We would introduce the transverse magnetization M_y , as the stream function such that the flow in the region is steady and $f + \mathcal{G}$ is consequently a function of M_y alone [16]. Hence we have:

$$M_y = V_1 y \quad \text{and} \quad f + \mathcal{G} = \frac{h_1(f_o + \beta y)}{h} = \frac{h_1(f_o + \frac{\beta}{V_1} M_y)}{h}$$

This must be the relationship between $f + \mathcal{G}$ and M_y that is valid over the whole of the region $x > 0$. Hence in that region we have [18]

$$\nabla^2 M_y = -\mathcal{G} = f_o \left(\frac{h - h_1}{h} \right) + \beta y - \rho^2 M_y \quad (1b)$$

where $\rho^2 = \beta \frac{h_1}{V_1 h}$.

Our choice of conditions has led to a linear equation for M_y . A form of solution which contains the linearity in M_y has been presented [18] where:

$$M_y = (y - a)\psi(x) + \frac{[f_o((h - h_1)/h) + \beta y]}{\rho^2}$$

here, a is a constant and $\psi(x)$ satisfies eqn (1a) with

$$\rho^2 = \frac{\gamma^2 B_1^2}{V^2} \quad \text{and} \quad x(y + a) = 1.$$

Eqn (1b) can be written in 3D as:

$$\nabla^2 M_y + \frac{\alpha}{v} M_y = 0 \quad (2a)$$

where v is the speed of fluid, α is a constant to be determined and the relative vorticity \mathcal{G} , is:

$$\mathcal{G} = \frac{\alpha}{v} M_y \quad (2b)$$

3. THE CYLINDRICAL MODEL

We consider a plaque which takes the shape of a cylinder as shown in Figure 1. A cylindrical coordinate (Fig. 2) is constructed such that its maximum radius indicates a

totally blocked blood vessel. For a blood molecule which tunnels through the plaque and could be located at the centre of the plaque or any other point within the plaque eqn (2) becomes,

$$\frac{\partial^2 M_y}{\partial r^2} + \frac{1}{r} \frac{\partial M_y}{\partial r} + \frac{1}{r^2} \frac{\partial^2 M_y}{\partial \theta^2} + \frac{\partial^2 M_y}{\partial Z^2} + \frac{\alpha}{v} M_y = 0 \quad (3)$$

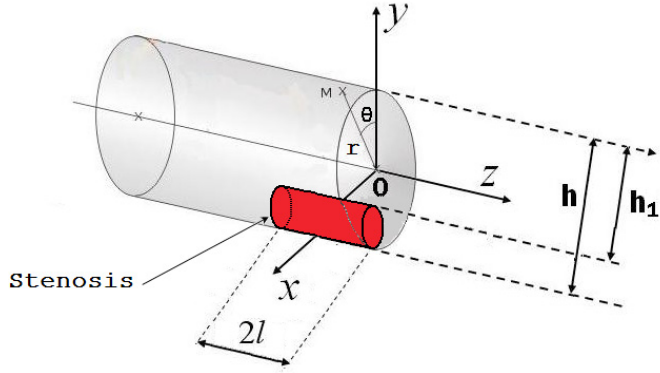


Figure 2. Stenosis with cylindrical geometry.

Eqn (3) will be solved by the method of separation of variables

$$M_y(r, \theta, z) = F(r, \theta(Z(z))) \quad (4)$$

satisfying the following three independent eqns in terms of $R(r)$, $\Theta(\theta)$ and $Z(z)$ respectively:

$$r^2 \frac{d^2 R}{dr^2} + r \frac{dR}{dr} + (k^2 r^2 - m^2) R = 0 \quad (5)$$

$$\frac{d^2 \Theta}{d\theta^2} - m^2 \Theta = 0 \quad (6)$$

$$\frac{d^2 Z}{dz^2} + \left(\frac{\alpha}{v} - k^2\right) Z = 0 \quad (7)$$

The solution of eqn (7) is

$$Z(z) = a_1 e^{i\lambda z} + a_2 e^{-i\lambda z} = B_1 \cos \lambda z + B_2 \sin \lambda z \quad (8)$$

where

$$\lambda = \pm i \sqrt{\left(\frac{\alpha}{v} - k^2\right)}, \quad B_2 = i(a_1 + a_2) \text{ and } B_1 = (a_1 + a_2) \quad (9)$$

The solution of eqn (6) is given as

$$\Theta(\theta) = C_1 e^{im\theta} + C_2 e^{-im\theta} = C_1 \cos m\theta + C_2 \sin m\theta \quad (10)$$

Eqn (5) is the Bessel differential equation of order m and the general solution is given by:

$$R(r) = D_1 J_m(kr) + D_2 Y_m(kr) \quad (11)$$

From eqns (9–11), we can write

$$M_y(r, \theta, z) = D_1 J_m(kr) \{C_1 \cos m\theta + C_2 \sin m\theta\} \{B_1 \cos \lambda z + B_2 \sin \lambda z\} \quad (12)$$

It should be noted in eqn (11) that within the region containing $r = 0$, it is required that $D_2 = 0$, due to the $Y_m(r)$ behaviour at the origin ($r = 0$).

4. NMR TRANSVERSE MAGNETIZATION AT ZERO RELATIVE VORTICITY

Since the stream approaching the stenosis is assumed to have zero relative vorticity ($\vartheta = 0$) and uniform velocity with components (V, V_1), and the coriolis force being balanced by a uniform pressure gradient, it is required that at the point $Z = -l$, we have:

$$M_y(r, \theta, -l) = R(r)(\Theta(\theta)Z(-l)) = 0 \text{ and } Z(-l) = 0 \quad (13a)$$

and

$$B_1 \cos(-\lambda l) + B_2 \sin(-\lambda l) = 0 \quad (13b)$$

provided that the length of the cylindrical plaque is $2l$.

From eqn (13b), if we choose $B_1 = 0; B_2 \neq 0$, we can write:

$$Z(-l) = -B_2 \sin \lambda l = 0. \quad (14)$$

Eqns (9) and (14) give:

$$\sqrt{\left(\frac{\alpha}{v} - k^2\right)} = \frac{m\pi}{l} \quad (15a)$$

$$Z_m(z) = B \sin \frac{m\pi z}{l}; \quad B = -B_2 \quad (15b)$$

and

$$\alpha = v \left(\frac{m^2 \pi^2}{l^2} + k^2 \right) \quad (16)$$

5. NMR TRANSVERSE MAGNETIZATION AT THE ABSOLUTE RADIUS OF THE CYLINDER

It is important to consider the NMR magnetization at $r = r_o$, (r_o is the absolute radius of the cylinder). At this point the blood vessel is totally blocked which implies that much collision of blood particles of the incoming stream (the spins) within the region is expected. There would be a considerable level of incoherence and the magnetic moments of blood cell spins will mostly cancel each other out. Hence we do not expect any NMR blood flow signal to be detected or at most, the flow signal should be minimum and very close to zero. Classically, it is not expected that many blood particles would tunnel through the plaque because most of them remain and circle around the plaque at the point $Z(-l)$. We can write;

$$M_y(r_o, \theta, -l) = R(r_o)(\Theta(\theta)Z(-l)) = 0 \quad (17a)$$

and

$$Z(-l) = R(r_o) = D_1 J_m(kr_o) = J_m(kr_o) = 0 \quad (17b)$$

where $k_{mp} r_o$ is defined as the positive roots of $J_m(k_{mp} r_o)$ and $p = 1, 2, 3, \dots$

Summing over both m and p gives:

$$M_y(r, \theta, z) = \sum_{m=0}^{\infty} \left[\sum_{p=1}^{\infty} A_{mp} J_m \left(\frac{\chi_{mp} r}{r_o} \right) \cos m\theta + \sum_{p=1}^{\infty} B_{mp} J_m \left(\frac{\chi_{mp} r}{r_o} \right) \sin m\theta \right] \sin \frac{m\pi z}{l} \quad (18)$$

where

$$\begin{cases} \chi_{mp} = k_{mp} r_o \\ A_{mp} = BC_1 D_1 \\ B_{mp} = BC_2 D_1 \end{cases} \quad (19)$$

6. NMR TRANSVERSE MAGNETIZATION DETECTABLE AT $Z=l/2m$

It is interesting to consider the NMR signal detectable at the point $z = l/2m$, ($m \neq 0$) when the following boundary condition is imposed:

$$M_y(r, \theta, \frac{l}{2m}) = M_c(r\theta) \quad (20)$$

we can write:

$$M_y(r, \theta, \frac{l}{2m}) = \sum_{m=1}^{\infty} [C_m \cos m\theta + D_m \sin m\theta] \quad (21)$$

where

$$\begin{cases} C_m = \lim_{N_0 \rightarrow \infty} \left(\frac{1}{2N_0} \sum_{p=1}^{N_0} \mu_{mp} B_{4m} \left(\frac{r}{r_o} \xi_{mp} \right) \right) \\ D_m = \lim_{N_0 \rightarrow \infty} \left(\frac{1}{2N_0} \sum_{p=1}^{N_0} \mu'_{mp} B_{4m} \left(\frac{r}{r_o} \xi_{mp} \right) \right) \end{cases} \quad (22)$$

where B_{4m} are the $4m$ -order Boubaker polynomials [26–34], ξ_{mp} is the B_{4m} polynomial p th order positive root, N_0 is a prefixed integer, $\mu_{mp}|_{p=1..N_0}$ and $\mu'_{mp}|_{p=1..N_0}$ are unknown real coefficients calculated using the properties of the Boubaker polynomial expansion scheme (BPES) [27–30]:

$$\mu_{mp} = \frac{2}{\pi r_o^2 [B_{4(m+1)}(\xi_{mp})]^2} \int_0^{r_o} \int_0^{2\pi} r M_c(r, \theta) B_{4m} \left(\frac{r}{r_o} \xi_{mp} \right) \cos m\theta dr d\theta \quad (23)$$

$$\mu'_{mp} = \frac{2}{\pi r_o^2 [B_{4(m+1)}(\xi_{mp})]^2} \int_0^{r_o} \int_0^{2\pi} r M_c(r, \theta) B_{4m} \left(\frac{r}{r_o} \xi_{mp} \right) \sin m\theta dr d\theta \quad (24)$$

Hence the required solution for the NMR transverse magnetization at $z = l/2m$ is:

$$M_y(r, \theta, z) \Big|_{z=\frac{l}{2m}} = \sum_{m=1}^{\infty} \left[\sum_{p=1}^{\infty} \mu_{mp} B_{4m} \left(\frac{r}{r_o} \xi_{mp} \right) \cos m\theta + \sum_{p=1}^{\infty} \mu'_{mp} B_{4m} \left(\frac{r}{r_o} \xi_{mp} \right) \sin m\theta \right] \quad (25)$$

7. CONCLUSION

The variation of NMR magnetizations and signals over a stenosis with a neat cylindrical geometry as presented in this study illustrates how the mathematical procedure can be useful in exciting ways for the general blood flow analysis especially in evaluation of coronary heart diseases. Eqns (16) and (19) define three dynamical properties which may be clinically significant:

(i) if we define the value of constant k or k_{mp} within the interval $0 < k < 1$ or $0 < k_{mp} < 1$, the parameter $\alpha\mu$ is directly proportional to the dynamic viscosity (in g/cm s) of blood flow under consideration where μ is the mass of a blood particle. The viscosity of a fluid is an important property in the analysis of liquid behaviour and fluid motion near vessel boundaries.

(ii) since the stream approaching the stenosis is assumed to have zero relative vorticity ($\mathcal{G} = 0$) with uniform velocity, equations (2b), (16) or (19) imply that constant k is inversely proportional to the length of the stenosis and χ_{mp} measures the ratio of stenosis diameter and length.

(iii) the values of m and k allow the evaluation of the NMR transverse magnetization and signal in terms of Bessel and Boubaker polynomials and trigonometrical functions depending on the diameter of the stenosis.

With the details provided in this study, we may begin to learn of blood flow behaviour in small blood vessels. Not only could this lead to new insights into fundamental problems, but it could point the way towards developing new strategies for understanding and treating coronary heart disease by MRI.

MRI is a sensitive method visualizing structural and functional changes in biological tissues. For example as blood particles (cells) develop, die, or regenerate, the local environment of the tissue and fluid changes and thus the NMR signal-changes. Such changes are reflected in MRI through local variations in the amount of water, its physical state (e.g. freely diffusing or protein bound), and its nuclear magnetic resonance (NMR) relaxation times (T_1 and T_2). The dynamics of these changes in the fluid are captured in MRI by the Bloch NMR flow equations [17–25]. The parameters that are derived in this presentation play a prominent role in the solution of the Bloch equations.

One key component of an MRI-sequence that is often used in the context of cardiovascular imaging, namely black blood imaging as mentioned above helps to distinguish blood from plaques in arteries. Apparently the transverse magnetization behaves in such a way that flow or other features can be detected. At present phase contrast imaging is the method of choice for measuring velocity in arteries. The method presented in this study, can have applications similar to phase contrast imaging but with more accurate information [17, 18]. How the NMR parameters derived in the present model are linked to a practical measurement in terms of an MRI sequence will be developed separately.

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