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APPLICATION OF BLOCH NMR EQUATION AND PENNES BIOHEAT EQUATION TO THERANOSTICS

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INTRODUCTION Theranostics has been regarded as a key part of personalized medicine and requires considerable advances in predictive medicine; novel theranostic agents are developed and carefully designed for in vivo quantitative assessment of the amount of drug reaching a pathological region and the visualization of molecular changes due to the therapeutic effects of the delivered drug. This study intends to mathematically model a closely knitted theranostic method in which a specially selected RF field is used to heat up a tissue and at the same time cause the spins of the tissue to emit MR signals. MATHEMATICAL FORMULATION We consider bioheat flow in one direction [1, 2] given in eqn(1); where ρ is tissue density, c is the specific heat of tissue, T is the tissue temperature, t is the time, w_b is the blood perfusion rate, c_b is the specific heat of blood, T_b is the supplying arterial blood temperature, k is the thermal conductivity of tissue, and x is the distance from the skin surface. SAR is the applied RF power per unit volume. If T changes very slowly with x, we have eqn(2) and solution to eqn(2) is given in eqn(3). If the T before the application of the RF field does not defer significantly from T_b, the initial the condition for this problem is given in eqn(4) and the final solution is given in eqn(5). The RF power for the voxel volume V_{vox} is P_{rf} = (SAR) V_{vox} . The energy of the oscillating radio wave is given as $E_{rf} = (1.055 \times 10^{-34}\,\mathrm{Js})\gamma B_1$, whose rate of change is expressed as in eqns(6) and (7). We can relate time dependent MRI signal to SAR using the time independent NMR equation [2] given by eqn (8) and (9). If we sample the signal when the M_y has the largest amplitude, we write $M_0 \approx 0$. Provided that the condition in eqn (10) holds, we have [2] eqn (11). From eqns (10) and (11), we obtain eqn (12). If the RF B_1 field is applied at time $t_0 = 0$, we have eqn (13). This solution is valid for the condition in eqn (14). It is always required that the M_y be finite as time tends to infinity; therefore, the solution to the problem is given by eqn (15). ANALYSIS OF RESULTS The results obtained in this study have been simulated with relaxation parameters of human liver at 1.5T [3] and the corresponding thermal properties [1, 3]: T_1 = 0.610s, T_2 = 0.057s, w_b = 2.86kg/m^{3}s, c_b = 3960J/kg.K, ρ = 1060kg/m^3, c = 3600J/kg.K. Plots a and b (SAR = 4W/m^3) give the distribution of the T and M_y on a log scale while plot c (SAR = 40000W/m^3) gives the density plot of M_y as a function of time and tissue temperature. CONCLUSION The temperature distribution and the RF power needed to generate RF B_1(t) field within the medically acceptable SAR limit during MRI scanning procedure have been investigated by solving the Pennes Bioheat equation in terms of MRI parameters. The relationship between T, SAR and RF B_1(t) at any given time is clearly shown in eqn (5), eqn (10) and Plots a, b, c. REFRENCES [1] Tzu-Ching Shih, Ping Yuan, Win-Li Lin, Hong-Sen Kou. MEP 29 (2007) 946-953. [2] O. B. Awojoyogbe, M. Dada, O.P. Faromika, O.E. Dada. CMRA. Vol. 38 A (3) 85-101 (2011). [3] Bottomley PA, Foster TH, Argersinger RE, Pfeifer LM (1984). Med Phys 11:425-448.

$$\rho c \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} + SAR - w_b c_b (T - T_b) \quad (1)$$

$$\rho c \frac{\partial T}{\partial t} = SAR - w_b c_b (T - T_b) \quad (2)$$

$$T(t) = T_b + \frac{SAR}{w_b c_b} + A \exp\left(-\frac{w_b c_b}{\rho c} t\right) \quad (3)$$

$$T(t=0) = T_b \quad (4)$$

$$T(t) = T_b + \frac{SAR}{w_b c_b}\left\{1 - \exp\left(-\frac{w_b c_b}{\rho c} t\right)\right\} \quad (5)$$

$$P_{rf} = \frac{dE_{rf}}{dt} \text{ and } E_{rf} = \int_{t_0}^{t} (SAR) V_{vox} dt \quad (6)$$

$$\gamma B_1(t) = \frac{V_{vox}}{\hbar}(SAR)(t - t_0) \quad (7)$$

$$\frac{d^2 M_y}{dt^2} + T_o \frac{dM_y}{dt} + \left(T_g + \gamma^2 B_1^2(t)\right) M_y = \frac{M_o}{T_1} \gamma B_1(t) \quad (8)$$

$$\text{where } T_g = \frac{1}{T_1 T_2} \text{ and } T_0 = \frac{1}{T_1} + \frac{1}{T_2} \quad (9)$$

$$T_g << \gamma^2 B_1^2(t) \quad (10)$$

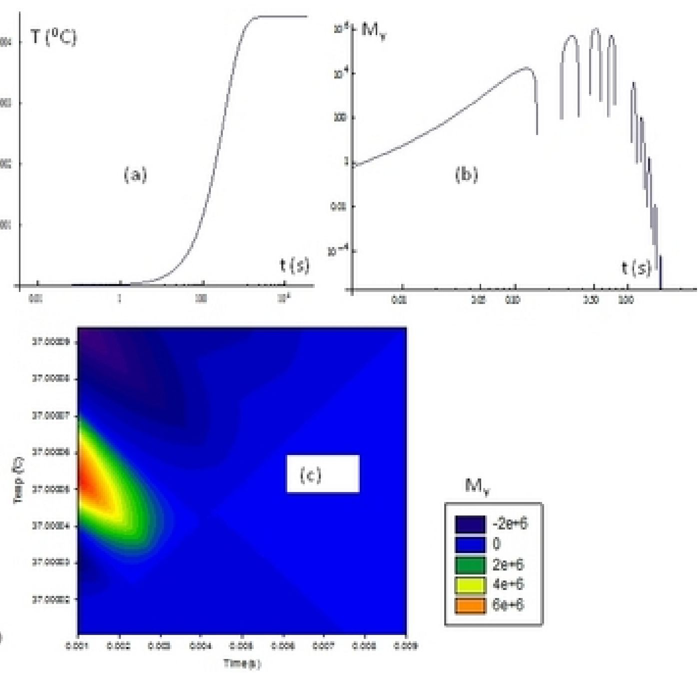
$$\frac{d^2 M_y}{dt^2} + T_o \frac{dM_y}{dt} + \gamma^2 B_1^2(t) M_y = 0 \quad (11)$$

$$\frac{d^2 M_y}{dt^2} + T_o \frac{dM_y}{dt} + \left(\frac{V_{vox}}{\hbar}\right)^2 (SAR)^2 (t - t_0)^2 M_y = 0 \quad (12)$$

$$M_y(t) = (\beta)^n (t)^n \left[C_1 J_n\left(\frac{V_{vox}}{2\hbar} t^2\right) + C_2 Y_n\left(\frac{V_{vox}}{2\hbar} t^2\right)\right], \; n = \frac{1 - T_g}{2} \quad (13)$$

$$T_0(t - t_0) \leq 1 \quad (14)$$

$$M_y(t) = C_1(\beta)^{\frac{1-T_g}{2}} (t)^{\frac{1-T_g}{2}} J_{\frac{1-T_g}{2}}\left(\frac{V_{vox}}{2\hbar} t^2\right) \quad (15)$$



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