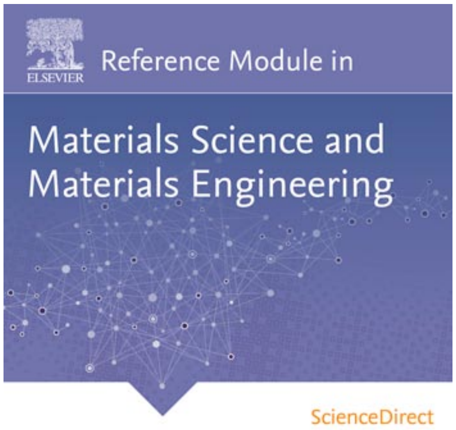

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Ice and Frozen Earth as Construction Materials☆

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1 Introduction

Systematic studies of the engineering properties of ice and frozen soil (IFS) have proliferated since the 1950 s. Cold War-era military concerns stimulated a great deal of interest in Arctic and sub-Arctic construction of military installations, resource extraction (e.g., the Trans-Alaska Pipeline, off-shore oil recovery), and now climate change issues have, in turn, provided the impetus for continuous improvements in our understanding of the physical, mechanical, and thermal properties of IFS. There has been a long-term interest in topics related to freshwater ice engineering (e.g., understanding and mitigating ice jams and extending the navigational season for inland waterways in northern regions). The logistics and support needs of Antarctic research activities have necessitated the use of both sea ice and freshwater ice surfaces for runways, and the new South Pole Station requires construction of a complex facility, tunnels, and runways on densely packed snow. A wealth of practical information exists in the circum-polar nations as they have dealt with the engineering problems associated with developing their infrastructures on permafrost. Then, the physical, thermal, and mechanical properties of snow, ice, and permafrost, specifically frozen ground, are responsible for the structure and behavior of these cryospheric materials (Arenson *et al.*, 2015).

Engineering interest in ice and frozen ground may be defined broadly in terms of expedient and long-term uses. Expedient or short-term applications include the use of seasonal ice covers for surface transportation, platforms for sub-sea drilling operations, and as aircraft runways. Ice pads have been constructed on permafrost for service as temporary drilling platforms and equipment storage areas. Perhaps the most common expedient use of frozen soil is termed *artificial ground freezing*. This technique uses freezing by mechanical refrigeration to temporarily stabilize ground adjacent to a construction site. In another application, some consideration is being given to the use of this technique to temporarily inhibit the spread of contaminants in ground water. Long-term engineering applications of frozen ground include building foundations and dams.

Ground that remains frozen through annual cycles is termed *permafrost*. The depth to which permafrost occurs is quite variable, but it can extend to 100 m at higher latitudes. The ground overlaying a permafrost deposit typically thaws and re-freezes annually. This is termed the *active layer*, and its thickness can extend up to several meters, depending on latitude, elevation, and microclimate. The active layer can experience a severe loss of strength when it thaws, depending on moisture and local drainage conditions. In the context of climate change studies, it has become evident that permafrost characteristics, and whether it is degrading (e.g., thawing – a common trend) or aggrading (growing in extent – currently rather rare), is highly dependent on local soil and moisture conditions and microclimate.

Dealing with the effects of annual freeze/thaw cycles and potentially significant climate changes over the design life of new structures significantly adds to the complexities of cold regions engineering. With regard to the issue of climate change, it is worth noting for the case of permafrost within a few degrees of the melting point, that its long-term mechanical behavior will be a very sensitive function of temperature. Consequently, engineering applications that require estimates of the long-term performance of warm frozen ground should account for possible changes in the thermal environment.

Permafrost deposits occur sporadically at lower latitudes (discontinuous permafrost), eventually giving way to seasonally frozen ground. Permafrost exists in alpine environments, and such deposits are currently of interest in climate studies because of their

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sensitivity to thermal fluctuations in the environment. Climate trends are such that permafrost deposits are warming and thawing in many areas, and parts of Alaska, for example, are warming much faster than the global average. This is having a disastrous effect on the engineering structures founded on the thawing material, many of which were constructed well before the current climate trends were recognized. Damage upon thawing is most severe when there is a high ice content in the frozen ground.

If the volume fraction of ice is small, the material will retain some strength when it thaws and settlement will be minimal. However, when the volume fraction of ice is high (so-called *ice-rich permafrost*), thawing results in excessive, and generally uneven, settlement and a nearly complete loss of shear strength. This in turn has catastrophic consequences on any overlying structures. As discussed below, the strength of frozen ground becomes increasingly more temperature sensitive as it approaches the melting point. As a result, relatively small increases in temperature cause dramatic increases in deformation rates in material that exists near the melting point. Such weakening can cause disastrous instabilities in alpine regions, for example, where the soil typically exists under a relatively high state of differential stress.

In addition to the extremes of either pure ice or dense frozen soil, there is considerable interest in the intermediate cases, referred to as *debris-laden ice* in a glaciological context, or ice-rich frozen ground in a geotechnical context. Interestingly, although an overall trend of particle strengthening is observed with increasing volume fraction of soil, the addition of small quantities of fine-grained particles has been found to actually accelerate the creep rate observed under a given stress level.

A challenging aspect of the engineering applications of IFS is that their mechanical properties must be understood over a wide range of environmental and loading conditions, for a variety of physical characteristics, and over a wide range of physical scales. Glaciological applications and the engineering of foundations in frozen materials, for example, deal with material behavior at time scales that cannot be achieved in the laboratory. To approach meaningful conditions, and thereby support the development of more reliable models, ice creep experiments have been conducted for periods of up to 4 years.

However, very few of such long-term experiments have been conducted, and creep rate predictions, for the most part, involve significant extrapolation. On the other extreme, to estimate peak loads in iceberg-structure interaction problems, the mechanical behavior of the same material must be known for strain rates in excess of $1 \times 10^{-2}\ \mathrm{s}^{-1}$. Applications dealing with floating ice sheets (either freshwater or sea ice) require knowledge of the deformation and fracture behavior of ice at higher strain rates, large scales, relatively warm temperatures (since the bottom of such sheets are always at the melting point), and frequently under complex loading.

2 Ice Fundamentals

The microstructure of ice, which is governed by the conditions of its formation and deformation history, strongly influences its mechanical properties. There are significant differences in the properties of, for example, freshwater lake ice and sea ice. As a result of this, ice mechanics applications are typically divided into several broad categories based on ice type. The most prominent difference is between freshwater ice and sea ice. Freshwater ice considerations may be subdivided into *granular* ice (in which the individual grains are roughly equidimensional), and *columnar* ice (in which the grains are elongated in the direction of growth). River and lake ice are examples of the latter; glacier and atmospheric ice are examples of the former.

Interest in glaciers has stimulated scientific work on freshwater granular ice, and transportation and navigational interests have stimulated a considerable amount of engineering work on river and lake ice. Sea ice research has been spurred significantly by naval requirements for operational capability in Arctic waters, the development of offshore oil fields, and more recently by climate modeling issues. With regard to the latter point, it has become increasingly evident that the predictions of global climate models depend critically on the treatment of the polar regions, and this in turn requires a thorough understanding of the growth, movement, and melting of sea ice in the polar regions.

Ice Ih is the common terrestrial form. It is hydrogen bonded and has a hexagonal close packed (HCP) structure. Although there are on average two hydrogen atoms per oxygen atom, the protons are disordered. This disorder gives rise to a dielectric and a weak mechanical relaxation. Basal plane slip dominates the inelastic deformation of ice unless a zero shear stress can be maintained on that plane. In polycrystals, basal dislocations typically nucleate at grain boundary irregularities and expand as semihexagonal loops. They experience an enormous drag – some five orders of magnitude larger than typically observed in metals. The high drag force is believed to result from the hydrogen disorder.

Because of this disorder, a basal dislocation cannot move an arbitrary distance without creating an excessive number of point defects as it moves. Rather, its motion is strongly influenced by the rate at which the protons in its path can arrange themselves such that the dislocation can pass without creating a defect. Nonbasal dislocations have been observed in ice. These consist of relatively fast but narrow edge segments that trail immobile screw segments. Although nonbasal slip is difficult to activate, and does not make a substantial contribution to overall straining, nonbasal dislocations have been observed to cross slip and operate as sources of basal dislocations.

2.1 Overview of the Mechanical Behavior of Ice

Emerging experimental evidence indicates that the flow of ice is controlled by basal dislocation drag, which is not unexpected given the high drag force on the predominant slip system in this material. A large body of evidence indicates that ice follows a power law (strain rate $\propto \sigma^n$) with $n = 3$–4, for strain rates of approximately 1×10^{-8}–1×10^{-3} s^{-1}. Values of n approaching four seem to be associated with the influence of microcracking on the strain rate. At usual environmental temperatures, power law break down occurs near a strain rate of approximately 1×10^{-3} s^{-1} for fine-grained ice (e.g., $d \sim \leqslant 1$–2 mm) and somewhat lower strain rates for larger-grained material. A fall-off to linear ($n = 1$) behavior has frequently been observed at strain rates in the approximate range of 1×10^{-8} s^{-1}–1×10^{-9} s^{-1}.

With regard to its overall creep response, ice typically exhibits a decelerating primary creep phase, which transitions relatively quickly (usually near an axial strain of 1×10^{-2}) into an accelerating tertiary creep phase. Ice does not exhibit a significant secondary creep phase (in which the strain rate is nominally constant), under typical environmental conditions because of the influence of dynamic recrystallization or microcracking. The term *secondary creep* is usually given to the strain rate minimum that occurs near 1×10^{-2} strain, and typically amounts to little more than an inflection point in strain rate versus strain or strain rate versus time space.

Although the strain rate minimum is typically used to quantify stress–strain rate relationships, it is an appropriate failure point only when analyzing short-term behavior. Applications dealing with long-term deformation to high strains should consider the behavior of the recrystallized material. In this regard is it worth emphasizing that terrestrial ice exists at high homologous temperatures and recrystallization is thus an important consideration in many applications. Additionally, because ice has a low fracture strength, cracks develop quite readily and have a significant influence on the deformation and failure under a wide range of conditions.

Microcracking influences the mechanical properties of ice to varying degrees at strain rates above approximately 1×10^{-6} s^{-1}. For the case of uniaxial compression, the extent of microcracking increases until the transition to brittle behavior is reached (typically at strain rates in the range of 1×10^{-3}–1×10^{-2} s^{-1}). The brittle failure strain in granular freshwater ice subjected to tension or compression is on the order of 1×10^{-3}, which is significantly lower than the ductile failure strains noted above. Confinement suppresses the cracking activity, and shifts the transition to brittle behavior to higher strain rates. Both the size of microcracks and the stress levels associated with their nucleation depend on grain size. It has been shown that tensile failure of pure polycrystalline ice prepared in the laboratory is controlled by the propagation of subcritical flaws at low strain rates, and controlled by the nucleation of critical sized flaws at higher strain rates.

Individual crystals have mildly anisotropic elastic properties, but because basal dislocation slip is so strongly favored over other slip systems, the inelastic properties are highly anisotropic. Ice exhibits significant anelastic straining due to basal dislocation activity primarily, and to grain boundary sliding to a lesser extent. The anelastic strain can be up to 10 times the elastic strain level, and can thus produce a significant modulus defect. Polycrystalline ice exhibits a relatively minor grain boundary anelasticity that amounts to approximately 10–20% of the elastic strain for grain sizes that typically occur in nature. There has been considerable debate over the mechanism of viscous straining in ice – primarily whether it is glide or climb controlled. Recent experimental results that provide insight into the effective dislocation density as a function of prior strain history indicate that $n = 3$ power law is observed when the dislocation density increases during straining, and that $n \sim 1$ results when there is a relatively constant dislocation density during straining. Although the issue requires more attention, this and other findings support a glide-controlled mechanism of viscous straining in ice.

2.2 Microstructural Considerations

The microstructure of ice depends on its formation process and its deformational history. Glacier ice forms from consolidated snow, and generally retains a granular structure even after considerable grain growth and dynamic recrystallization. The initial ice skim that forms on bodies of salt water or freshwater frequently has a granular structure, but because the preferred growth direction is perpendicular to the *c* axis, it develops a columnar structure with continued growth, and the c axes of the individual grains tend to lie in the horizontal direction. Whereas freshwater ice grows with a planar interface and tends to reject impurities, sea ice grows dendritically, forming a microstructure with highly saline brine inclusions aligned along the interdendritic surfaces within each grain.

The subgrain structure exerts a strong influence on the mechanical, optical, and dielectric properties of sea ice. Either the growth process itself, or subsequent thermal fluctuations, apparently causes sea ice to develop a very high grown-in dislocation density relative to freshwater ice. The high dislocation density makes first year sea ice a much more compliant material than freshwater ice. Additionally, the liquid brine inclusions in sea ice provide a large flaw population that has a significant effect on the fracture properties of this complex material.

The existence of a liquid phase in sea ice leads to an important difference relative to freshwater ice. Although freshwater and sea ice sheets exist with a temperature gradient, the flaw structure of freshwater ice remains relatively stable until the onset of melt season. Conversely, the liquid-filled inclusions in sea ice form a complex flaw structure when the ice initially forms, and undergo a nearly constant series of changes in shape and size as the growth and melt seasons progress. Brine migrates in the sheet in response to the overall temperature gradient, seasonal thermal fluctuations, and gravity. The resulting changes in the flaw structure have profound effects on its mechanical and optical properties as well as the overall permeability of the sheet.

Fabric has an important influence on the properties of ice, and is of greatest interest in studies of glaciers and sea ice. As noted above, ice that forms on a body of water develops a c axis horizontal structure, and in freshwater ice, the c axes generally remain randomly oriented in the horizontal plane. However, deformational recrystallization in glacier ice produces a fabric (a preferred c axis direction), and under-ice currents during the growth of sea ice give a growth advantage to grains with their c axis aligned with the current direction. In either case, a fabric results that imparts a significant anisotropy to the mechanical properties of these materials. In ice mechanics, fabric is typically expressed in terms of the c axis direction only, due to the fact that there is no apparent directional dependence of slip on the basal plane.

2.3 Enhancement of Mechanical Properties

Engineers have applied various means to improve the mechanical properties of ice for specific applications. Sea ice crossings are sometimes required to gain access to offshore installations. If the naturally occurring ice cover is not sufficiently thick to sustain the necessary vehicular loads, the sheet can be built up by methods such as pumping or spraying to achieve the desired thickness. Although this method avoids a bearing capacity failure, problems have been encountered with surface gouging from heavy truck traffic. A successful engineering solution to this problem called for the placement of geosynthetics near the surface layer to serve as reinforcement. In earlier work, the addition of other reinforcing materials such as wood chips has been examined.

Freshwater ice crossings are common in northern regions, and tens of thousands of kilometers of ice roads are in seasonal use. In addition to the maximum load criteria, vehicular spacing, and speed limitations, there is the important but little-explored issue of fatigue failure. Heavy vehicle loads can produce low cycle fatigue behavior, and this has caused a number of breakthroughs, some of which have resulted in loss of life. Unfortunately, the fatigue failure process is not well understood for ice.

3 Frozen Soil Fundamentals

Under most environmental conditions, frozen soil consists of soil particles, gas inclusions, and both frozen and unfrozen water. An important characteristic of frozen soil is that even though it exists below the freezing point of pure bulk water, not all of the water is necessarily frozen. The amount of unfrozen water depends on the specific surface area and composition of the soil, the presence of dissolved salts or other chemicals, and the temperature. Laboratory observations indicate that there is hysteresis in the unfrozen water-temperature relationship. As a consequence, the unfrozen water content at a given subfreezing temperature will be higher upon cooling than during warming. This unfrozen water occurs primarily at the ice-particle interface region and forms what is sometimes referred to as a *liquid-like layer*. The water molecules in this region are sufficiently disordered by the influence of the soil particle that they retain some liquid-like characteristics (e.g., a lack of shear strength and a high rate of self-diffusion).

As a consequence, unfrozen water exerts a profound influence on the mechanical properties of frozen soil. For pure systems, the actual amount of unfrozen water at a given temperature increases as the average particle diameter decreases. Thus clays typically have the highest and sands the lowest amounts of unfrozen water at a given temperature near the freezing point. Silts are usually intermediate between those two. With regard to temperature sensitivity, coarse-grained materials have little, if any, detectable unfrozen water once the temperature drops to a few degrees below freezing. However, clays can retain a substantial quantity of unfrozen water at temperatures as low as -5 to $-10\ ^{\circ}\mathrm{C}$. The practical effect of this circumstance is that a saturated sand, for example, will realize most of its strength gain, and become relatively impermeable at substantially higher temperatures than silt or clay.

Although it might be expected that the temperature sensitivity of frozen soil deformation would closely parallel that of ice, this is not the case. The creep and strength properties of frozen soil generally exhibit a much stronger temperature dependence (e.g., a higher activation energy) than pure polycrystalline ice. Moreover, the activation energy increases dramatically as the melting point is approached. It is difficult to interpret the observed temperature dependence of frozen soil because the precise mechanisms of deformation have not been clearly identified.

The liquid-like layer associated with the soil particles form a communicating network in dense, frozen soils, so this relatively strong and seemingly impervious material can actually support diffusion, and thus the migration of contaminants. Prior to the time when this aspect of frozen soil was recognized, it was common practice in permafrost regions to isolate waste products by burial. Field studies now indicate that the contaminants associated with such sites are actually much more mobile than previously believed. Unfortunately, the effects of climate change are now exacerbating the situation.

3.1 Frost Heave and Thaw Weakening

Freezing soils can exhibit the phenomenon known as *frost heave*, whereby a gradient in the soil moisture tension is established that draws water at depth to the freezing front (the pore ice–pore water interface), where it joins the forming ice lattice. This process can force the soil particles apart to produce what are termed *ice lenses* (lenticular regions of segregated ice). Silt-sized particles are particularly prone to frost heaving because they are coarse enough to allow a sufficient flow of pore water to the freezing front, yet fine enough to support the significant capillary rise needed to access the water supply at depth. Frost heaving can generate tremendous pressures and cause differential surface displacements. It also leads to a phenomenon known as *thaw weakening*. When the material thaws, the excess water that was drawn up during the heaving process can result in positive pore water pressure, and thereby greatly reduce the soil strength.

Soils that experience this phenomenon are termed *frost susceptible*, and are particularly problematic in pavement systems where the strength loss can result in excessive amounts of damage to pavement surfaces. This strength loss subsides as the excess moisture drains subsequent to thawing. Since the rate at which the soil drains and reconsolidates governs the rate of strength recovery, pavements containing frost susceptible soil layers in poorly drained areas experience a protracted period of thaw weakening, and can accumulate a great deal of damage under traffic loading. Excessive damage can occur during the initial stages of thawing, when a relatively impermeable frozen layer underlies the thawed soil containing excess moisture. Most, if not all, of the soil's prefreezing strength is usually recovered once the excess pore water drains away and the soil reconsolidates. The postfreezing strength of newly compacted soils usually stabilizes after one to three freeze-thaw cycles.

As a consequence of the influence of freeze-thaw cycling on the mechanical properties of soil, it is frequently necessary to control the thermal regime of foundations and support structures. In the case of the elevated portions of the Trans-Alaska Pipeline, for example, devices known as *thermal siphons* were installed in many of the vertical support members to extract heat and thereby prevent thawing and loss of ground strength. The foundations of buildings founded on permafrost can be designed to minimize heat transfer to the ground where thaw settlement is a concern, and active refrigeration can be employed as well. Conversely, pre-construction thawing is carried out to stabilize the ground in cases where thawing is inevitable, or where re-freezing can be controlled or eliminated.

Since engineering decisions regarding structures on frozen ground depend critically on the consequences of thaw, knowledge of the physical state (primarily the thermal profile and ice content) of the underlying ground is essential. Information on the extent and thermal regime of permafrost has been complied and is available to the public (refer to the International Permafrost Association (IPA)). However, because of the extreme variability in subsurface conditions, site-specific information is necessary. Although coring provides a direct indication of subsurface conditions, it is time consuming and expensive in frozen ground. More expedient, indirect methods, such as ground penetrating radar, work well under many circumstances, and can provide useful information regarding the presence of massive ground ice.

The fact that frozen soil has a significant strength advantage over unfrozen soil has prompted the use of artificial ground freezing to aid in construction projects where it is critical to avoid deformation of the ground adjacent to an excavation. In applying such methods, however, it is important to remain cognizant of the potential for frost damage to occur – particularly in cases where the soil type and moisture conditions can support frost heave/thaw weakening. Since the presence of salts can dramatically increase the amount of unfrozen water in frozen soil, and thereby cause a loss in strength, it is important to exercise caution in the application of artificial ground freezing techniques in saline sediments.

3.2 Overview of the Mechanical Behavior of Frozen Soil

Engineering construction in cold regions and artificial ground freezing projects require an understanding of the mechanical properties of frozen soils. An understanding of properties such as strength and deformation are important for engineering design purposes (Lai *et al.*, 2013). Overall consideration of the mechanical properties of frozen soil may be divided into two thermal regimes: warm frozen (typically -10 to $-8\ ^{\circ}\mathrm{C}$ and the melting point) and cold frozen ($\leqslant -10\ ^{\circ}\mathrm{C}$). The boundary is somewhat arbitrary, varies with soil type, and is influenced by the presence of dissolved salts. The reason for considering two thermal regimes is based largely on the unfrozen water effects described above. The mechanical properties are much more temperature sensitive in the warm frozen regime than in the cold frozen regime and it has usually proven simpler to develop temperature-dependent constitutive relationships only for the regime of interest in a particular application.

There is a spectrum of composition that must be considered, ranging from ice rich to dense frozen soil. Experimental results on sand, for example, indicate that particle strengthening becomes important only after 42 vol.% of soil grains is exceeded. This is presumably a critical content for direct particle-to-particle contact to occur generally throughout the specimen. The strength (or creep) properties are only mildly influenced by the presence of the sand grains below this threshold.

The strength of dense frozen soil is higher than either of its components. This is now believed to be the result of soil particle interaction effects and strengthening of the pore ice (probably through the suppression of cracking). The effective elastic modulus of frozen soil is intermediate between that of pure ice and that of the solid soil mineral, and the stress–strain behavior in the nominally elastic stages of deformation appears to be largely governed by the pore ice matrix. In strength tests, particle interaction becomes important after an initial yield, and most frozen soils exhibit strain hardening, followed by strain softening at a relatively high axial strain (e.g., 0.20) compared to that found in pure ice. The dynamic parameters of frozen soil are important for both engineering design and numerical simulation, which directly affect the accuracy of calculations (Liu *et al.*, 2014). **Figure 1** (Xie *et al.*, 2014) illustrates the strain–stress curves of frozen soil obtained at different strain rates. It is seen that in the process of impacting, as the specimen deforms within a small strain, the stress–strain curves which obtained from dynamic tests are nearly linear; however, as the strain becomes larger and the specimen deforms within a larger strain, the strain-hardening behavior of the frozen soil specimen becomes notable. Although, different from the results obtain by Zhang *et al.* (2013); there is not an obvious plateau when the stress reaches to its peak value.

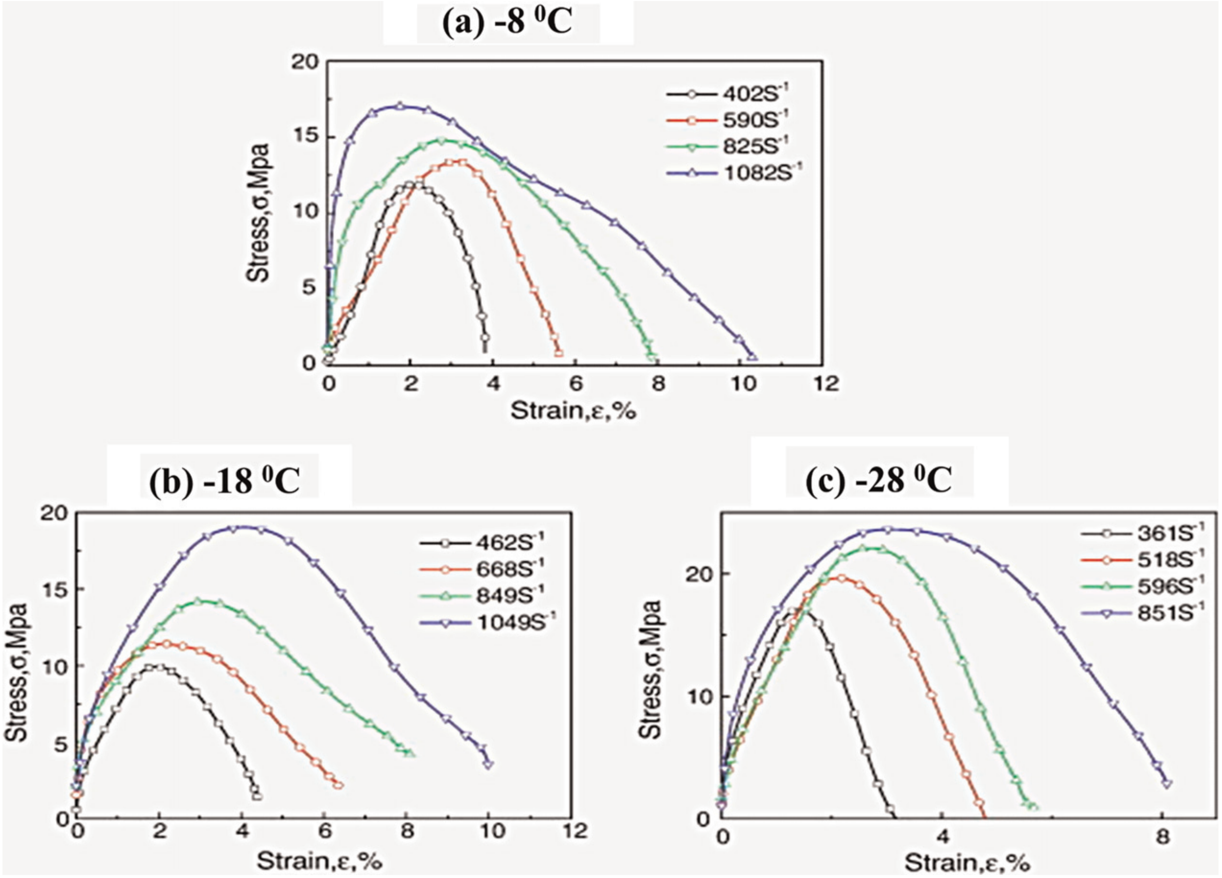

Figure 1 Strain–stress curves of frozen soil at different stain rates. Reproduced from Xie, Q., Zhu, Z., Kang, G., 2014. Dynamic stress–strain behavior of frozen soil: Experiments and modeling. Cold Regions Science and Technology 106–107, 153–160.

The extent of strain hardening depends upon the loading conditions and the initial relative density of the material. Frozen soils exhibit similar phases of creep behavior as polycrystalline ice, although a nominally constant secondary creep phase is somewhat more pronounced and the strains associated with primary creep and the strain at which the minimum creep rate occur are significantly greater (failure strains of ~0.20 are typical). Note that despite the identification of a failure point (e.g., either the peak stress achieved in a strength test or the minimum strain rate achieved in a creep test) for IFS, a considerable level of postpeak strength remains when the material is in the ductile regime.

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