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Non-Dimensionalized Parameter Development for Class 2B-lpl Compliant Constant-Force Compression Slider Mechanism

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ABSTRACT
This research work focuses on the development of non-dimensionalized parameter for class 2B-lpl compliant constant-force compression slider mechanism. It also expresses the desire to simplify the behavioral model for easy usage. Results obtained indicated an average non-dimesionalized parameter value of 1.2573, 1.2991, 1.3483, and 1.4081 for a 10, 20, 30, and 40% displacement respectively. The result also shows that the average force generated by the mechanism for a 10, 20, 30, and 40% displacement were 901.23N, 316.56N, 171.17N, and 110.44N respectively using the maximum flexible segments parameter values for the different percentages of mechanism slider displacement. This indicates clearly that using the non-dimensionalized parameter, the average force generated by this class of mechanism can easily be determined which greatly simplifies its usage.

Keywords: *Compliant, constant-force, mechanism, non-dimensionalized, parameter, slider.*

1 INTRODUCTION

A constant-force mechanism (CFM) can be defined as one that generates a constant, unidirectional force at any given point on a hinged lever, for all positions of the lever (Nathan, 1985). Constant-force mechanisms (CFMs) can be rigid-body mechanisms with linear and/or torsional springs or they can be compliant mechanisms CMs (Weight, 2001). Alternatively, CFM can be defined as a mechanism that produces a constant output force for a large range of input displacements. Such mechanisms are important in applications with varying displacements, but requiring a constant resultant output force (Nahar and Sugar, 2003). Traditionally, engineered devices are designed to be strong and stiff, and the systems are usually assembled from discrete components. On the other hand, designs in nature are strong, but compliant, and the systems in nature developed as one connected whole. CMs are relatively new class of mechanism that utilize compliance of their constituent elements to transmit motion/or force (Kota et al., 1999). CMs are single-piece flexible structures that deliver the desired motion by undergoing elastic deformation as opposed to rigid body motions of conventional mechanisms. They are particularly suited for applications with a small range of motions, as their unitized construction without joints makes their manufacture extremely simple, eliminating assembly operations altogether (Kota et al., 1999).

Designing CMs for specific applications can be a complex problem with many considerations. The basic trade-off is between the flexibility to achieve deformed motion and the rigidity to sustain external load (Li and Kota, 2002). The pseudo-rigid-body-model (PRBM) technique is a design tool that approximates the force-deflection relationships of CMs by assigning a rigid-body, lumped compliance counterpart to every flexible segment comprising the mechanism (Howell, 2001). What makes it so useful is its ability to transform a CM requiring in-depth nonlinear analysis into an equivalent rigid-body mechanism, for which well-known rigid-body kinematics techniques are already in place.

The compliant constant-force compression slider mechanism (CCFCSM) configurations as presented by Boyle (2001) and Weight (2001) have at least one rigid link, and at most one long fixed-pinned flexible beam. The CCFCSM $Class\ 2B-lpl$ configuration developed by Ugwuoke (2010) and considered in this work incorporates two long fixed-pinned flexible segments. The use of long flexible segments is important in gaining stiffness without increasing stress. This CCFCSM configuration is new, it is uncomplicated, and it can be adapted to various practical applications in which a constant reaction force is desired in response to a linear displacement. The result obtained for this class of mechanism as presented by Ugwuoke (2010) indicated that the $Class\ 2B-lpl$ CCFCSM generated the maximum constant-force within allowable slider displacement and stress limits when compared to the other classes of compliant constant-force compression slider mechanisms (CCFCSMs).

Using the PRBM technique, the behavioral model equations developed in most cases rely heavily upon the PRBM making the design of CCFCSMs difficult for engineers who have little or no experience with the PRBM technique (Weight, 2001). This research work attempts to further simplify the behavioral model equations by introducing into the model equations non-dimensionalized mechanism parameters specifically for $Class\ 2B-lpl$ CCFCSM configuration to simplify its usage and extend its application.

2 METHODOLOGY

Figure 1 shows the *Class* $2B-lpl$ CCFCSM and its PRBM. This class of mechanism maintains a constant force regardless of input displacement which is accomplished by determining specific geometric ratios that allow for equal increases in stored strain energy and mechanical advantage. *Class* $2B-lpl$ CCFCSM as shown in Figure 1 have two flexible segments located at the first and third pivot points. Dividing the CCFCSM shown in Figure 1 along its line of symmetry shows that it consists of a pair of the same CCFCSM mounted to the same ground and sharing the same slider. Having two mechanisms opposite each other is useful because each cancels the moment induced by the other and the issue of friction between slider and ground is eliminated.

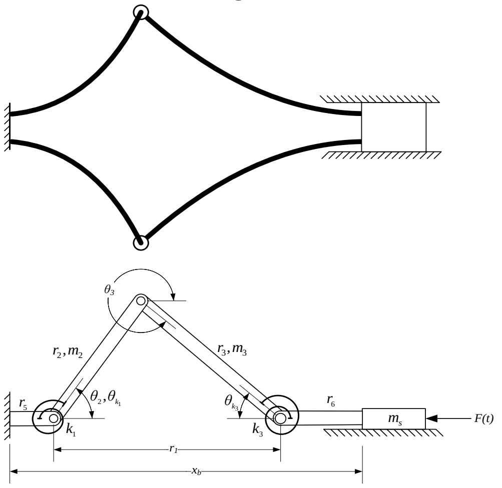

Figure 1: *Class* $2B-lpl$ Compliant Constant-Force Slider Mechanism, and its PRBM

Application of the principle of virtual work to the PRBM of mechanism and taking θ_2 as the generalized coordinate gives the following expression (Ugwuoke 2010; Ugwuoke, 2011)

$$F_{VW} = \frac{k_1}{r_2\left(\frac{r_3}{r_2}\right)\sin(\theta_2-\theta_3)} \times \left(\left(\frac{r_3}{r_2}\right)\theta_{k1}\cos\theta_3 + \left(\frac{k_3}{k_1}\right)\theta_{k3}\cos\theta_2\right) \quad (1)$$

Where,

r_2 *and* r_3 is the PRBM lengths of links 2 and 3

k_1 *and* k_3 is the PRBM torsional spring constants

θ_2 *and* θ_3 is the angular displacements of PRBM links

θ_{k1} *and* θ_{k3} is the angular displacements of PRBM torsional springs

Inspection of equation (1) shows that it relies on many independent PRBM variables. It would be of great benefit to simplify the behavioral model by introducing into the model equation dimensionless mechanism parameters in order to simplify its usage. In an attempt to do this, we try to replace all independent variables with dimensionless parameters. In the work done by Millar et al. (1996), Weight (2001), Ugwuoke (2010) and Ugwuoke (2011), three dimensionless parameters $R, K_1,$ *and* K_2 were chosen. These parameters, when substituted into equation (1) give the expression below

$$F_{VW} = \frac{k_1}{r_2 R\sin(\theta_2-\theta_3)} \times \left(R\theta_{k1}\cos\theta_3 + K_2\theta_{k3}\cos\theta_2\right) \quad (2)$$

Where,

$$R = \frac{r_3}{r_2};\ K_1 = \frac{k_2}{k_1} = 0;\ K_2 = \frac{k_3}{k_1} \quad (3)$$

R is the dimensionless geometric parameter ratio

K_1 *and* K_2 is the dimensionless stiffness parameter ratio

Associated with the CCFCSM's pin joints are (Boyle, 2001; Ugwuoke, 2010)

1) Coulomb friction in the pins
2) Possible binding of the pins due to misalignment
3) Unmodeled tolerances in the pin joints and
4) The effect of heating of the pins as they rotate

These effects are compensated for by introducing the term τ_{CFE} (torque due to Coulomb friction effects). Torque τ_{CFE} may be approximated using the following simple relation (Ugwuoke, 2010)

$$\tau_{CFE} = C\theta_2 sign\left(\dot{\theta}_2\right)\left(1+\frac{\cos\theta_2}{\sqrt{R^2-\sin^2\theta_2}}\right) \quad (4)$$

Torque τ_{CFE} is transformed to force F_{CFE} using the power relationship given as

$$F_{CFE}\,\dot{r}_1 = F_{CFE}\,\dot{\theta}_2\left(\frac{\partial r_1}{\partial\theta_2}\right) =$$

$$-r_2\sin\theta_2\left(1+\frac{\cos\theta_2}{\sqrt{R^2-\sin^2\theta_2}}\right)\dot{\theta}_2 \quad (5)$$

$$\times F_{CFE} = \tau_{CFE}\,\dot{\theta}_2$$

$$\tau_{CFE} = C\theta_2 sign\left(\dot{\theta}_2\right)\times\left(1+\frac{\cos\theta_2}{\sqrt{R^2-\sin^2\theta_2}}\right)$$

$$= -r_2\sin\theta_2\left(1+\frac{\cos\theta_2}{\sqrt{R^2-\sin^2\theta_2}}\right)\times F_{CFE} \quad (6)$$

$$F_{CFE} = -\frac{C\theta_2}{r_2\sin\theta_2}sign\left(\dot{\theta}_2\right) \quad (7)$$

Associated with the CCFCSM's links/segments are (Boyle, 2001; Ugwuoke 2010; Ugwuoke, 2011)

1) Possible flexing of the rigid links of CCFCSMs and
2) Possible flexing of the portion of the CCFCSM's compliant segments that the PRBM assumed to be rigid.

These possibilities are compensated for by introducing the term τ_{AFE} (torque due to axial force effects). Torque τ_{AFE} may be approximated using the following expression (Ugwuoke, 2010; Ugwuoke 2011)

$$\tau_{AFE} = F_{VW}\delta e = F_{VW} r_2 \alpha_{AFE}\left(1+\frac{r_2}{r_3}\right) \quad (8)$$

α_{AFE} *is the angle of axial force effect*

Similarly, torque τ_{AFE} is transformed to force F_{AFE} using the power relationship given as

$$F_{AFE}\,\dot{r}_1 = F_{AFE}\,\dot{\theta}_2\left(\frac{\partial r_1}{\partial \theta_2}\right)$$

$$= -r_2\sin\theta_2\left(1+\frac{r_2\cos\theta_2}{\sqrt{r_3^2 - r_2^2\sin^2\theta_2}}\right)\dot{\theta}_2 \quad (9)$$

$$\times F_{AFE} = \tau_{AFE}\,\dot{\theta}_2$$

$$\tau_{AFE} = F_{VW} r_2 \alpha_{AFE}\left(\frac{R+1}{R}\right)$$

$$= -r_2\sin\theta_2\left(1+\frac{\cos\theta_2}{\sqrt{R^2-\sin^2\theta_2}}\right)\times F_{AFE} \quad (10)$$

$$F_{AFE} = -\frac{\alpha_{AFE}\left(\frac{R+1}{R}\right)}{\sin\theta_2\left(1+\frac{\cos\theta_2}{\sqrt{R^2-\sin^2\theta_2}}\right)} \quad (11)$$

The value of the angle of axial force effect α_{AFE} is chosen using experimental data (Ugwuoke, 2010; Ugwuoke 2011). The generalized equation is therefore a combination of the three forces F_{VW}, F_{CFE} *and* F_{AFE} which may be expressed mathematically as

$$F = F_{VW} + F_{CFE} + F_{AFE} \quad (12)$$

$$F = F_{VW}\left(1-\frac{\alpha_{AFE}\left(\frac{R+1}{R}\right)}{\sin\theta_2\left(1+\frac{\cos\theta_2}{\sqrt{R^2-\sin^2\theta_2}}\right)}\right) \quad (13)$$

$$-\frac{C\theta_2}{r_2\sin\theta_2}sign\left(\dot{\theta}_2\right) = \left(\frac{k_1}{r_2}\Phi - \frac{C\theta_2}{r_2\sin\theta_2}sign\left(\dot{\theta}_2\right)\right)$$

Where,

$$\Phi = \left(\frac{R\theta_{k1}\cos\theta_3 + K_2\theta_{k3}\cos\theta_2}{R\sin(\theta_2-\theta_3)}\right)$$

$$\times\left(1-\frac{\alpha_{AFE}\left(\frac{R+1}{R}\right)}{\sin\theta_2\left(1+\frac{\cos\theta_2}{\sqrt{R^2-\sin^2\theta_2}}\right)}\right) \quad (14)$$

$$\theta_3 = \sin^{-1}\left(-\frac{1}{R}\sin\theta_2\right) \quad (15)$$

$$\theta_{k1} = \theta_2 = \cos^{-1}\left(\frac{r_1^2+r_2^2-r_3^2}{2r_1r_2}\right) \quad (16)$$

$$\theta_{k2} = \theta_{k1} + \sin^{-1}\left(\frac{1}{R}\sin\theta_2\right) \quad (17)$$

$$\theta_{k3} = \sin^{-1}\left(\frac{1}{R}\sin\theta_2\right) \quad (18)$$

$$L_{Tot} = L_1 + L_3 = Total\ CCFCSM\ length \quad (19)$$

$$r_{Tot} = \frac{L_{Tot}}{\lambda} = r_2 + r_3 = Total\ PRBM\ length \quad (20)$$

$$r_2 = \frac{r_{Tot}}{(R+1)} = 0.85\times L_1;\ r_5 = 0.15\times L_1 \quad (21)$$

$$r_3 = \frac{r_{Tot}}{\left(\frac{1}{R}+1\right)} = 0.85\times L_3;\ r_6 = 0.15\times L_3 \quad (22)$$

The Value of the length parameter λ for a 10, 20, 30, and 40% mechanism slider displacement is 1.1765 for *Class* $2B-lpl$ CCFCSM (Ugwuoke, 2010). L_1 and L_3 are the lengths of the flexible segments of the actual mechanism. Neglecting the effect of Coulomb friction in the mechanism pin joint, equation (13) reduces to

$$F = F_{VW}\left(1-\frac{\alpha_{AFE}\left(\frac{R+1}{R}\right)}{\sin\theta_2\left(1+\frac{\cos\theta_2}{\sqrt{R^2-\sin^2\theta_2}}\right)}\right) = \frac{k_1}{r_2}\Phi \quad (23)$$

A close examination shows that equation (14) is dimensionless indicating that the introduction of the non-dimensionalized mechanism parameter Φ in equation (13) greatly simplified the model. Examination of equation (23), indicates that F depends only on the parameters Φ, k_1 and r_2. The spring constant k_1 is considered to be the stiffness parameter, while the link length r_2 is known as the geometric parameter. Thus, the development of non-dimmensionalized mechanism parameter Φ reduced the number of independent variables, making the model easier to use. Once the non-dimensionalized mechanism parameter Φ is known, the average force generated by *Class* $2B-lpl$ CCFCSM can easily be computed using equation (23).

3 RESULTS AND DISCUSSION

Table 1 gives the relevant mechanism parameters and variable values of the *Class* $2B-lpl$ CCFCSM for a 40% mechanism slider displacement. Tables 2, 3, and 4 gives the maximum flexible segments parameter values for a 10, 20, and 30% mechanism slider displacement for a material yield strength of 1400 MPa. Table 5 gives the CCFCSM's extended length, fully compressed length, nominal constant-force and average non-dimensionalized parameter value for a 10, 20, 30, and 40% mechanism slider displacement. Plate 1 shows the photograph of the fabricated *Class* $2B-lpl$ CCFCSM in its fully compressed state.

Table 1: Parameters and Values for CCFCSM

Parameter	*Class* $2B-lpl$
r_2	64.7619 mm
r_3	71.2381 mm
r_5	11.4286 mm
r_6	12.5714 mm
m_2	24.8075 g
m_3	29.4264 g
m_S	116.7138 g
b	25.40 mm
h_1	0.4611 mm
h_2	-
h_3	0.5817 mm
I_1	2.0747×10^{-13} m^4
I_2	-
I_3	4.1672×10^{-13} m^4
E	207 Gpa
l_1	76.1905 mm
l_2	-
l_3	83.8095 mm
k_1	2.5393 Nm
k_2	-
k_3	4.6368 Nm

Table 2: Maximum Flexible Segments Parameter Values for a 10% Displacement

Parameter	*Class* $2B-lpl$
h_1	0.9639 mm
h_2	-
h_3	1.1746 mm
k_1	23.2016 Nm
k_2	-
k_3	38.1649 Nm

Table 3: Maximum Flexible Segments Parameter Values for a 20% Displacement

Parameter	*Class* $2B-lpl$
h_1	0.6727 mm
h_2	-
h_3	0.8270 mm
k_1	7.8875 Nm
k_2	-
k_3	13.3198 Nm

Table 4: Maximum Flexible Segments Parameter Values for a 30% Displacement

Parameter	*Class* $2B-lpl$
h_1	0.5413 mm
h_2	-
h_3	0.6729 mm
k_1	4.1095 Nm
k_2	-
k_3	7.1769 Nm

As indicated in Table 5, using the maximum flexible segments parameter values for the different percentages of mechanism slider displacement, this particular class of mechanism generated a force of 901.23N, 316.56N, 171.17N, and 110.44N for a 10, 20, 30, and 40% mechanism slider displacement respectively. Table 5 also indicates an average value of non-dimesionalized mechanism parameter Φ of 1.2573, 1.2991, 1.3483, and 1.4081 for the different percentages of mechanism slider displacement. Values of Φ were determined for several displacement points, but only the average value was indicated in Table 5. With the average values of Φ for the various percentage slider displacement of the *Class* $2B-lpl$ CCFCSM as indicated in Table 5, the average forces were computed easily using equation (23) which greatly simplifies the models applicability for predicting the average amount of force generated by this class of mechanism for the various percentages of slider displacements indicated in Table 5. This will further extend the use of the *Class* $2B-lpl$ CCFCSM for engineering applications.

Table 5: CCFCSM's Extended Length, Fully Compressed Length, Nominal Constant-Force, and Average Non-Dimensionalized Parameter Value

Parameter	Mechanism *Class* $2B-lpl$ $F_{Nom}=\frac{2k_1}{r_2}\Phi$			
	10%	20%	30%	40%
x_b max	160.00 mm	160.00 mm	160.00 mm	160.00 mm
x_b min	146.40 mm	132.80 mm	119.20 mm	105.60 mm
F_{Nom}	901.23 N	316.56 N	171.17 N	110.44 N
Φ	1.2573	1.2991	1.3483	1.4081

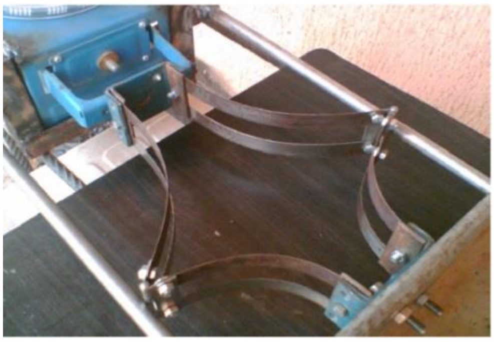

Plate 1: *Class* $2B-lpl$ test mechanism

4 CONCLUSION

The traditional compliant constant-force compression slider mechanism (CCFCSM) configurations have at least one rigid link, and at most one long fixed-pinned flexible beam. The CCFCSM *Class* $2B-lpl$ configuration considered in this work incorporates two long fixed-pinned flexible segments. The use of long flexible segments is important in gaining stiffness without increasing stress. This new CCFCSM configuration is relatively uncomplicated, with capacity to maintain a maximum constant-force within allowable stress limits for up to 40% of its slider displacement when compared to the other classes of CCFCSM and can be adapted to various practical applications in which a constant reaction force is desired in response to a linear displacement. This research work focuses on the development of non-dimensionalized mechanism parameter Φ for this class of CCFCSM mechanism. It also expresses the desire to simplify the behavioral model equation for easy usage. Results obtained indicated an average value of non-dimesionalized mechanism parameter of 1.2573, 1.2991, 1.3483, and 1.4081 for 10, 20, 30, and 40% mechanism slider displacement respectively. The result also indicated that using the maximum flexible segments parameter values for the different percentages of mechanism slider displacement as contained in this work, shows that the average force generated by the mechanism for a 10, 20, 30, and 40% displacement were 901.23N, 316.56N, 171.17N, and 110.44N respectively. This shows clearly that using the non-dimensionalized mechanism parameter, the average force generated by this class of mechanism can easily be determined which greatly simplifies the models applicability and further extends the use of the $Class\ 2B-lpl$ CCFCSM for engineering applications.

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