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Method of lines analysis of Dufour and Soret effects on unsteady MHD free convective heat and mass transport flow past a porous infinite plate

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Abstract

The problem of coupled heat and mass transfer by free convection of a chemically reacting viscous incompressible and electrically conducting fluid past an infinite vertical porous plate under the influence of uniform transverse magnate field in the presence of diffusion-thermo (Dufour), thermal-diffusion (Soret) and internal heat source or sink is studied. A magnetic field of uniform strength is applied perpendicular to the plate and the fluid is subjected to a normal suction velocity while the heat flux at the plate is constant. The dimensionless governing equations were solved numerically in MATLAB using the Method of Lines (MOL). Some of the results indicating the influence of various thermos-physical parameters of fluid flow, heat and mass transfer characteristics were obtained and presented graphically. The numerical values of skin-friction coefficient and Nusselt number at the plate are derived, discussed numerically for various values of physical parameters and presented through tables. The presence of a generating heat source has effects on the fluid velocity, as well as on temperature.

Keywords: convective flow; heat and mass transfer; heat source; mhd; porous medium; unsteady flows; viscous dissipation.

1 Introduction

Convective heat transfer in fluid occurs in many industrial applications and is an important aspect in the study of heat transfer. If stratification occurs, the fluid temperature is function of distance and convection in such environment exists in lakes, oceans, nuclear reactors where coolant (generally liquid

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metals) is present in magnetic field etc. Cheesewright [1] examined the natural convection along an isothermal vertical surface in non-isothermal surroundings. Chen and Eichhorn [2] studied natural convection along an isothermal vertical plate in thermally analyzed the effect of magnetic field on natural convection in liquid metal (NaK) used as coolant in nuclear reactor. Venkatachala and Nath [17] obtained the non-similarity solution for natural convection in thermally stratified fluid. Uotani [14] experimentally studied the natural convection in thermally stratification for liquid metal (PbBi). Kulkarni et al. [5] investigated the problem of natural convection from an isothermal flat plate suspended in a linearly stratified fluid medium. Ostrach [7] presented the similarity solution of natural convection along vertical isothermal plate.

The study of magnetohydrodynamic (MHD) flows plays an important role in agriculture, engineering and petroleum industries. The problem of free convention under the influence of a magnetic field has attracted the interest of many researchers in view of its applications in geophysics and astrophysics. Ugwu *et al*. [12] investigated the problem of MHD flow of continuous Dusty particle in a Non-newtonian Darcy fluid between parallel plates. Soundalgekar et al. [9] analyzed the problem of free convection effects on Stokes problem for a vertical plate under the action of transversely applied magnetic field. Helmy [4] presented an unsteady two-dimensional laminar free convection flow of an incompressible, electrically conducting (Newtonian or polar) fluid through a porous medium bounded by an infinite vertical plane surface of a constant temperature. Zueco [18] analyzed the hydromagnetic convection past a flat plate. Ugwu *et al* [10] studied the MHD effects on convective flow of dusty viscous fluid. The problem was solved numerically using finite difference method under the influence of magnetic field. The properties of solution and uniqueness of solution was established. The effects of heat and mass transfer on free convective flow of micropolar fluid were studied over an infinite vertical porous plate in the presence of an inclined magnetic field with a constant suction velocity and taking Hall current into account have been discussed by Veera Krishna *et al.* [15] Veera Krishna *et al.* [16] discussed the heat and mass transfer on unsteady, MHD oscillatory flow of second-grade fluid through a porous medium between two vertical plates under the influence of fluctuating heat source/sink, and chemical reaction. Ugwu *et al* [11] studied the effects of MHD flow on convective fluids incorporating viscous dissipation energy though it was a Newtonian fluid. This problem was analysed numerically using method of lines and various fluid parameter and that of the particles were obtained.

In recent times, many researchers have given considerable interest to the study of MHD in the presence of radiation. Mishra *et al*. [6] studied the effect of radiation and non-uniform heat source on the unstable, MHD viscous fluid through a heated plate with time-dependent suction and viscous dissipation. Chiranjeevi *et al*. [3] investigated the problem of the MHD boundary layer flow analysis in the presence of thermal absorption with heat generation and chemical reaction over a plate. Sadiq and Nagarathna [8] studied the heat and mass transport on MHD Free convective flow through a porous medium past and infinite plate with time dependent permeability neglecting viscous dissipation. This was done analytically.

Ugwu *et al.* [13] investigated the unsteady free convective flow of a viscous incompressible electrically conducting fluid over an infinite vertical porous plate under the influence of uniform transverse magnate field with time dependent permeability and oscillatory suction embedded with viscous energy dissipation using numerical approach (MOL).

In this paper, we are considering the MOL analysis of Dufour and Soret effects of unsteady MHD free convective fluid of heat and mass transport flow past an infinite porous late under the influence of transverse magnate field with time dependent permeability and oscillatory suction embedded with viscous energy dissipation and heat source.

2 Formulation of the problem

We considered the unsteady MHD free convection flow of an incompressible viscous electrically conducting fluid with simultaneous heat and mass transfer over an infinite vertical plate through porous medium with time dependent permeability and oscillatory suction. The *y*-axis is taken along the plate and *x*-axis perpendicular to it and *u* is the velocity along the *x*-direction.

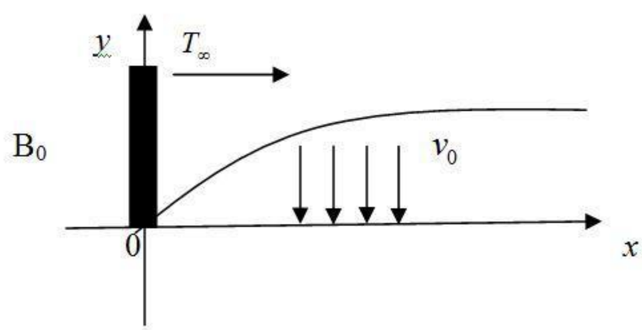

Figure 1: Physical configuration of the problem

The basic assumptions are:

1. All fluid proportions are constant.
2. The plate and the fluid are to be at the same temperature and the species concentration is raised or lowered.
3. The magnetic Reynolds number is so small that the induced magnetic field are neglected in comparison to the applied magnetic field.
4. The permeability of the porous medium is

$$k(t) = k\left(1+\varepsilon e^{iwt}\right) \tag{1}$$

5. The suction velocity is

$$v(t) = -v_0\left(1+\varepsilon e^{iwt}\right) \tag{2}$$

where, v_0 represents the suction / injection velocity at the plate.

6. If the plate is extended to infinite length, then all the physical variables are functions of *y* and *t* alone.

The governing equations are given by

$$\frac{\partial u}{\partial y}=0 \tag{3}$$

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)+g\beta^*(C-C_\infty)-\frac{\sigma B_0^2}{\rho}u-\frac{v}{k(t)}u \tag{4}$$

$$\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-S_1(T-T_\infty)+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{D_m K_T}{c_s c_p}\frac{\partial^2 c}{\partial y^2} \tag{5}$$

$$\frac{\partial c}{\partial t}+v\frac{\partial c}{\partial y}=D\frac{\partial^2 c}{\partial y^2}-k_1(C-C_\infty)+\frac{D_m K_T}{T_m}\frac{\partial^2 T}{\partial y^2} \tag{6}$$

Subject to

$$u(y,t)=T(y,t)=C(y,t)=f(t)=1+\varepsilon e^{iwt} \quad @\, y=0 \tag{7}$$

$$u(y,t)=T(y,t)=C(y,t)=0 \quad @\, y\to\infty$$

where

u - component of the velocity, t – time, c_p - the specific heat at constant pressure, C – dimensional concentration, D_m - mass diffusivity, g – acceleration due to gravity, k_1 - chemical reaction parameter, α - fluid thermal diffusivity, β - thermal expansion coefficient, β^* - concentration expansion coefficient, μ - coefficient of viscosity, ρ - fluid density, K_T - thermal diffusion ratio, T_m - mean fluid temperature, T_∞ - free stream dimensional temperature, C_∞ - free stream dimensional concentration, σ - electrical conductivity of the fluid, B_0 - the external imposed magnetic field strength, c_s - concentration susceptivity, v - viscosity, T_w - wall dimensional temperature, C_w - wall dimensional concentration.

From equation (3), it is obvious that the continuity equation is a constant. Also ε is a small value less than unity i.e., $\varepsilon \ll 1$ and v_0 is a non-zero positive constant, where the negative sign indicates that the suction is towards the plate.

Using the following dimensionless variables, we dimensionalise (3) – (7), taking the Boussinesq's approximation into account in equation (7),

$$u^*=\frac{u}{v_0};\quad y^*=\frac{v_0 y}{v};\quad t^*=\frac{{v_0}^2 t}{v};\quad v^*=\frac{v}{v_0};\quad w^*=\frac{vw}{{v_0}^2};\quad \theta=\frac{T-T_\infty}{T_w-T_\infty};\quad \phi=\frac{C-C_\infty}{C_w-C_\infty}; \tag{8}$$

Thus, we obtain,

$$\frac{\partial u}{\partial y}=0 \tag{9}$$

$$\frac{\partial u}{\partial t}-v_0\left(1+\varepsilon e^{iwt}\right)\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}+Gr\theta+Gm\phi-\left(M^2+\frac{1}{K}\right)u \tag{10}$$

$$\frac{\partial \theta}{\partial t}-v_0\left(1+\varepsilon e^{iwt}\right)\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2}-S\theta+Ec\left(\frac{\partial u}{\partial y}\right)^2+D_u\frac{\partial^2 \phi}{\partial y^2} \tag{11}$$

$$\frac{\partial \phi}{\partial t}-v_0\left(1+\varepsilon e^{iwt}\right)\frac{\partial \phi}{\partial y}=\frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2}-Kc\phi+S_r\frac{\partial^2 \theta}{\partial y^2} \tag{12}$$

subject to

$$u(y,t)=\theta(y,t)=\phi(y,t)=1+\varepsilon e^{iwt} \quad @\, y=0 \tag{13}$$

$$u(y,t)=\theta(y,t)=\phi(y,t)=0 \quad @\, y\to\infty$$

where,

$M^2=\frac{\sigma{\beta_0}^2 v}{\rho {v_0}^2}$ is the Hartmann number (Magnetic field parameter), $\Pr=\frac{v}{\alpha}$ is the Prantl number, $K=\frac{k}{v^2}$ is the permeability parameter (Porosity or Darcy parameter), $Gm=\frac{\upsilon g\beta^*(T_w-T_\infty)}{{v_0}^3}$ is the Prandtl number, $Sc=\frac{\upsilon}{D}$ is the Schmidt number, $Kc=\frac{k_1\upsilon}{{v_0}^2}$ is the chemical reaction parameter, $S=\frac{S_1\upsilon}{{v_0}^2}$ is the Heat Source parameter, $Gr=\frac{\upsilon g\beta(T_w-T_\infty)}{{v_0}^3}$ is the thermal Grashof number, $Gm=\frac{\upsilon g\beta^*(T_w-T_\infty)}{{v_0}^3}$ is the mass Grashof number and $Ec=\frac{{v_0}^2}{Cp(T_w-T_\infty)}$ is the Eckert number, $D_u=\frac{D_m K_T(C_w-C_\infty)}{vc_s c_p(T_w-T_\infty)}$ is the Dufour number, $S_r=\frac{D_m K_T(T_w-T_\infty)}{vT_m(C_w-C_\infty)}$ is the Soret number.

3 Method of lines

The basic idea of the MOL is to replace the special (boundary value) derivatives in the PDE with algebraic approximations. Once this is done, only the initial value variable, typically time in a physical problem remains. Then, with only one remaining independent variable, we have a system of ODEs

that approximates the original PDE. Any suitable integration algorithm for the initial value ODEs can now be used to compute an approximate numerical solution to the PDE.

Before applying the method of lines to equation (9) – (12) subject to boundary conditions (13), we adopt the approximation below to decouple and linearize equations (9) – (12). In view of the approximation adopted, equations (9) – (12) reduce to;

$$\frac{\partial u}{\partial t}=\nu_0\left(1+\varepsilon e^{iwt}\right)\frac{\partial u}{\partial y}+\frac{\partial^2 u}{\partial y^2}+Gr+Gm-\left(M^2+\frac{1}{K}\right)u \tag{14}$$

$$\frac{\partial \phi}{\partial t}=\nu_0\left(1+\varepsilon e^{iwt}\right)\frac{\partial \phi}{\partial y}+\frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2}-Kc\phi+S_r \tag{15}$$

$$\frac{\partial \theta}{\partial t}=\nu_0\left(1+\varepsilon e^{iwt}\right)\frac{\partial \theta}{\partial y}+\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2}-S\theta+Ec+D_u \tag{16}$$

Applying the method of lines to equation (14), we discretize the partial derivative in space variable y, to result in approximating system of ODEs in variable t, thus we have;

$$\left(\frac{\partial u}{\partial t}\right)_i=\nu_0\left(1+\varepsilon e^{iwt}\right)\frac{u_{i+1}-u_{i-1}}{2h}+\frac{u_{i+1}-2u_i+u_{i-1}}{h^2}+Gr+Gm-\left(M^2+\frac{1}{k}\right)u_i \tag{17}$$

Simplifying the right-hand side of equation (17) gives:

$$\left(\frac{\partial u}{\partial t}\right)_i=\alpha_1 u_{i-1}+\alpha_2 u_i+\alpha_3 u_{i+1}+\alpha_4 \tag{18}$$

where,

$$\alpha_1=\left(\frac{1}{h^2}-\frac{\nu_0\left(1+\varepsilon e^{iwt}\right)}{2h}\right);\ \alpha_2=-\left(\frac{2}{h^2}+M^2+\frac{1}{K}\right);\ \alpha_3=\left(\frac{1}{h^2}+\frac{\nu_0\left(1+\varepsilon e^{iwt}\right)}{2h}\right);\ \alpha_4=Gr+Gm,$$

$$i=1,2,3,\ldots,N \tag{19}$$

Equations (18) – (19) can be solved iteratively using the boundary conditions $u(0,t)=1+\varepsilon e^{nt}$ and $u(\alpha,t)=0$ in equations (13).

For $i=1,2,3,...,N$; $u(0,t)=u(y,t)=1+\varepsilon e^{iwt}$;& $u(\infty,t)\approx u(N+1,t)=0$; in equation (18) can be written as;

$$\begin{aligned}
\dot{u}_1 &= \alpha_1+\alpha_2 u_1+\alpha_3 u_2+\alpha_4\\
\dot{u}_2 &= \alpha_1 u_1+\alpha_2 u_2+\alpha_3 u_3+\alpha_4\\
\dot{u}_3 &= \alpha_1 u_2+\alpha_2 u_3+\alpha_3 u_4+\alpha_4\\
\dot{u}_4 &= \alpha_1 u_3+\alpha_2 u_4+\alpha_3 u_5+\alpha_4\\
\dot{u}_N &= \alpha_1 u_{N-1}+\alpha_2 u_N+\alpha_4
\end{aligned} \tag{20}$$

The system in equation (20), in matrix form is given as

$$\begin{bmatrix} \dot{u}_1\\ \dot{u}_2\\ \vdots\\ \dot{u}_{N-1}\\ u_N \end{bmatrix} = \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 & 0 & 0 & ... & 0 & 0\\ 0 & \alpha_1 & \alpha_2 & \alpha_3 & 0 & ... & 0 & 0\\ 0 & 0 & \alpha_1 & \alpha_2 & \alpha_3 & ... & 0 & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & 0 & 0 & ... & \alpha_1 & \alpha_2 \end{bmatrix} \begin{bmatrix} 1+\varepsilon e^{iwt}\\ u_1\\ u_2\\ \vdots\\ u_{N-1}\\ u_N \end{bmatrix} + \begin{bmatrix} \alpha_4\\ \alpha_4\\ \alpha_4\\ \vdots\\ \alpha_4\\ \alpha_4 \end{bmatrix} \tag{21}$$

where the coefficient of α_1, α_2, α_3 and α_4 are given by (19) and $\dot{u}_i=\left(\frac{\partial u}{\partial t}\right)_i$

In a similar way, equation (15) becomes;

$$\left(\frac{\partial\phi}{\partial t}\right)_i=\nu_0\left(1+\varepsilon e^{iwt}\right)\frac{\phi_{i+1}-\phi_{i-1}}{2h}+\frac{1}{Sc}\left(\frac{\phi_{i+1}-2\phi_i+\phi_{i-1}}{h^2}\right)-Kc\phi_i+S_r \tag{22}$$

Simplifying the right-hand side, we obtain;

$$\left(\frac{\partial\phi}{\partial t}\right)_i=\beta_1\phi_{i-1}+\beta_2\phi_i+\beta_3\phi_{i+1}+\beta_4 \tag{23}$$

where,

$$\beta_1 = \left(\frac{1}{h^2 Sc} - \frac{v_0\left(1+\varepsilon e^{iwt}\right)}{2h} \right);\ \beta_2 = -\left(\frac{2}{h^2 Sc} + Kc \right);\ \beta_3 = \left(\frac{1}{h^2 Sc} + \frac{v_0\left(1+\varepsilon e^{iwt}\right)}{2h} \right);\ \beta_4 = S_r; \quad (24)$$

Equations (23) – (24) can be solved iteratively using the boundary conditions $\phi(0,t) = 1+\varepsilon e^{iwt}$ and $\phi(\infty,t) = 0$ in equations (13).

For $i = 1,2,3,\dots,N;\ \phi(0,t) = 1+\varepsilon e^{iwt};\ \&\ \phi(\infty,t) \approx \phi(N+1,t) = 0;$ in equation (23) can be written in matrix form as;

$$\begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \\ \vdots \\ \dot{\phi}_{N-1} \\ \dot{\phi}_N \end{bmatrix} = \begin{bmatrix} \beta_1 & \beta_2 & \beta_3 & 0 & 0 & \dots & 0 & 0 \\ 0 & \beta_1 & \beta_2 & \beta_3 & 0 & \dots & 0 & 0 \\ 0 & 0 & \beta_1 & \beta_2 & \beta_3 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & \beta_1 & \beta_2 \end{bmatrix} \begin{bmatrix} 1+\varepsilon e^{iwt} \\ \phi_1 \\ \phi_2 \\ \vdots \\ \phi_{N-1} \\ \phi_N \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \quad (25)$$

where the coefficient β_1, β_2, and β_3 are given in equation (24) and $\dot{\phi}_i = \left(\frac{\partial \phi}{\partial t} \right)_i$.

Similarly, equation (16) becomes;

$$\left(\frac{\partial \theta}{\partial t} \right)_i = \left(\frac{1}{h^2 \Pr} - \frac{v_0\left(1+\varepsilon e^{iwt}\right)}{2h} \right)\theta_{i-1} - \left(\frac{2}{h^2 \Pr} + S \right)\theta_i + \left(\frac{1}{h^2 \Pr} + \frac{v_0\left(1+\varepsilon e^{iwt}\right)}{2h} \right)\theta_{i+1} + Ec + D_u \quad (26)$$

$$\left(\frac{\partial \theta}{\partial t} \right)_i = \gamma_1\theta_{i-1} + \gamma_2\theta_i + \gamma_3\theta_{i+1} + \gamma_4 \quad (27)$$

where

$$\gamma_1 = \left(\frac{1}{h^2 \Pr} - \frac{v_0\left(1+\varepsilon e^{iwt}\right)}{2h} \right);\ \gamma_2 = -\left(\frac{2}{h^2 \Pr} + S \right);\ \gamma_3 = \left(\frac{1}{h^2 \Pr} + \frac{v_0\left(1+\varepsilon e^{iwt}\right)}{2h} \right);\ \gamma_4 = Ec + D_u \quad (28)$$

Equations (26) – (28) can be solved iteratively using the boundary conditions $\theta(0,t) = 1+\varepsilon e^{iwt}$ and $\theta(\infty,t) = 0$ in equations (13).

For $i = 1,2,3,\dots,N;\ \theta(0,t) = 1+\varepsilon e^{iwt};\ \&\ \theta(\infty,t) \approx \theta(N+1,t) = 0;$ in equation (27) can be written in matrix form as;

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \vdots \\ \dot{\theta}_{N-1} \\ \dot{\theta}_N \end{bmatrix} = \begin{bmatrix} \gamma_1 & \gamma_2 & \gamma_3 & 0 & 0 & \dots & 0 & 0 \\ 0 & \gamma_1 & \gamma_2 & \gamma_3 & 0 & \dots & 0 & 0 \\ 0 & 0 & \gamma_1 & \gamma_2 & \gamma_3 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & \gamma_1 & \gamma_2 \end{bmatrix} \begin{bmatrix} 1+\varepsilon e^{iwt} \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_{N-1} \\ \theta_N \end{bmatrix} + \begin{bmatrix} \gamma_4 \\ \gamma_4 \\ \gamma_4 \\ \vdots \\ \gamma_4 \\ \gamma_4 \end{bmatrix} \tag{29}$$

where the coefficient $\gamma_1, \gamma_2, \gamma_3$ and γ_4 are given by the equation (28) and $\dot{\theta}_i = \left(\frac{\partial \theta}{\partial t}\right)_i$.

The three physical quantities at the plate are given as,

Skin friction coefficient: $$C_f = \frac{\tau' \omega}{\rho u_0 v_0} = \left(\frac{\partial u}{\partial y}\right)\Bigg|_{y=0} \tag{30}$$

Nusselt Number: $$Nu = -\frac{x\left(\frac{\partial T}{\partial y}\right)}{\left(T_w - T_\infty\right)}\Bigg|_{y=0} \tag{31}$$

Sherwood Number: $$Sh = -\frac{x\left(\frac{\partial C}{\partial y}\right)}{\left(C_w - C_\infty\right)}\Bigg|_{y=0} \tag{32}$$

4 Results and discussion

We have considered the effects of some thermo-physical properties of unsteady free convective flow of a viscous incompressible electrically conducting fluid over an infinite vertical porous plate under the influence of uniform transverse magnate field with time dependent permeability and oscillatory suction in the presence Soret and Dufour parameter numerically using MATLAB simulations. Some graphical and numerical representation are thus given below;

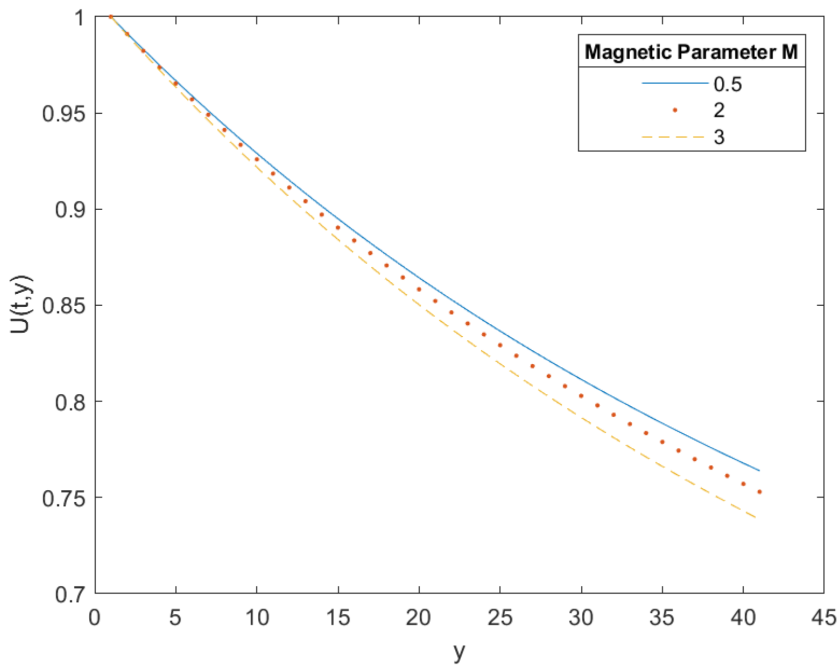

Figure 2: Effects of Magnetic Parameter on the velocity profile.

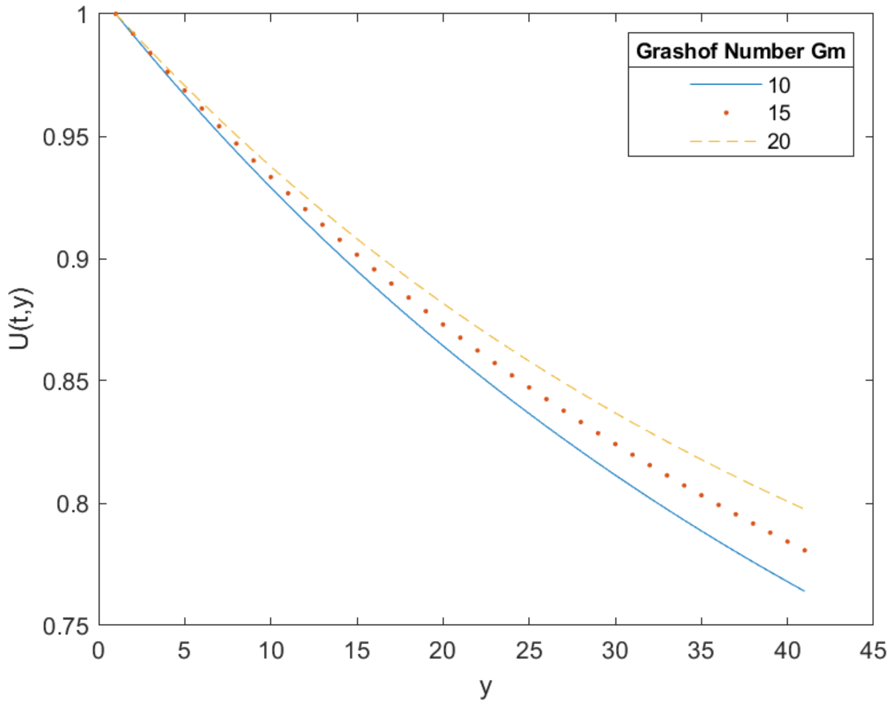

Figure 3: Effects of Mass Grashof Number on the velocity profile

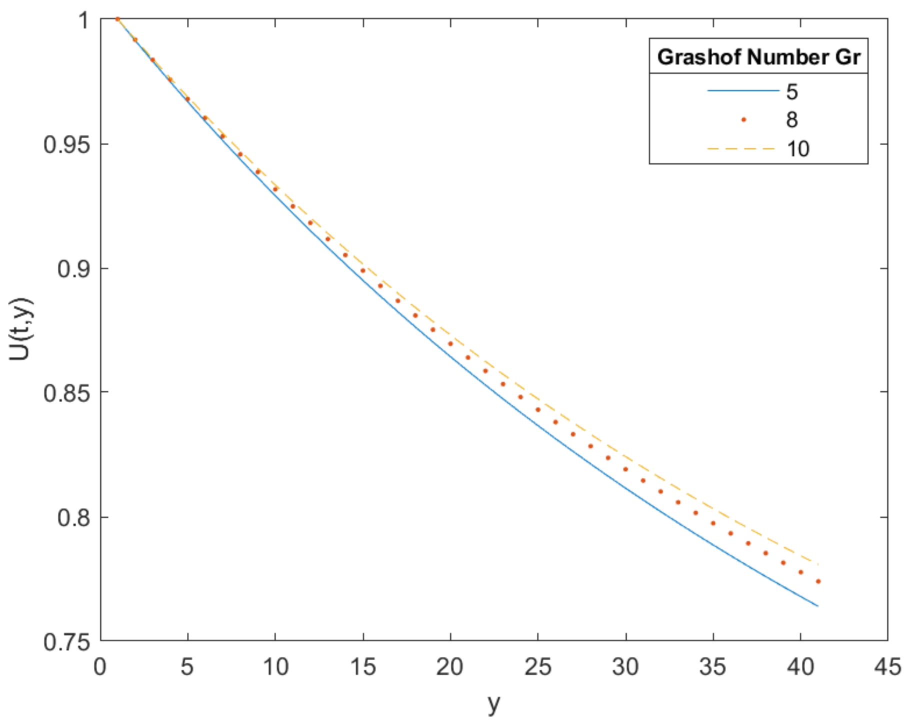

Figure 4: Effects of Thermal Grashof Number on the velocity profile

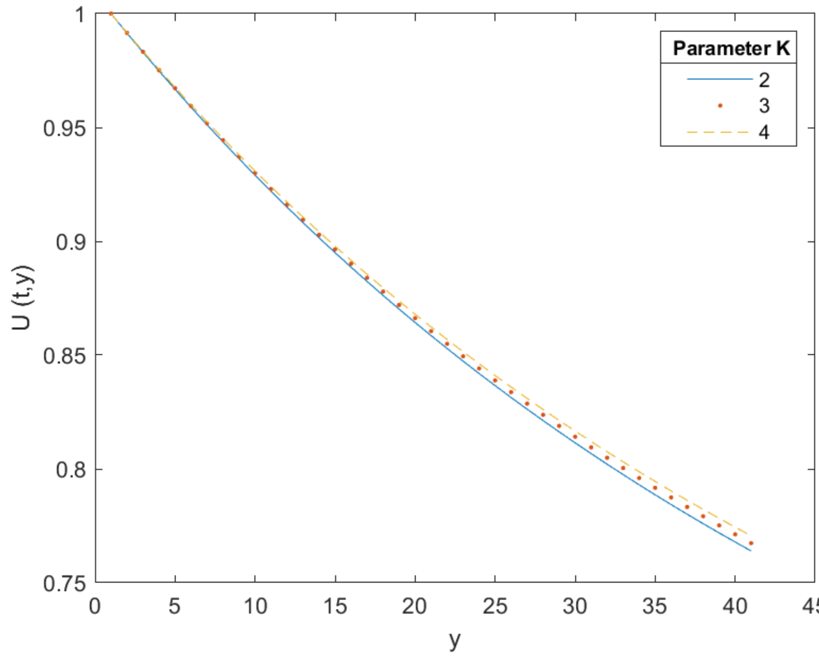

Figure 5: Effects of Permeability (Porosity) on the velocity profile

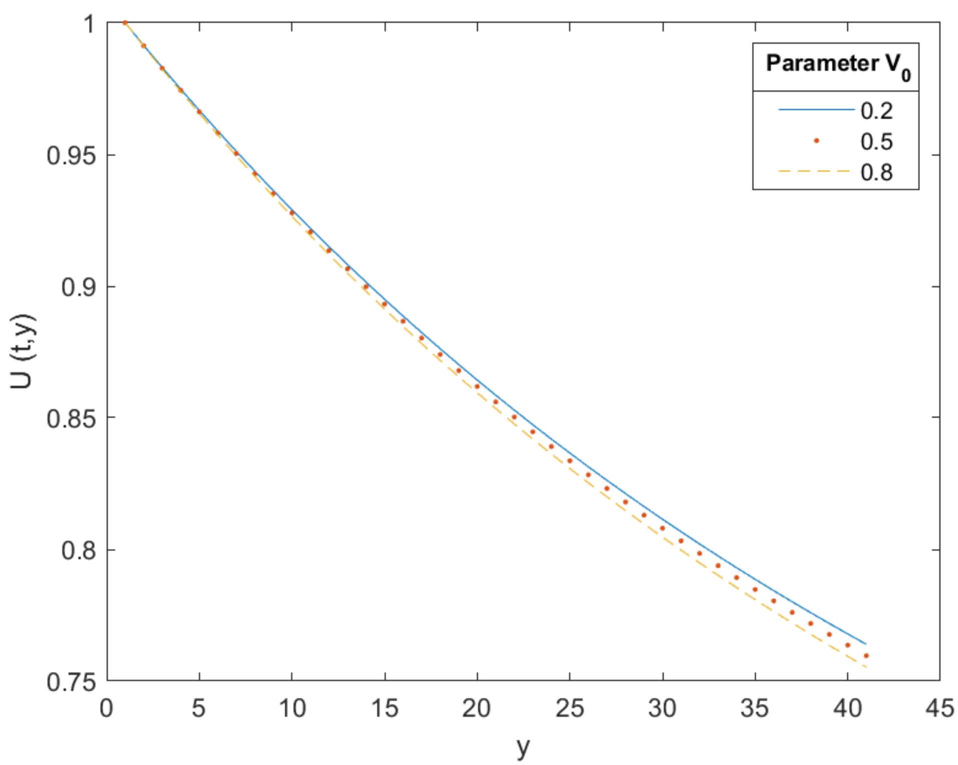

Figure 6: Effects of Parameter V_0 on the velocity profile

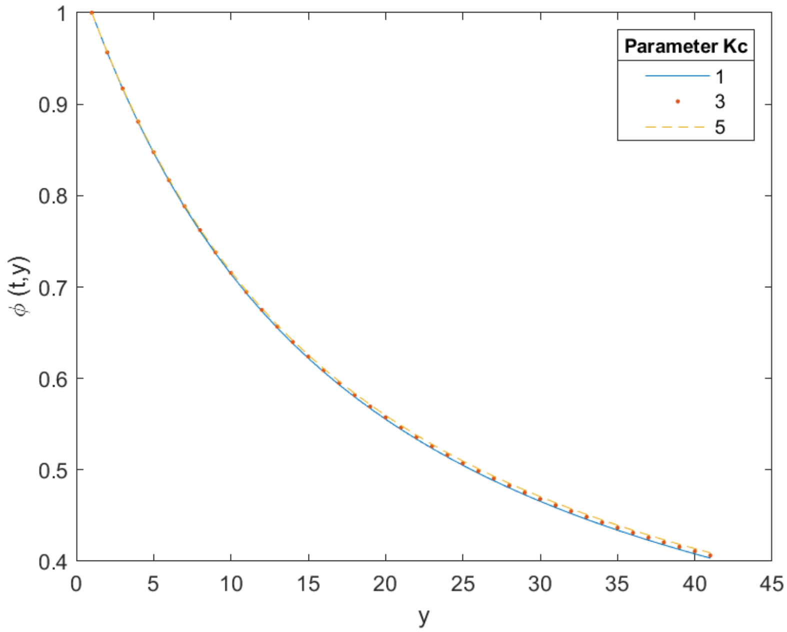

Figure 7: Effects of Parameter Kc on the Concentration profile

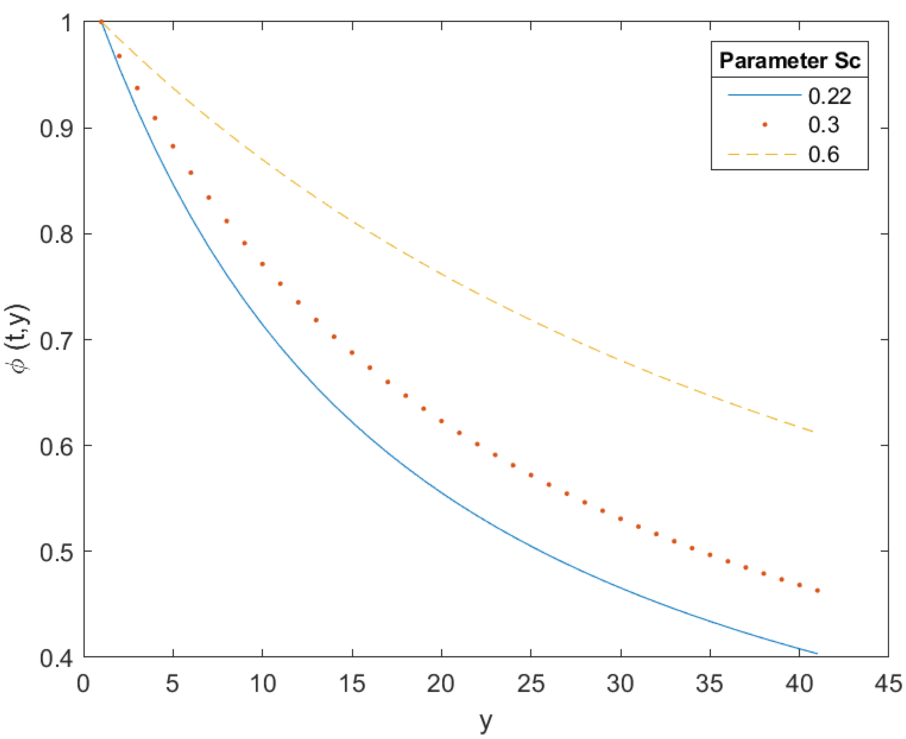

Figure 8: Effects of Schmidt Number, Sc on the Concentration profile

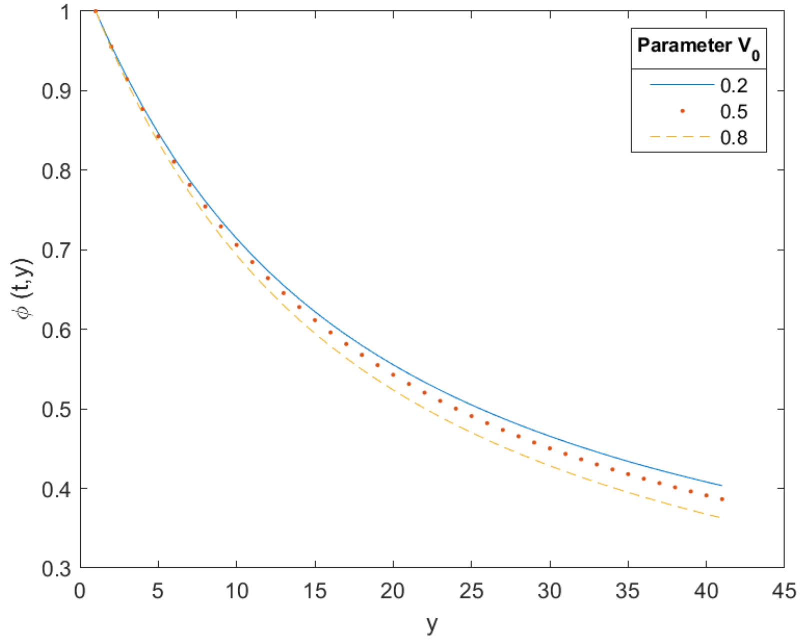

Figure 9: Effects of Parameter V_0 on the Concentration profile

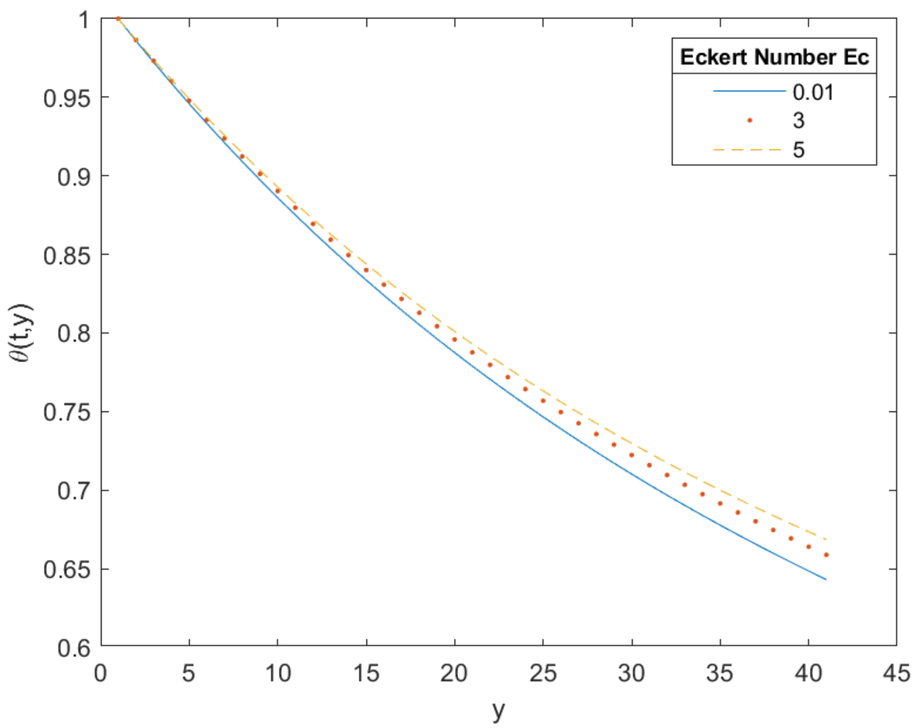

Figure 10: Effects of Eckert Number on the Temperature profile

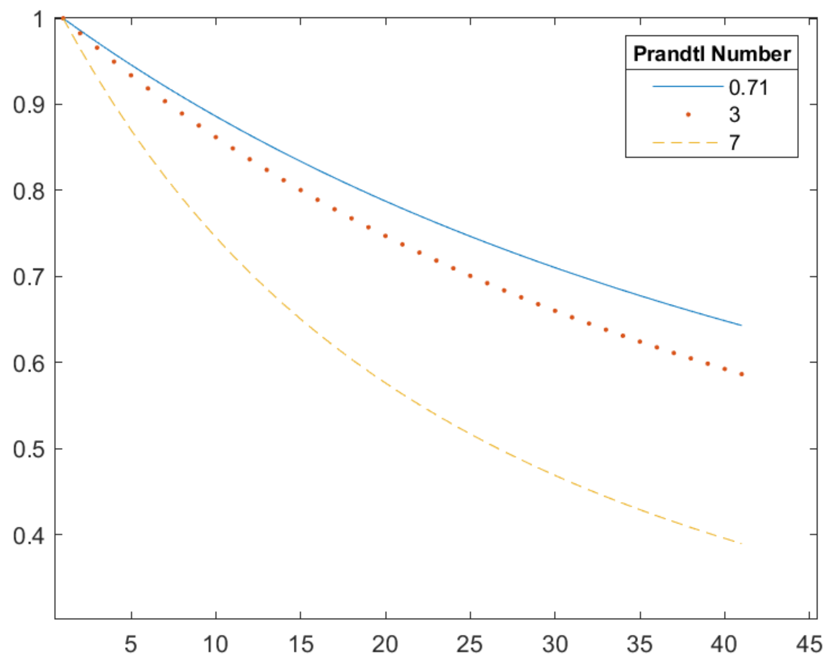

Figure 11: Effects of Prantl number on the Temperature profile

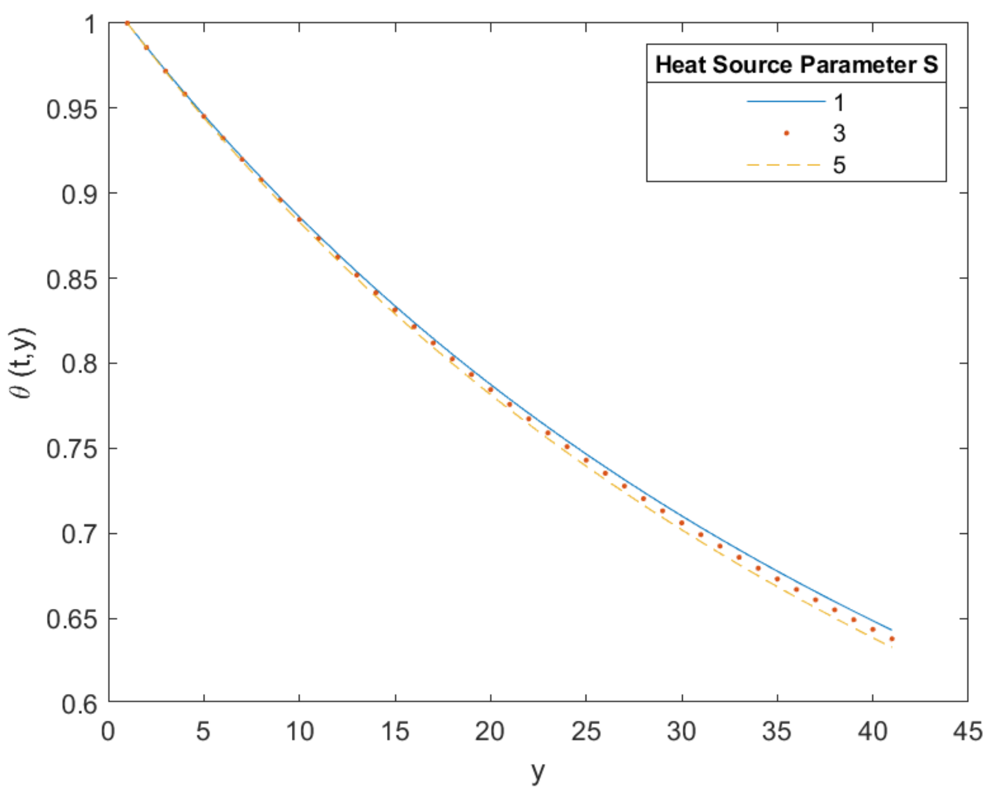

Figure 12: Effects of Heat Source on the Temperature profile

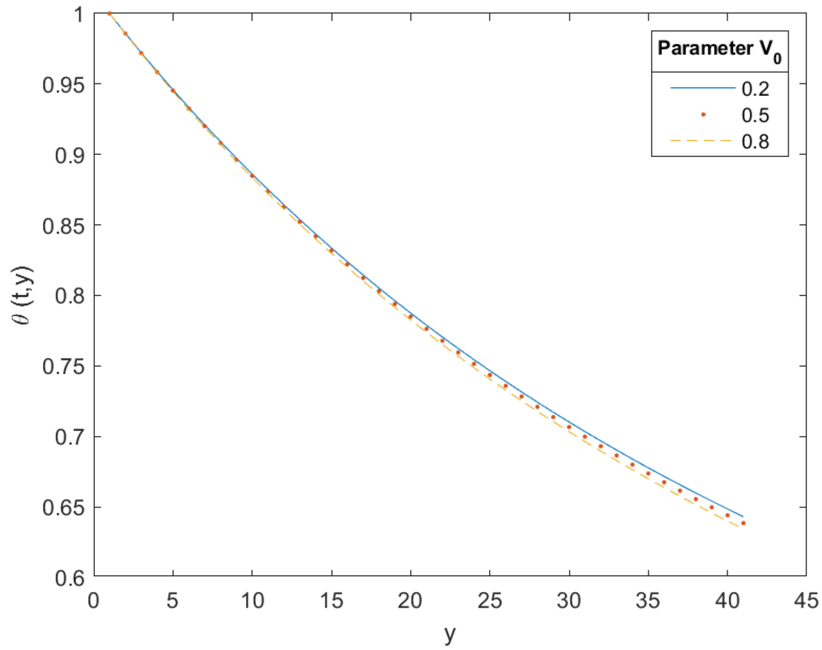

Figure 13: Effects of Parameter V_0 on the Temperature profile

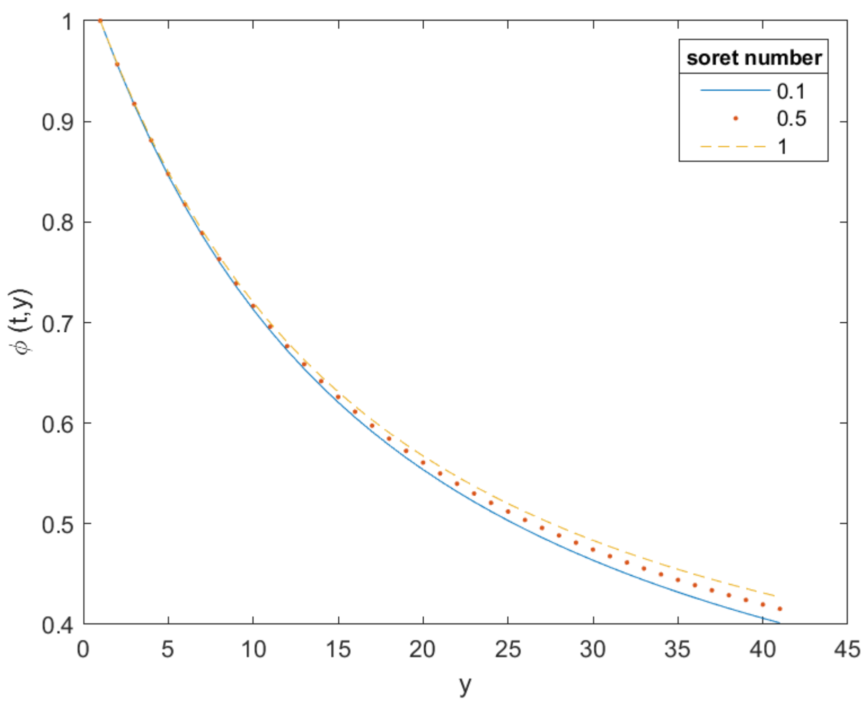

Figure 14: Effects of Soret number on the concentration profile

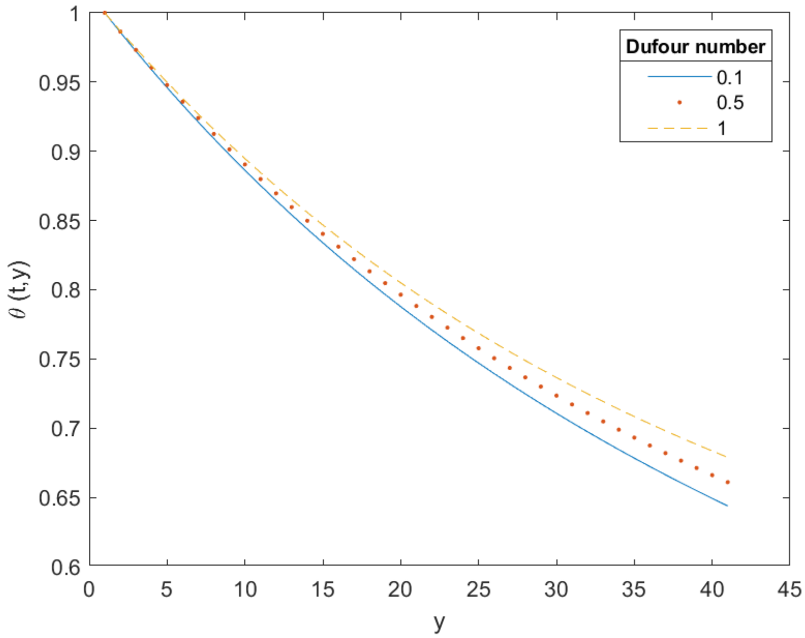

Figure 15: Effects of Dufour number on the temperature profile

Table 1. Computational values for local skin friction $u'(0)$ for different values of M, K, Gm, and Gr where $t := 0.2 : h := 0.1 : \nu[0] := 0.2 : \varepsilon := 0.001 : S := 1 : Sc := 0.22 : Kc := 1 : Pr := 0.71 : Ec := 0.001 : \omega := \pi/6 : Du := 0.1 : Sr := 0.1 :$

M	K	Gm	Gr	Skin friction (stress)
0.5	2	10	5	12.9134
2				7.6352
3				3.8917
	3			12.1057
	4			10.4123
		15		15.3607
		20		17.5112
			8	13.7213
			10	14.3443

Table 2. Computational values for local Sherwood number (Sh) for different values of Sc, Kc, V_0, w, and Sr where $t := 0.2 : h := 0.1 : \varepsilon := 0.001 : S := 1 : M := 0.5 : K := 2 : Gr := 5 : Gm := 10 : Pr := 0.71 : Ec := 0.001 : \omega := \pi/6 : Du := 0.1 :$

Sc	Kc	V_0	w	Sr	Sherwood Number (Sh)
0.22	1	0.2	$\pi/6$	0.1	0.5239
0.3					0.6683
0.6					0.9675
	3				0.7551
	5				0.9354
		0.5			0.6816
		0.8			0.8147
			$\pi/4$		0.5254
			$\pi/3$		0.5279
				0.5	0.3917
				1	0.1782

Table 3. Computational values for Nusselt number (Nu) for different values of *Pr, S, Ec, Du,* V_0, and where $t := 0.2 : h := 0.1 : \varepsilon := 0.001 : n := 0.01 : M := 0.5 : K := 2 : Gr := 5 : Gm := 10 : Sc := 0.22 : Kc := 1 : Sr := 0.1 :$

Pr	S	Ec	V_0	Du	Nusselt Number (Sh)
0.71	1	0.01	0.2	0.1	1.1209
3					2.9979
7					4.0122
	2				1.7231
	3				2.0464
		0.5			1.5926
		3			1.9917
			0.5		1.4461
			0.8		1.6022
				0.5	0.9341
				1	0.7533

Table 4. Comparison of Results when Sc=0, Sr=0 in present study with Basha *et al.* (2019)

Sc	V_0	w	Sherwood Number (Sh)	
			Basha et al. [19]	Present study
0.22	0.2	$\pi/6$	0.044020	0.051062
0.3			0.060238	0.065387
0.6			0.120325	0.196711
	0.5		0.110179	0.189233
	0.8		0.176133	0.242128
		$\pi/4$	0.044236	0.054688
		$\pi/3$	0.044253	0.056167

Figure (2) shows that the magnitude of the velocity component increases with decreasing the intensity of the magnetic field, M. This is due to the Lorentz force the velocity retards continuously throughout the fluid region. Figure (3) shows that the velocity component decreases with decrease in mass Grashof number. Figure (4) shows that the velocity component also increases with increase in thermal Grashof number. Figure (5) depicts that increasing the permeability of the porous medium increases the fluid speed in the entire region. Figure (6) depicts that the velocity component experiences retardation in the flow field with increasing suction parameter. Figure (7) and (8) shows that the Concentration distribution decreases with increase in chemical reaction or Schmidt number respectively, thus causing a retardation in the fluid flow. Also figure (9) shows that decreasing the suction parameter leads to increase in the concentration distribution. These results in the concentration distribution show that the heavier diffusing species have a greater retarding effect on the concentration distribution of the flow field. Figure (10) show that the temperature of the system decreases with a decrease in Eckert number. Figure (11) shows that the temperature of the flow field reduces as the Prantl number increases. This is consistent with the fact that the thermal boundary layer thickness decreases with increasing Prantl number. Figure (12) depicts that with increasing heat source, the temperature of the flow field reduces. This may be as a result of the elastic property of the fluid. It is observed that in figure (13) increasing the suction parameter reduces the temperature of the system. Figure (14) depicts that as Soret number increases the concentration of the fluid increases as well, also figure (15) shows that a decrease in the Dufour number leads to a considerable decrease in the temperature of the system.

The frictional force is a significant phenomenon which characterizes the frictional drag force at the solid surface. From Table 1, it is observed that the 'τ'increases with the increase in K, Gr, Gm and v_0, but it is interesting to note that the 'τ'decreases with the increase in M. From Table 2, It is observed that as Sc, Kc, w and v_0 increases, 'Sh' also increases at the surface of the plate but decreases with increase in Soret number. From Table 3, we see that all the entries are positive. It is seen that increase in S, Pr, Ec, and v_0 increases Nu at the surface of the plate. Also, Nu decreases with increase in Du. Table 4 represents the comparison of the results with Basha *et al.* (2019).

5 Conclusion

The study has examined the Method of Lines analysis (MOL) on the problem of an unsteady heat and mass transfer flow of an MHD fluid past an infinite porous plate under the influence of Dufour, Soret and other pertinent flow parameters. The following conclusions are drawn from the study; as soret number (S_r) and dufour number (D_u) increases, the skin friction coefficient (C_f) increases while the Nusselt number (N_u) decreases. As soret number (S_r) decreases, the Sherwood number (S_h) increases drastically while decrease in Dufour number has a negligible or no influence on it
As soret number (S_r) increases, both the velocity $u(y,t)$ profile and the concentration profile $\phi(y,t)$ increase, while there is negligible or no effect on the temperature distributon. As dufour number (D_u) increases, both the velocity profile $u(y,t)$ and temperature profile $\theta(y,t)$ increase. As the thermal Grashof number (G_r) and modified Grashof number (G_m) increases, the velocity profile $u(y,t)$ increases. As Schmidt number (S_c) and Chemical reaction (K_c) increases, the velocity profile $u(y,t)$ and the concentration profile $\phi(y,t)$ decrease. As Eckert number (E_c) increases, both the velocity profile $u(y,t)$ and temperature profile $\theta(y,t)$ increase. An Increase in the magnetic parameter (M) decreases the velocity profile $u(y,t)$ as a result of a resistive Lorentz force produced by the magnetic field. It has been observed that, in designing a system where high temperature is needed, such as glass production, propulsion system, plasma physics etc., the effects of Soret, Dufour, Eckert number, Grashof number, Chemical reaction parameter, Schmidt number and Magnetic parameter should be considered carefully for optimal performance of such system.

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