PREDICTION OF VEHICULAR ACCIDENTS ALONG LOKOJA-ABUJA-KADUNA EXPRESS WAY, NIGERIA, USING GREY-SYSTEM MODEL GM(1,1)

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Abstract
The Lokoja-Abuja-Kaduna express way is one of the strategic roads that connect the northern and southern parts of the country. The express way, is one of the busiest roads in the country, because of high volume of vehicular movements on this road, hundreds of road accidents that mostly result in loss of many lives and properties are recorded every year. The World Health Organization (WHO) reported that road traffic accident is one of the top causes of deaths worldwide. Hence, the need to assist the government with information that could be used to reduce the accident rate on this Express way becomes paramount. A Grey-System Model GM(1,1) for the prediction of vehicular accidents has been developed and implemented on Lokoja-Abuja-Kaduna Express Way, Nigeria. The data used in this research were collected from the archive of Federal Road Safety Corps of Nigeria for a period of ten years (2010-2019). The fitted model showed 85.97% *accuracy, this indicates reliability of the model. Therefore, results from this model could be used for decision making process and policy formulation for road safety management.*

Keywords: Accident, Crashes, Express Way, Grey Model, Lokoja-Abuja-Kaduna, Road,

Introduction

Road traffic crashes prediction is an important component of road management decision-making processes. Information from road prediction models is used for road safety policy management formulation to reduce the loss of lives and properties on the roads. Nigeria has roughly 195,000 km of surfaced road, making it largest road network in West Africa and the second largest in south of the Sahara out of which a proportion of about 32,000 km are federal roads while 31,000km are state roads. Roadways are basically the means of transportation on land (Enwerem and Ali, 2016). This includes the modern highway system, streets feeder roads and village roads, which take care of various vehicles and human movements from one point to the other. Road transport is the commonest means of transportation in Nigeria due to the almost total collapse of the rail system and the nearly non-existence and exploitation of the inland waterways. Road transport is also the only means of transportation available to the masses of the country as the political elites can easily afford the high-cost of air transport that seems exclusive to the elites and political class. So, road transport accounts for over 90 percent of the sub-sector's contribution to the Gross Domestic Product (GDP). Road traffic entirely depends on the human settlements, and in Nigeria where the settlement is agrarian, roads transportation for both humans and goods, especially agricultural produces or products of various kinds. This important infrastructure is almost in bad shape due to largely lack of maintenance culture, corruption in the system, and the inadvertent huge resort to trucks and articulated vehicles for haulage. There are several types of roads worldwide that can be used by motorized traffic (Enwerem and Ali, 2016). These include: Driveway, Arterial Road, Highway Expressway and so on. Nigeria as a country is not

benefitting from the enormous benefits of good roads. The reason being that most roads in Nigeria are poorly constructed, and not maintained, hence, it is not durable. According to The Director-General, Infrastructure Concession Regulatory Commission (ICRC), Mr Chidi Izuwah in 2017, Out 195,000 km of surfaced road in Nigeria, only about 60,000km are paved. Of the paved roads, a large proportion is in very poor unacceptable condition due to insufficient investment and lack of adequate maintenance. Roads are supposed to be constructed in such a way that it will last for between thirty to fifty years. However, in Nigeria, this is not the case. The surface of most Nigerian roads do not even last up to six months or a year, hence, the reason for so many Road Traffic crashes over the years. Road traffic crash occur when a vehicle collides with another vehicle, pedestrian, animal, road debris, or other stationary obstruction, such as a tree or utility pole. Road traffic crash has been found to be one of the most common causes of death worldwide. In 2013, approximately 1.24 people died from road a traffic crash, which is one of the reason it claimed the ninth spot on the list of WHO top ten causes of death (John *et.al*, 2017). Every year 20 to 50 million people sustained injuries many resulting in disabilities worldwide with, more than 90% of death in low and middle income countries (Ismail and Abdelmajeed, 2010). Road traffic crashes have been found to be influenced by many complex factors such as weather, driver factor, speed, rear by vehicular density, risk location, driver consciousness, driver fatigue nature of the rode and so on. Since Road Traffic Crash is generally a random occurrence it is therefore, quite important to select an appropriate forecasting model that will fully capture the behaviour of the system. Grey system stochastic model has been selected for this purpose. Successful application of this model has been reported in (Mao and Sun, 2011), (Xin*et al*., 2018), (Jian-Yi and Ying, 2014). Equally, successful studies of traffic accidents on major road have been reported in the works of (Bamidele *et al.,* 2016; Nyothiri *et al.,* 2018 and Xiaoxia *et al.,* 2018). A GM(1,1) forecasting model has been selected in this research to determine the long time trend of accidents rate on Lokoja-Abuja-Kaduna express way with a view to helping the government with information that will be put to use to reduce the loss of lives and properties. GM(1,1) forecasting model is a viable and powerful mathematical tool because of its ability to use small sample size and make long time forecasting with minimal error (Jian-Yi and Ying, 2014; Wei and Jian-Min, 2013;Yong *et al.,* 2016).

Methodology

The Grey-System Model GM(1,1)

Let $x^{(0)}(k), k = 1,2,3,...,n, x^{(0)}(k) \geq 0$ be the raw data series which can also be represented by:

$$X^{(0)}(k) = \left(x^{(0)}(1), x^{(0)}(2), x^{(0)}(3),..., x^{(0)}(n)\right) \quad (1)$$

(Li *et al.,* 2007)

the accumulated generating sequence is represent as:

$$X^{(1)}(k) = \left(x^{(1)}(1), x^{(1)}(2), x^{(1)}(3),..., x^{(1)}(n)\right) \quad (2)$$

Where $X^{(0)}(k) = \sum_{i=1}^{k} x^{(0)}(i), k = 1,2,...,n \quad (3)$

$$x^{(0)}(k) + ax^{(1)}(k) = b \quad (4)$$

Equation (4) represents the original form of the GM(1,1) model. The symbol GM(1,1) stands for first order Grey Model in one variable.

The Grey Model differential equation can also be represented as:

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b \tag{5}$$

Where a and b are parameters to be identified. a is called developing coefficient and b is grey input.

Equation (6) is the solution of equation (5)

$$\hat{x}^{(1)}(k+1) = \left(x^{(1)}(0) - \frac{b}{a} \right) e^{-ak} + \frac{b}{a} \tag{6}$$

Equation (6) is the time response function while parameters a and b are estimated using Least Square Method as follows:

$$\begin{bmatrix} a \\ b \end{bmatrix} = \left[B^T B \right]^{-1} B^T Y \tag{7}$$

Where, $B = \begin{bmatrix} \frac{-[x^{(1)}(1) + x^{(1)}(2)]}{2} & 1 \\ \frac{-[x^{(1)}(2) + x^{(1)}(3)]}{2} & 1 \\ \frac{-[x^{(1)}(3) + x^{(1)}(4)]}{2} & 1 \\ \frac{-[x^{(1)}(4) + x^{(1)}(5)]}{2} & 1 \\ \frac{-[x^{(1)}(5) + x^{(1)}(6)]}{2} & 1 \\ \vdots & \vdots \\ \frac{-[x^{(1)}(n-1) + x^{(1)}(n]}{2} & 1 \end{bmatrix}$ (8)

$$Y = \left[x^{(0)}(2), x^{(0)}(3), x^{(0)}(4), \ldots, x^{(0)}(n) \right]^T \tag{9}$$

The reduction value of equation (6) is given below:

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) = \left(1 - e^{a}\right) \left(x^{(0)}(1) - \frac{b}{a} \right) e^{-ak} \tag{10}$$

Prediction Accuracy Test

To determine the accuracy of our prediction, we shall adopt mean absolute percentage error (MAPE).

This tool is often used for determining prediction accuracy showing the same characteristics i.e. the smaller the value, the higher the prediction accuracy.

$$MAPE = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{y_i - \hat{y}_i}{y_i} \right| \times 100\% \tag{11}$$

Where;

$\hat{y}_i$ is the Grey Model predicted value.

y_i is the Grey-Model actual value.

n is the number of prediction samples (Xin Z *et al.,* 2018)

Lewis (1982) divided the prediction accuracy of models into four grades and the division of prediction accuracy grades is shown in the table below:

Table 1: Classification of Prediction Accuracy

MAPE	Prediction Accuracy
$<10\%$	High
$10\%-20\%$	Good
$20\%-50\%$	Feasible
$>50\%$	Low

Results

Application of Grey Model for Prediction of Vehicular Accidents along Lokoja-Abuja-Kaduna Expressway Nigeria

The data used in this research were collected from the archive of Federal Road Safety of Nigeria for the period of ten years (2010-2019).

The summary of the data is presented in table 2 below:

Table 2: Summary of Number of Vehicular Crashes for Ten Years

S/N	Year of Crash	Number of Crash Within The Year
1	2010	877
2	2011	806
3	2012	560
4	2013	1170
5	2014	929
6	2015	672
7	2016	654
8	2017	655
9	2018	780
10	2019	701

Using equation (1) and table (2), we obtain equation 12 below

$$X^{(0)}=(877,\ 806,\ 560,\ 1170,\ 929,\ 672,\ 654,\ 655,\ 780,\ 701) \quad (12)$$

From equation (2) we obtain the accumulated generating sequence as given below:

$$X^{(1)}=(877,\ 1683,\ 2243,\ 3413,\ 4342,\ 5014,\ 5668,\ 6323,\ 7103,\ 7804) \quad (13)$$

Equation (14) below is obtained using equation (6) with help of Maple 17 software.

$$\hat{a}=\begin{bmatrix}0.02241\\869.607\end{bmatrix}=\begin{bmatrix}a\\b\end{bmatrix} \quad (14)$$

Where $a=0.02241$, $b=869.607$

Substituting for a and b in equation (6), we obtained equation (20) below:

$$\hat{x}(k+1)=38804.42-37927.42e^{-0.02241k} \quad (15)$$

Evaluating equation (15) for $k=0,1,...,9$ we obtained the following values below:

$$\hat{X}^{(1)}=(877,\ 1718,\ 2539,\ 3343,\ 4129,\ 4897,\ 5649,\ 6383,\ 7102,\ 7805) \quad (16)$$

We compute the simulated value using equation (17) below:

$$\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1) \quad (17)$$

$$\hat{X}^{(0)} = (877,\ 841,\ 821,\ 804,\ 786,\ 768,\ 752,\ 734,\ 719,\ 703) \quad (18)$$

Equation (18) is the simulated values from 2010-2019

Table 3: Comparison of Actual and Grey Simulated Value for Vehicular Accidents along Lokoja-Abuja-Kaduna Express Way, Nigeria from 2010-2019

S/N	YEAR OF CRASH	ACTUAL NUMBER OF CRASH	GREY MODEL SIMULATED VALUES	RESIDUAL ERROR	RELATIVE ERROR (%)
1	2010	877	877	0	0
2	2011	806	841	-35	-4.3
3	2012	560	821	-261	-46.6
4	2013	1170	804	366	31.3
5	2014	929	786	143	15.4
6	2015	672	768	-96	-14.3
7	2016	654	752	-98	-14.1
8	2017	655	734	-79	-12.1
9	2018	780	719	61	7.8
10	2019	701	703	-2	-0.3

Using equation (11), we observed from table (3) that:

$MAPE = 14.62\%$

$ACCURACY = 100\% - 14.62\% = 85.97\%$

The figure 85.97% indicates that the prediction accuracy is good

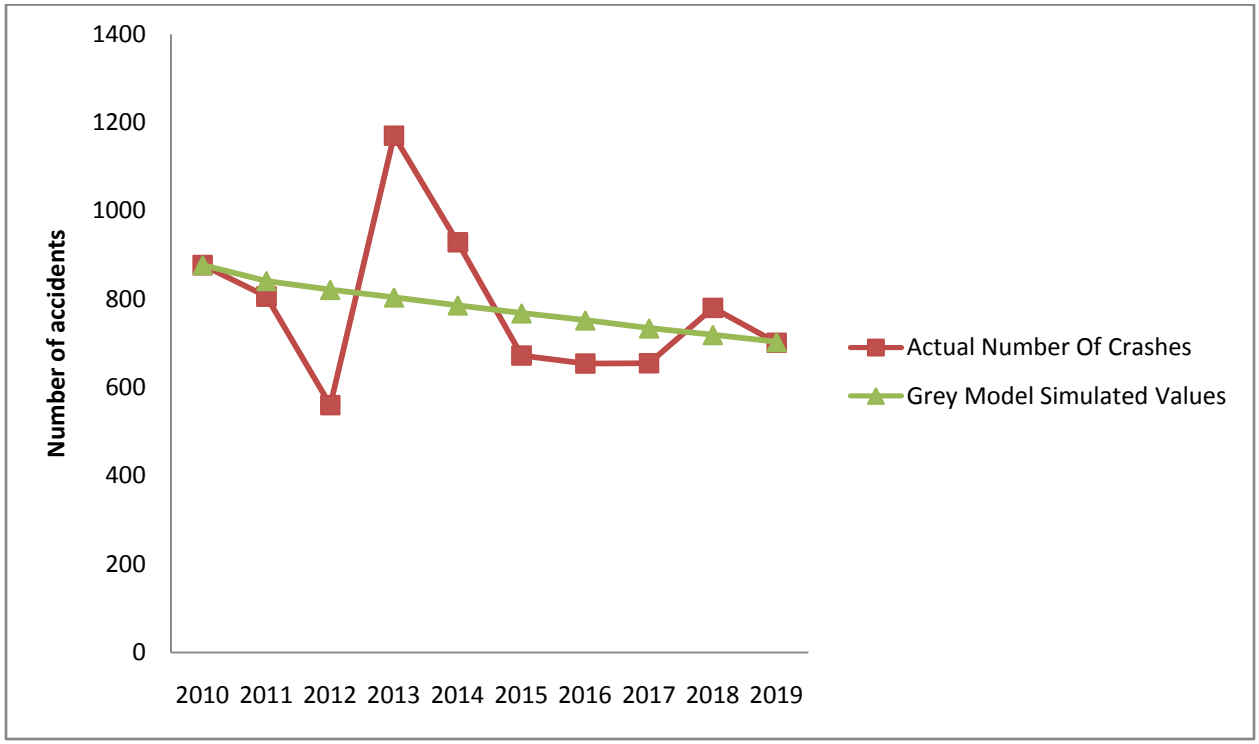

Figure 1: Comparison of Actual and Grey Simulated Value for Vehicular Accidents along Lokoja-Abuja-Kaduna Express Way, Nigeria from 2010-2019

Grey Forecasting for Road Traffic Crash from Year 2020-2023

Using equation (15), that is;

$\hat{x}^{(1)}(k+1) = 38804.42 - 37927.42e^{-0.0224k}$

Evaluating the above equation fro $k = 9,10,11,12,13$

We obtained the following values.

$x^{(1)}(10) = 8491$

$x^{(1)}(11) = 9163$

$x^{(1)}(12) = 9820$

$x^{(1)}(13) = 10462$

$x^{(1)}(14) = 11091$

We compute the predicted values using the equation below:

$\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1)$

Hence, we have;

$x^{(0)}(11) = 672$

$x^{(0)}(12) = 657$

$x^{(0)}(13) = 642$

$x^{(0)}(14) = 629$

The values above are presented in table 4

Table 4: Grey-Model predicted values from year 2020-2023

YEAR OF CRASH	GREY MODEL PRDICTION VALUE
2020	672
2021	657
2022	642
2023	629

Conclusion

A Grey system model has been successfully applied in this research to predict the number of occurrence of vehicular crashes along Lokoja-Abuja-Kaduna Expressway Nigeria. The fitted model showed accuracy of 85.97%.This indicate that the model is good and reliable, therefore, information from this research could be used by government for road safety policy formulation that could reduced the number of loss of lives and properties on the express way.

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