DETERMINATION OF EXTERIOR ORIENTATION PARAMETERS FROM A SINGLE OBLIQUE PHOTOGRAPH :- LEAST SQUARES APPROACH

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ABSTRACT:
This paper presents a simple least squares approach to determination of exterior orientation parameters using a parametric form solution to the conventional collinearity-condition equation for a single oblique photograph. A MATLAB based program has been written to perform the required computation by two methods namely; "iterative model" and "non-iterative tilt photo-generator equation". With a Standard error of 10.77m, 10.77m and 279.31m for theX_0, Y_0 and Z_0 respectively, the "Non-iterative Tilt Photo Generator Equation" Model was chosen as a better fit for the Solution although the accuracy achieved is unacceptable for higher order survey tasks. This therefore confirms that stereo images are better suited for higher order survey tasks.

Keywords: Ground Control Points (GCP), Oblique Photographs, Collinearity Equation, exterior orientation,

1.0 INTRODUCTION

Though one of the fastest methods of spatial data acquisition because of its capacity to capture an infinite number of points in a single exposure of the camera, photogrammetric method of data gathering still suffers set back on the grounds of cost and computational difficulty. Reconstruction of original object scene from photographs require certain steps known as Orientation (Fundamentals of Computational Photogrammetry).

Therefore, the fundamental photogrammetric problem is the determination of the interior and exterior orientation parameters of the camera and the coordinates of object space points measured on photos (M^c^Glone, 1989). The six elements of exterior orientation are: 3D Object Space Co-ordinates of the principal Point and Three Rotation angles.

Exterior orientation could be performed either in two separate steps known as "Relative and Absolute Orientation" or in a combined solution called "Bundle Adjustment". While relative Orientation is the process of bringing corresponding rays to intersect at model points thereby recreating the same parallactic angles as existed between successive exposures, absolute orientation establishes the mathematical relationship between the stereo-model and the ground control co-ordinate system. Odumosu and Ajayi, (2014) examined some of the existing models used for absolute orientation and their suitability for third – Order Planimetric mapping.

Several techniques exist for determining the exterior orientation parameters of which the analytical (empirical method) shall be considered in this paper. The other techniques being the Graphical and Numerical techniques. Analytical photogrammetry focuses on the rigorous mathematical calculation of coordinates of points in object space based upon camera parameters, measured photo coordinates and ground control.

Traditionally, the collinearity, coplanarity and co-angularity conditions are used to determine exterior orientation parameters based on point co-ordinates as input data (Grussenmeyer, 2008). However, in recent times, softwares are available that automate the easy computation of the exterior orientation parameters and just supply users the computed ground co-ordinates of desired points, the initial cost of acquisition of these equipment is very high and uneconomical.

Besides, the time consuming ground survey of control points can be reduced by block adjustment or even more by combined block adjustment with projection center coordinates from relative kinematic GPS-positioning. It is also possible to avoid control points like the measurement of image coordinates of tie points by direct sensor orientation with a combination of GPS and an Inertial Measurement Unit (IMU) (Karsten, 2001).

This paper presents a simple user-friendly least squares technique for solving exterior orientation parameters using MATLAB software.

2.0 MATHEMATICAL MODELS:

The transformation (Projective equation) describing the relationship between two mutually associated three dimensional system of co-ordinates can easily be illustrated by the collinearity equation:

$$x_a = x_o - f\left[\frac{m_{11}(X_A - X_L) + m_{12}(Y_A - Y_L) + m_{13}(Z_A - Z_L)}{m_{31}(X_A - X_L) + m_{32}(Y_A - Y_L) + m_{33}(Z_A - Z_L)}\right] \qquad \text{Equ. 1.0}$$

$$y_a = y_o - f\left[\frac{m_{21}(X_A - X_L) + m_{22}(Y_A - Y_L) + m_{23}(Z_A - Z_L)}{m_{31}(X_A - X_L) + m_{32}(Y_A - Y_L) + m_{33}(Z_A - Z_L)}\right] \qquad \text{Equ. 1.1}$$

For ease of mathematical and programming manipulations, Equations 1.1 and 1.2 above can easily be re-written in Vector Form as:

$$\begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \end{pmatrix} = \boldsymbol{KM} \begin{pmatrix} X_A - X_L \\ Y_A - Y_L \\ Z_A - Z_L \end{pmatrix} \qquad \text{Equ. 2.0}$$

Where x, y, z are co-ordinates of point A in the image space co-ordinate system.

K = Scale Factor

M = a Rotation Matrix

X_L , Y_L, Z_L = Co-ordinates of the Principal Point in the Object Space co-ordinate System.

X_A , Y_A, Z_A = Co-ordinates of Point A in the Object Space co-ordinate System.

The Rotation Matrix "M" can be further written as:

$$\mathbf{R} = \begin{bmatrix} \cos\phi\cos\kappa & -\cos\phi\sin\kappa & \sin\phi \\ \cos\omega\sin\kappa + \sin\omega\sin\phi\cos\kappa & \cos\omega\cos\kappa - \sin\omega\sin\phi\sin\kappa & -\sin\omega\cos\phi \\ \sin\omega\sin\kappa - \cos\omega\sin\phi\cos\kappa & \sin\omega\cos\kappa + \cos\omega\sin\phi\sin\kappa & \cos\omega\cos\phi \end{bmatrix}$$

Where ω, Φ and κ represent rotations or angular shifts in the x, y and z axis respectively.

Also for ease of numerical manipulations, Equation 3.0 above is commonly represented as:

$$M = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \qquad \text{Equ. 3.1}$$

Equation 2 above thus becomes the fundamental equation from where subsequent derivations are performed.

Olaleye, (2010) re-arranged equation 2 above into "Parametric form" and called it the "Vertical Photo-Generator" Equation for Vertical Photographs. The vertical Photo Generator Equation assumes that the aerial photograph is truly or near Vertical, thus the rotation matrix is completely eliminated. The equation in Parametric form thus becomes:

$$(x_a, y_a, -f) = \frac{1}{s}[(X_A, Y_A, Z_A) - (X_L, Y_L, Z_L)] \qquad \text{Equ. 4.0}$$

However, when the photograph is taken such that the optical axis is deviated from the vertical, the reconstruction of original image geometry from photograph will require that the rotations of the camera be efficiently modelled. Therefore, there is need for the rotation matrix to be fully implemented in such solutions.

The "Tilt Photo-Generator" Equation for tilted Photographs thus utilises the Rodriguez approximation for rotation matrix rather than using the full rotation matrix in-order to reduce the mathematical complexity of the resulting equation. Thus, K in equations 2 and 3 above are replaced as:

$$\mathbf{R} \cong \begin{bmatrix} 1 & -k & \phi \\ k & 1 & -\omega \\ -\phi & \omega & 1 \end{bmatrix} \qquad \text{Equ. 5.0}$$

Thus applying the Rodriguez approximation rather than the full rotation matrix Equation

Equation 2 becomes:

$$(x_a, y_a, -f) = \frac{\mathbf{1}}{s}\begin{pmatrix} \mathbf{1} & \kappa & -\varphi \\ -\kappa & \mathbf{1} & \omega \\ \varphi & -\omega & \mathbf{1} \end{pmatrix}\begin{pmatrix} X_A - X_o \\ Y_A - Y_o \\ Z_A - Z_o \end{pmatrix} \qquad \text{Equ. 6.0}$$

$$(x_a, y_a, -f) = \frac{\mathbf{1}}{\boldsymbol{s}}\begin{pmatrix} \mathbf{1} & \kappa & -\varphi \\ -\kappa & \mathbf{1} & \omega \\ \varphi & -\omega & \mathbf{1} \end{pmatrix}\begin{pmatrix} \Delta\boldsymbol{X_A} \\ \Delta\boldsymbol{Y_A} \\ \Delta\boldsymbol{Z_A} \end{pmatrix} \qquad \text{Equ. 6.1}$$

Therefore re-arranging in parametric solution form Olaleye (2010) gives the “Tilt Photo-Generator” Equation as:

$$(x_a, y_a, -f) = \frac{\mathbf{1}}{\boldsymbol{s}}(\Delta\boldsymbol{X_A} + \kappa\Delta\boldsymbol{Y_A} - \varphi\Delta\boldsymbol{Z_A}, -\kappa\Delta\boldsymbol{X_A} + \Delta\boldsymbol{Y_A} + \omega\Delta\boldsymbol{Z_A}, \varphi\Delta\boldsymbol{X_A} - \omega\Delta\boldsymbol{Y_A} + \Delta\boldsymbol{Z_A}) \qquad \text{Equ. 7.0}$$

2.1 LEAST SQUARES APPROACH

Conventionally, the least Squares Observation equation is given as:

$$V = AX + L^b \qquad \text{Equ. 8.0}$$

$$X = (A^T PA)^{-1} A^T L^b \qquad \text{Equ. 9.0}$$

$$\textstyle\sum_{\hat{X}} = \sigma_O^2 (A^T PA)^{-1} \qquad \text{Equ. 10}$$

$$\sigma_O^2 = \frac{V^T PV}{n-m} \qquad \text{Equ. 11}$$

Where;

V = vector of residuals

A = the design or coefficient matrix

X = vector of unknowns

L = the vector of observations

P = weight matrix of observation

σ_O^2 = a-posteriori variance of unit weight

n = the number of observations

m = the number of unknowns

In vector space representation, Equ. 4(a) can be re-written as:

$$\begin{pmatrix} X_A \\ Y_A \\ Z_A \end{pmatrix} = \begin{pmatrix} X_L - sx_a \\ Y_L - sy_a \\ Z_L + sf \end{pmatrix} \qquad \text{Equ. 12.0}$$

Therefore, the required Matrix for the Observation Equation Solution to solve for the Exterior Orientation parameters from the Vertical Photo-Generator Equation is:

$$(Design\ Matrix)A = \begin{pmatrix} 1 & 0 & 0 & x_a \\ 0 & 1 & 0 & y_a \\ 0 & 0 & 1 & -f \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix} \qquad \text{Equ. 12.1}$$

$$(Parameters)X = \begin{bmatrix} X_L \\ Y_L \\ Z_L \\ S \end{bmatrix} \qquad \text{Equ. 12.2}$$

$$(Observation\ Matrix)L = \begin{pmatrix} X_A \\ Y_A \\ Z_A \\ \vdots \end{pmatrix} \qquad \text{Equ. 12.3}$$

The solution for Equations 12.1 – 12.3 gives the Object Space Co-ordinates of the principal Point for a vertical photograph. However, in a tilted photograph as this, the obtained solution serves as initial guess for subsequent iterations which now incorporate the full tilt equation.

The Matrix formulations for the subsequent iterations are as given below:

$$(Observation\ Matrix)L = \begin{pmatrix} x_a \\ y_a \\ -f \\ \vdots \end{pmatrix} \qquad \text{Equ. 12.4}$$

$$(Design\ Matrix)A = \begin{pmatrix} (Y_A - Y_L) & -(Z_A - Z_L) & 0 & (X_A - X_L) \\ -(X_A - X_L) & 0 & (Z_A - Z_L) & (Y_A - Y_L) \\ 0 & (X_A - X_L) & -(Y_A - Y_L) & (Z_A - Z_L) \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix} \qquad \text{Equ. 12.5}$$

$$(Parameters)X = \begin{bmatrix} \kappa.\frac{1}{S} \\ \varphi.\frac{1}{S} \\ \omega.\frac{1}{S} \\ \frac{1}{S} \end{bmatrix} \qquad \text{Equ. 12.6}$$

Equ. 11.4 – 11.6 are used for determination of the rotational Parameters.

The Design Matrix for the Tilt Photograph Model in the Non-iterative Least Squares Solution thus becomes

$$(Design\ Matrix)A = \begin{pmatrix} 1 & 0 & 0 & x_a & -y_a & f & 0 \\ 0 & 1 & 0 & y_a & x_a & 0 & f \\ 0 & 0 & 1 & -f & 0 & -x_a & y_a \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix} \qquad \text{Equ. 12.7}$$

3.0 DATA USED

The Image used for this experiment is an aerial photograph of part of Minna, Niger-State. The Photograph was captured on 20th April 2013, during the surveying camping exercise of the Surveying and Geo-Informatics Students, Federal University of Technology Minna.

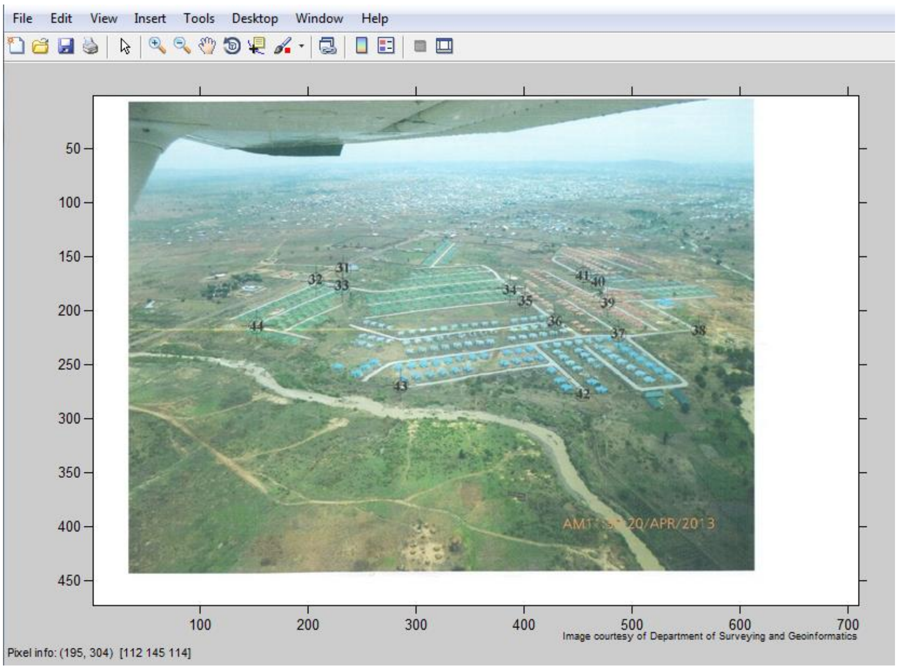

Figure. 1: Aerial Photo of part of Minna, Niger State.

4.0 METHODOLOGY

The collinearity condition earlier described in section 2.0 above was utilised in the determination of the exterior orientation parameters. Two kinds of solutions are herein proposed; the first being an iterative solution that requires an initial guess for the X_L, Y_L, Z_L values (Initial Guess is first computed using the Vertical Photo Generator Equation before subsequent iterations are done till the solution converges) and the second a direct solution implemented via the tilt Photo-Generator Equation.

The captured image was then read into MATLAB environment (Trial Version) where digital co-ordinate values were assigned for all control points. The codes for reading in the image into MATLAB and for subsequent computations are contained in the appendix.

The design matrix was formulated alongside the other relevant vectors and then MATLAB codes written to solve for the Exterior Orientation parameters of the Photograph by applying simple least squares technique to the collinearity equation as earlier described.

The list of Ground controls and their corresponding Digital Photo controls are as shown in Table 1.0.

Table 1: List of control points and their corresponding digital photo co-ordinates.

Ground Co-ordinates		**Digital Photo Co-ordinates**	
226230.128	1059899	231	161
226163.936	1059828	207	172
226219.49	1059766	232	177
226562.352	1059635	388	179
226559.683	1059554	402	189
226559.787	1059433	428	210
226634.523	1059360	487	221
226713.313	1059506	478	192
226779.621	1059636	468	173
226780.687	1059706	455	166
226480.357	1059212	455	276
226263.845	1059297	287	269
226056.051	1059581	153	213

(Source: Authors)

5.0 RESULTS OBTAINED AND DISCUSSION OF RESULTS

A total of five (5) Ground control points have been selected for the computation. The GCP's were chosen to have a good spread across the study area so as to efficiently model the correct ground-photo relationship at every point within the image. The results obtained after running the programs are summarised in Tables 2.0, 3.0 and 4.0. Table 2.0 shows the solution obtained via the iterative program (the result reveals that the solution converged after the third iteration). Table 3.0 presents the Exterior Orientation parameters obtained from the non-iterative solution using "Tilt Photo-Generator" Equation while Table 4.0 contains the summary of the result obtained from the two approaches and the differences between them.

Table 2.0: Exterior Orientation parameters obtained from the iterative solution. (Author's Research)

Parameters	1st Iteration	2nd Iteration	Diff 1	3rd Iteration	Diff 2	4th Iteration	Diff 3
Xo	226031.030	225780.890	-250.140	225780.880	-0.010	225780.880	0.000
Yo	1059375.200	1060600.000	1224.800	1060611.900	11.900	1060611.900	0.000
Zo	214.683	0.000	-214.683	256.000	256.000	256.000	0.000
Scale	1.162	1.162		1.162	0.000	1.162	0.000
kappa	-0.0005876514	-0.0005872794	0.0000003720	-0.0005872743	0.0000000051	-0.0005872743	0.00000
phi	0.0000001278	0.0000001276	-0.0000000002	0.0000001276	0.0000000000	0.0000001276	0.00000
omega	-0.0005410179	-0.0000001553	0.0005408626	-0.0000001553	0.0000000000	-0.0000001553	0.00000

Table 3.0: Exterior Orientation parameters obtained from the non- iterative solution using "Tilt Photo-Generator" Equation.

Parameters	Tilt Model
Xo	225909.310
Yo	1059581.000
Zo	239.675
Scale	1.162
Kappa	-0.526411591
Phi	-0.004551548
Omega	-0.1161281561

Table 4.0: Summary and differences between the Results obtained from the iterative and non-iterative (Using Tilt Model) process (Authors Research).

Parameter	Tilt Model	Iterative Model	Differences
Xo	225909.310	225780.880	128.430
Yo	1059581.000	1060611.900	-1030.900
Zo	239.675	256.000	-16.325
Scale	1.162	1.162	0.000
Kappa	-0.526411591	-0.0005872743	-0.526
Phi	-0.004551548	0.0000001276	-0.005
Omega	-0.1161281561	-0.0000001553	-0.116

Considering the Large variance between the obtained results, the standard error of both models was computed to determine the best fit.

The computation of the standard error of measurements derived reveal a statistically unsatisfactory result for the iterative solution and a fairly acceptable solution for the "Tilt-Photo generator" Model as shown in Table 5.0 and 6.0 respectively.

Table 5.0: Standard Error of Computation for Iterative Solution (Author's Research).

	Xo	Yo	Zo	Scale	kappa	Phi	Omega
Xo	-288922.844	379834.862	2.44E12	-60.696	-0.176	6.98E13	-288922.844
Yo	379834.862	-399950.337	-2.9E12	0.036	-299.43	-8.50E13	379834.862
Zo	2.44E12	-2.97E12	-1.9E20	0.000	0.000	-5.47E19	2.44E12
Scale	-60.696	0.036	0.000	0.180	0.000	0.000	-60.696
kappa	-0.176	-299.432	0.000	0.000	1.505	-0.001	-0.176
Phi	6.98E12	-8.50E13	-5.47E20	0.000	-0.001	-1.564	6.98E12
Omega	-288922.844	379834.862	2.44E12	-60.696	-0.176	6.98E13	-288922.844

Table 6.0: Standard Error of Computation of Non-Iterative "Tilt Photo Generator" Model Solution (Author's Research).

	Xo	Yo	Zo	Scale	kappa	Phi	Omega
Xo	10.77	0.00	13.03	-213.57	126.34	0.03	0.02
Yo	0.00	10.77	42.27	-126.34	-213.57	-0.02	-0.22
Zo	13.03	42.27	279.31	-0.02	0.00	158.66	-108.14
Scale	-213.57	-126.34	-0.02	0.63	0.00	0.00	0.00
kappa	126.34	-213.57	0.00	0.00	0.63	0.00	0.00
Phi	0.03	-0.02	158.66	0.00	0.00	0.75	0.46
Omega	0.02	-0.22	-108.14	0.00	0.00	0.46	6.22

Also, the Standard Error obtained reveal that both models do not provide optimum results for the computation of the Ground Height of points. This could be as a result of the single photograph used. More reliable height values are thus expected when a stereo-pair of images is used. This will be verified in subsequent research works.

6.0 CONCLUSION

A simple least squares approach to solving exterior orientation parameters of a single photograph has been presented. The Standard Error obtained suggest that the direct usage of the "Tilt Photo-Generator" Equation is most efficient rather than an iterative solution with an initial guess obtained from the vertical Photo Generator equation. Besides, the use of single image rather than a stereo pair also reduce the ability of the model to effectively compute the Height value of the Photo Principal Point.

It can also be concluded that the use of Single Photographs for determination of the exterior orientation parameters is not suitable for high order accuracy photogrammetric tasks. Therefore, similar techniques could be employed for an overlapping pair of images as better results are anticipated in such an event.

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APPENDIX

Sub-routine to read in the Unregistered Image into the Matlab environment.

```
%Sample Image Registration Part 1
%read in the base and Unregistered Image
%base = imread('ImReg01.jpg');
unregistered = imread('TalbaEst.jpg');
% Display the Unregistered Image
iptsetpref('ImshowAxesVisible','on')
A = imshow(unregistered)
text(size(unregistered,2),size(unregistered,1)+30,'Image courtesy of Department of Surveying and Geoinformatics','FontSize',7,'HorizontalAlignment','right');
```

All other Algorithms (Source Codes) used for the determination can be requested for from the author by sending a request to his email:

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