Application of Adomian Decomposition Method (ADM) for Solving Mathematical Model of Measles

[1]Somma S. A., [2]Ayegbusi F. D., [3]Gana P. [4]Adama P. W., [5]Abdurrahman N. O. and [6]Eguda F. Y.

[1]Department of Mathematics, Federal University of Technology, Minna, Nigeria
[2]Department of Mathematics, First Technical University, Ibadan, Nigeria
[3]Department of Mathematics & Statistics, Niger State Polytechnic, Zungeru, Nigeria
[4]Department of Mathematics, Federal Polytechnic, Bida, Nigeria
[6]Department of Mathematics, Federal University Dutse, Dutse, Nigeria

*sam.abu@futminna.edu.ng

Abstract

Adomian Decomposition Method (ADM) is a semi-analytical method that give the approximate solution of the linear and non-linear differential equations. In this paper the Adomian Decomposition Method (ADM) was used to solve the mathematical model of measles. The ADM solution was validated with Runge-Kutta built-in in Maple software. The graphical solutions show the decrease and increase in the classes with time. It was revealed from the graphical solution that the ADM is in agreement with Runge-Kutta.

Keywords: Mathematical modeling, Adomian Decomposition Method, numerical solution

1 Introduction

Measles is extremely transmissible infectious disease caused by measles virus [1, 2]. Measles is an airborne disease which spreads easily through the coughs and sneezes of infected people [3]. It may also be spread through direct contact with mouth or nasal secretions [4]. Measles affects about 20 million people a year, [1] primarily in the developing areas of Africa and Asia [3]. While often regarded as a childhood illness, it can affect people of any age [5]. In 2018 there were 142,300 measles related deaths globally. Most of the reported cases were from African and eastern Mediterranean regions. These estimates were a little higher than that of 2017, when 124,000 deaths were reported due to measles infection worldwide [6]. The reported cases of measles in the first quarter of 2019 were three times higher than in the first quarter of 2018. The outbreaks occurred in almost every region of the world, even in countries with high vaccination coverage where it spread among clusters of unvaccinated people [7].

The Adomian Decomposition Method (ADM) was introduced by George Adomian in 1989 [8]. The method has been applied by several researchers to solve various problems [9-12]. Adomian Decomposition Method (ADM) is very useful for solving linear and nonlinear ordinary and partial differential equations, algebraic equations, functional equations, integral differential equations. In [10] they solved system of ordinary differential equations using ADM.

In this paper, ADM was used to solve the Mathematical modeling of Measles. The Mathematical modeling of infectious diseases many a times resulted into system of autonomous differential equations which does not explicitly depend on the independent variable. The solution was validated graphically with Runge-Kutta in built-in in Maple software since it is difficult to have the exact solution of this system of equation. The values used for the numerical result was from [13] they estimated the demographic and the cases of measles of world using the data from World Health Organization (WHO).

2 Materials and Methods

Basics of Adomian Decomposition Method (ADM)

Consider the system of ordinary differential equation:

$$\left.\begin{aligned} \frac{dy_1}{dt} &= N_1(y_1, y_2, \ldots, y_n) \\ \frac{dy_2}{dt} &= N_2(y_1, y_2, \ldots, y_n) \\ &\vdots \\ \frac{dy_n}{dt} &= N_n(y_1, y_2, \ldots, y_n) \end{aligned}\right\} \quad (2.1)$$

where each equation represents the first derivatives of one of the unknowns functions depending on the independent variable and n unknown functions $N_1, N_2, \cdots, N_n$.

Equation (2.1) can be represented by using the ith equation as:

$$Ly_i = N_i(y_1, y_2, \cdots, y_n) \quad i = 1, 2, \cdots, n \quad (2.2)$$

Where L is the linear operator $\frac{d}{dt}$ with the inverse $L^{-1} = \int_0^t (\cdot)dt$ and N is the non-linear functions. Applying the inverse operator on (2.2) gives

$$y_i = y_i(0) + \int_0^t N_i(y_1, y_2, \cdots, y_n)dt \quad i = 1, 2, \cdots, n \quad (2.3)$$

In ADM the solution of (2.3) is consider to as the sum of a series

$$y_i = \sum_{j=0}^{\infty} y_{i,j} \tag{2.4}$$

And the integrand in (2.3), as the sum of the following series:

$$N_i(y_1, y_2, \cdots, y_n) = \sum_{j=0}^{\infty} A_{i,j}(y_{i,0}, y_{i,1}, \cdots, y_{i,j}) \tag{2.5}$$

Where $A_{i,j}(y_{i,0}, y_{i,1}, \cdots, y_{i,j})$ are called Adomian polynomials.
Substituting (2.5) into (2.3) gives

$$\begin{aligned}\sum_{j=0}^{\infty} y_{i,j} &= y_i(0) + \int_0^t \sum_{j=0}^{\infty} A_{i,j}(y_{i,0}, y_{i,1}, \cdots, y_{i,j})dt \\ &= y_i(0) + \sum_{j=0}^{\infty} \int_0^t A_{i,j}(y_{i,0}, y_{i,1}, \cdots, y_{i,j})dt\end{aligned} \tag{2.6}$$

From (2.6) we define

$$\left.\begin{aligned} y_{i,0} &= y_i(0) \\ y_{i,n+1} &= \int_0^t A_{i,n}(y_{i,0}, y_{i,1}, \cdots, y_{i,n})dt \end{aligned}\right\} \tag{2.7}$$

Model Equations

Consider the system of autonomous differential equation:

$$\frac{dM}{dt} = \Lambda - (\theta + \mu)M \tag{2.8}$$

$$\frac{dS}{dt} = \theta M - \frac{\alpha SI}{N} - (\mu + v)S \tag{2.9}$$

$$\frac{dI}{dt} = \frac{\alpha SI}{N} - (\gamma + \mu + \delta)I \tag{2.10}$$

$$\frac{dR}{dt} = \gamma I - \mu R + vS \tag{2.11}$$

With the initial conditions

$$M(0) = M_0,\ S(0) = S_0,\ I(0) = I_0 \text{ and } R(0) = R_0 \tag{2.12}$$

Applying $L^{-1} = \int_0^t (\cdot)dt$ to both side of equation (2.8) - (2.11) gives

$$M(t)=M(0)+\int_0^t \Lambda dt-\int_0^t K_1 M dt \tag{2.13}$$

$$S(t)=S(0)+\int_0^t \theta M dt-\int_0^t \frac{\alpha SI}{N}dt-\int_0^t K_2 S dt \tag{2.14}$$

$$I(t)=I(0)+\int_0^t \frac{\alpha SI}{N}dt-\int_0^t K_3 I dt \tag{2.15}$$

$$R(t)=R(0)+\int_0^t \gamma I dt+\int_0^t v S dt-\int_0^t \mu R dt \tag{2.16}$$

Where,

$$K_1=(\theta+\mu),\ K_2=(v+\mu) \text{ and } K_3=(\gamma+\mu+\delta) \tag{2.17}$$

Using the alternate algorithm for computing the Adomian polynomial as used by [14].

Let,

$$\left.\begin{aligned} &M=y_1,\ S=y_2,\ I=y_3 \text{ and } R=y_4 \\ &SI=\sum_{j=o}^{n} y_{2,j}y_{3,n-j} \end{aligned}\right\} \tag{2.18}$$

Substituting equations (2.12) and (2.18) into (2.13) to (2.16) gives

$$y_{1,n}=M_0+\int_0^t \Lambda dt-K_1\int_0^t y_{1,n}dt \tag{2.19}$$

$$y_{2,n}=S_0+\theta\int_0^t y_{1,n}dt-\frac{\alpha}{N}\int_0^t\left(\sum_{j=o}^{n} y_{2,j}y_{3,n-j}\right)dt-K_2\int_0^t y_{2,n}dt \tag{2.20}$$

$$y_{3,n}=I_0+\frac{\alpha}{N}\int_0^t\left(\sum_{j=o}^{n} y_{2,j}y_{3,n-j}\right)dt-K_3\int_0^t y_{3,n}dt \tag{2.21}$$

$$y_{4,n}=R_0+\gamma\int_0^t y_{3,n}dt+v\int_0^t y_{2,n}dt-\mu\int_0^t y_{4,n}dt \tag{2.22}$$

Equation (2.19) to (2.22) leads to the following scheme:

$$y_{1,0}=M_0+\Lambda t \qquad y_{1,n+1}=-K_1\int_0^t y_{1,n}dt \tag{2.23}$$

$$y_{2,0} = S_0 \qquad y_{2,n+1} = \theta\int_0^t y_{1,n}\,dt - \frac{\alpha}{N}\int_0^t\left(\sum_{j=0}^{n} y_{2,j}\,y_{3,n-j}\right)dt - K_2\int_0^t y_{2,n}\,dt \qquad (2.24)$$

$$y_{3,0} = I_0 \qquad y_{3,n+1} = \frac{\alpha}{N}\int_0^t\left(\sum_{j=0}^{n} y_{2,j}\,y_{3,n-j}\right)dt - K_3\int_0^t y_{3,n}\,dt \qquad (2.25)$$

$$y_{4,0} = R_0 \qquad y_{4,n+1} = \gamma\int_0^t y_{3,n}\,dt + v\int_0^t y_{2,n}\,dt - \mu\int_0^t y_{4,n}\,dt \qquad (2.26)$$

Where $n = 0, 1, 2, 3, \cdots$

$n = 0$

$$\left.\begin{aligned} y_{1,1} &= -K_1\int_0^t y_{1,0}\,dt \\ &= -K_1\left(M_0 t + \Lambda\frac{t^2}{2}\right) \\ y_{1,1} &= -E_1 \end{aligned}\right\} \qquad (2.27)$$

$$\left.\begin{aligned} y_{2,1} &= \theta\int_0^t y_{1,0}\,dt - \frac{\alpha}{N}\int_0^t y_{2,0}y_{3,0}\,dt - K_2\int_0^t y_{2,0}\,dt \\ &= \left(\theta M_0 - \frac{\alpha}{N}S_0 I_0 - K_2 S_0\right)t + \theta\Lambda\frac{t^2}{2} \\ y_{2,1} &= E_2 t + \theta\Lambda\frac{t^2}{2} \end{aligned}\right\} \qquad (2.28)$$

$$\left.\begin{aligned} y_{3,1} &= \frac{\alpha}{N}\int_0^t y_{2,0}y_{3,0}\,dt - K_3\int_0^t y_{3,0}\,dt \\ &= \left(\frac{\alpha}{N}S_0 I_0 - K_3 I_0\right)t \\ y_{3,1} &= E_3 t \end{aligned}\right\} \qquad (2.29)$$

$$\left.\begin{aligned} y_{4,1} &= \gamma\int_0^t y_{3,0}\,dt + v\int_0^t y_{2,0}\,dt - \mu\int_0^t y_{4,0}\,dt \\ &= (\gamma I_0 + vS_0 - \mu R_0)t \\ y_{4,1} &= E_4 t \end{aligned}\right\} \qquad (2.30)$$

where,

$$\left.\begin{aligned} E_1 &= K_1\left(M_0 t + \Lambda\frac{t^2}{2}\right),\ E_2 = \left(\theta M_0 - \frac{\alpha}{N}S_0 I_0 - K_2 S_0\right), \\ E_3 &= \left(\frac{\alpha}{N}S_0 I_0 - K_3 I_0\right),\ E_4 = (\gamma I_0 + v S_0 - \mu R_0) \end{aligned}\right\} \quad (2.31)$$

$n = 1$

$$\left.\begin{aligned} y_{1,2} &= -K_1\int_0^t y_{1,1}dt \\ &= K_1^2 M_0\frac{t^2}{2} + K_1^2\Lambda\frac{t^3}{6} \\ y_{1,2} &= F_{121}\frac{t^2}{2} + F_{122}\frac{t^3}{6} \end{aligned}\right\} \quad (1.32)$$

$$\left.\begin{aligned} y_{2,2} &= \theta\int_0^t y_{1,1}dt - \frac{\alpha}{N}\int_0^t (y_{2,0}y_{3,1} + y_{2,1}y_{3,0})dt - K_2\int_0^t y_{2,1}dt \\ &= -\left[\theta M_0 K_1 + \frac{\alpha}{N}(S_0 E_3 + I_0 E_2) + K_2 E_2\right]\frac{t^2}{2} - \Lambda\left(\theta K_1 + \frac{\alpha\theta I_0}{N} + \theta K_2\right)\frac{t^3}{6} \\ y_{2,2} &= -\left(F_1\frac{t^2}{2} + F_2\frac{t^3}{6}\right) \end{aligned}\right\}$$

(2.33)

$$\left.\begin{aligned} y_{3,2} &= \frac{\alpha}{N}\int_0^t (y_{2,0}y_{3,1} + y_{2,1}y_{3,0})dt - K_3\int_0^t y_{3,1}dt \\ &= \left[\frac{\alpha}{N}(S_0 E_3 + I_0 E_2) - K_3 E_3\right]\frac{t^2}{2} + \frac{\alpha\theta I_0\Lambda}{N}\frac{t^3}{6} \\ y_{3,2} &= F_3\frac{t}{2} + F_{32}\frac{t^3}{6} \end{aligned}\right\}$$

(2.34)

$$\left.\begin{aligned} y_{4,2} &= \gamma\int_0^t y_{3,1}dt + v\int_0^t y_{2,1}dt - \mu\int_0^t y_{4,1}dt \\ &= [\gamma E_3 + v E_2 - \mu E_4]\frac{t^2}{2} + v\theta\Lambda\frac{t^3}{6} \\ y_{4,2} &= F_4\frac{t^2}{2} + F_{42}\frac{t^3}{6} \end{aligned}\right\}$$

(2.35)

where,

$$
\left.
\begin{aligned}
&F_{121} = K_1^{\,2} M_0,\ F_{122} = K_1^{\,2}\Lambda,\ F_1 = \left[\theta K_1 M_0 + \frac{\alpha}{\Lambda}(S_0 E_3 + I_0 E_2) + K_2 E_2\right], \\
&F_2 = \Lambda\left(\theta K_1 + \frac{\alpha\theta I_0}{N} + \theta K_2\right),\ F_3 = \left[\frac{\alpha}{N}(S_0 E_3 + I_0 E_2) - K_3 E_3\right], \\
&F_{32} = \frac{\alpha\theta I_0 \Lambda}{N},\ F_4 = [\gamma E_3 + \nu E_2 - \mu E_4],\ F_{12} = \theta\nu\Lambda
\end{aligned}
\right\} \quad (2.36)
$$

$n = 2$

$$
\left.
\begin{aligned}
y_{1,3} &= -K_1 \int_0^t y_{1,2}\,dt \\
&= -K_1^{\,3} M_0 \frac{t^3}{6} - K_1^{\,3}\Lambda \frac{t^4}{24} \\
y_{1,3} &= -\left(G_{131}\frac{t^3}{6} + G_{132}\frac{t^4}{24}\right)
\end{aligned}
\right\} \quad (2.37)
$$

$$
\left.
\begin{aligned}
y_{2,3} &= \theta\int_0^t y_{1,2}\,dt - \frac{\alpha}{N}\int_0^t (y_{2,0}y_{3,2} + y_{2,1}y_{3,1} + y_{2,2}y_{3,0})\,dt - K_2\int_0^t y_{2,2}\,dt \\
&= \left[\theta F_{121} - \frac{\alpha}{N}(S_0 F_3 + 2E_2E_3 - I_0 F_1) + F_1 K_2\right]\frac{t^3}{6} + \left(\theta F_{122} - \frac{\alpha}{N}(S_0 F_{32} - I_0 F_2 + 3\Lambda\theta E_3) + F_2 K_2\right)\frac{t^4}{24} \\
y_{2,3} &= G_1\frac{t^3}{6} + G_2\frac{t^4}{24}
\end{aligned}
\right\} \quad (2.38)
$$

$$
\left.
\begin{aligned}
y_{3,3} &= \frac{\alpha}{N}\int_0^t (y_{2,0}y_{3,2} + y_{2,1}y_{3,1} + y_{2,2}y_{3,0})\,dt - K_3\int_0^t y_{3,2}\,dt \\
&= \left[\frac{\alpha}{N}(S_0 F_3 + 2E_2E_3 - I_0 F_1) - F_3 K_3\right]\frac{t^3}{6} + \left[\frac{\alpha}{N}(S_0 F_{32} + 3\Lambda\theta E_3 - I_0 F_2) - K_3 F_{32}\right]\frac{t^4}{24} \\
y_{3,3} &= G_3\frac{t^3}{6} + G_4\frac{t^4}{24}
\end{aligned}
\right\} \quad (2.39)
$$

$$\left.\begin{aligned} y_{4,3} &= \gamma\int_0^t y_{3,2}dt + v\int_0^t y_{2,2}dt - \mu\int_0^t y_{4,2}dt \\ &= (\gamma F_3 - vF_1 - \mu F_4)\frac{t^3}{6} + (\gamma F_{32} - vF_2 - \mu F_{42})\frac{t^4}{24} \\ y_{4,3} &= G_5\frac{t^3}{6} + G_6\frac{t^4}{24} \end{aligned}\right\} \quad (2.40)$$

where,

$$\left.\begin{aligned} &G_{131} = K_1^3 M_0,\ G_{132} = K_1^3\Lambda,\ G_1 = \left[\theta F_{121} - \frac{\alpha}{N}(S_0F_3 + 2E_2E_3 - I_0F_1) + F_1K_2\right], \\ &G_2 = \left(\theta F_{122} - \frac{\alpha}{N}(S_0F_{32} - I_0F_2 + 3\Lambda\theta E_3) + F_2K_2\right),\ G_3 = \left[\frac{\alpha}{N}(S_0F_3 + 2E_2E_3 - I_0F_1) - F_3K_3\right] \\ &G_4 = \left[\frac{\alpha}{N}(S_0F_{32} + 3\Lambda\theta E_3 - I_0F_2) - K_3F_{32}\right],\ G_5 = (\gamma F_3 - vF_1 - \mu F_4),\ G_6 = (\gamma F_{32} - vF_2 - \mu F_{42}) \end{aligned}\right\} \quad (2.41)$$

$n = 3$

$$\left.\begin{aligned} y_{1,4} &= -K_1\int_0^t y_{1,3}dt \\ &= K_1^4 M_0\frac{t^4}{24} + K_1^4\Lambda\frac{t^5}{120} \\ y_{1,4} &= H_{141}\frac{t^4}{24} + H_{142}\frac{t^5}{120} \end{aligned}\right\} \quad (2.42)$$

$$\left.\begin{aligned} y_{2,4} &= \theta\int_0^t y_{1,3}dt - \frac{\alpha}{N}\int_0^t (y_{2,0}y_{3,3} + y_{2,1}y_{3,2} + y_{2,2}y_{3,1} + y_{2,3}y_{3,0})dt - K_2\int_0^t y_{2,3}dt \\ &= \left[-\theta G_{131} - \frac{\alpha}{N}\begin{pmatrix} G_3S_0 + 3E_2F_3 \\ -3E_3F_1 + G_1I_0 \end{pmatrix} - K_2G_1\right]\frac{t^4}{24} + \left[\begin{matrix} -\theta G_{132} - \frac{\alpha}{N}\begin{pmatrix} S_0G_4 + 4E_2F_{32} \\ +6\Lambda\theta F_3 - 4E_3F_2 + G_2I_0 \end{pmatrix} \\ -K_2G_2 \end{matrix}\right]\frac{t^5}{120} \\ &+ \left(-\frac{\alpha}{N}\Lambda\theta F_{32}\right)\frac{t^6}{72} \\ y_{2,4} &= H_1\frac{t^4}{24} + H_2\frac{t^5}{120} + H_3\frac{t^6}{72} \end{aligned}\right\} \quad (2.43)$$

where,

$$\left.\begin{aligned} &H_{141} = K_1^{\ 4} M_0,\ H_{142} = K_1^{\ 4}\Lambda \\ &H_1 = \left[-\theta G_{131} - \frac{\alpha}{N}\left(G_3 S_0 + 3E_2 F_3 - 3E_3 F_1 + G_1 I_0\right) - K_2 G_1\right], \\ &H_2 = \left[-\theta G_{132} - \frac{\alpha}{N}\left(S_0 G_4 + 4E_2 F_{32} + 6\Lambda\theta F_3 - 4E_3 F_2 + G_2 I_0\right) - K_2 G_2\right], \\ &H_3 = -\frac{\alpha}{N}\Lambda\theta F_{32} \end{aligned}\right\} \quad (2.44)$$

$$\left.\begin{aligned} y_{3,4} &= \frac{\alpha}{N}\int_0^t \left(y_{2,0}y_{3,3} + y_{2,1}y_{3,2} + y_{2,2}y_{3,1} + y_{2,3}y_{3,0}\right)dt - K_3\int_0^t y_{3,3}\,dt \\ &= \left[\frac{\alpha}{N}\begin{pmatrix} G_3 S_0 + 3E_2 F_3 \\ -3E_3 F_1 + G_1 I_0 \end{pmatrix} - K_3 G_2\right]\frac{t^4}{24} + \left[\begin{matrix}\frac{\alpha}{N}\begin{pmatrix} S_0 G_4 + 4E_2 F_{32} \\ +6\Lambda\theta F_3 - 4E_3 F_2 + G_2 I_0 \end{pmatrix} \\ -K_3 G_4 \end{matrix}\right]\frac{t^5}{120} + \frac{\alpha}{N}\Lambda\theta F_{32}\frac{t^6}{72} \\ y_{3,4} &= H_4\frac{t^4}{24} + H_5\frac{t^5}{120} + H_6\frac{t^6}{72} \end{aligned}\right\} \quad (2.45)$$

where,

$$\left.\begin{aligned} &H_4 = \left[\frac{\alpha}{N}\left(G_3 S_0 + 3E_2 F_3 - 3E_3 F_1 + G_1 I_0\right) - K_3 G_3\right], \\ &H_5 = \left[\frac{\alpha}{N}\left(S_0 G_4 + 4E_2 F_{32} + 6\Lambda\theta F_3 - 4E_3 F_2 + G_2 I_0\right) - K_3 G_4\right],\ H_6 = \frac{\alpha}{N}\Lambda\theta F_{32} \end{aligned}\right\} \quad (2.46)$$

$$\left.\begin{aligned} y_{4,4} &= \gamma\int_0^t y_{3,3}\,dt + v\int_0^t y_{2,3}\,dt - \mu\int_0^t y_{4,3}\,dt \\ &= \left(\gamma G_3 + vG_1 - \mu G_5\right)\frac{t^4}{24} + \left(\gamma G_4 + vG_2 - \mu G_6\right)\frac{t^5}{120} \\ y_{4,4} &= H_7\frac{t^4}{24} + H_8\frac{t^5}{120} \end{aligned}\right\} \quad (2.47)$$

where,

$$H_7 =, \left(\gamma G_3 + vG_1 - \mu G_5\right)\ H_8 = \left(\gamma G_4 + vG_2 - \mu G_6\right) \quad (2.48)$$

Therefore, the solutions of the model equations are:

$$
\left.\begin{aligned}
M(t) &= M_0 + \Lambda t + y_{1,1} + y_{1,2} + y_{1,3} + y_{1,4} + \cdots \\
S(t) &= S_0 + y_{2,1} + y_{2,2} + y_{2,3} + y_{2,4} + \cdots \\
I(t) &= I_0 + y_{3,1} + y_{3,2} + y_{3,3} + y_{3,4} + \cdots \\
R(t) &= R_0 + y_{4,1} + y_{4,2} + y_{4,3} + y_{4,4} + \cdots
\end{aligned}\right\} \tag{2.49}
$$

Substituting (2.27) to (2.47) into (2.49) gives

$$
\left.\begin{aligned}
M(t) &= M_0 + M_1 t - M_2 \frac{t^2}{2} + M_3 \frac{t^3}{6} + M_4 \frac{t^4}{24} + M_5 \frac{t^5}{120} + \cdots \\
S(t) &= S_0 + S_1 t - S_2 \frac{t^2}{2} + S_3 \frac{t^3}{6} + S_4 \frac{t^4}{24} + S_5 \frac{t^5}{120} + S_6 \frac{t^6}{72} + \cdots \\
I(t) &= I_0 + I_1 t + I_2 \frac{t^2}{2} + I_3 \frac{t^3}{6} + I_4 \frac{t^4}{24} + I_5 \frac{t^5}{120} + I_6 \frac{t^6}{72} + \cdots \\
R(t) &= R_0 + R_1 t + R_2 \frac{t^2}{2} + R_3 \frac{t^3}{6} + R_4 \frac{t^4}{24} + R_5 \frac{t^5}{120} + \cdots
\end{aligned}\right\} \tag{2.50}
$$

Where,

$$
\begin{aligned}
&M_1 = (\Lambda - K_1 M_0),\ M_2 = (K_1 \Lambda - F_{121}),\ M_3 = (F_{122} - G_{111}),\ M_4 = (H_{14} - G_{112}),\ M_5 = H, \\
&S_1 = E_2,\ S_2 = (\theta\Lambda - F_1),\ S_3 = (G_1 - F_2),\ S_4 = (G_2 + H_1),\ S_5 = H_2,\ S_6 = H \\
&I_1 = E_3,\ I_2 = F_3,\ I_3 = (F_{32} + G_3),\ I_4 = (G_4 + H_4),\ I_5 = H_5,\ I_6 = H, \\
&R_1 = E_4,\ R_2 = F_4,\ R_3 = (F_{42} + G_5),\ R_4 = (G_6 + H_7),\ R_5 = H,
\end{aligned} \tag{2.51}
$$

3 Results and Discussion

Table 3.1 is the estimated world demographic and cases of measles from World Health Organization. Figure 3.1 to 3.4 are the graphical solutions of Runge-Kutta Method (RKM) and Adomian Decomposition Method (ADM) of equation (2.51)

Table 3.1: Values for Parameters used for Sensitivity Analysis

Variables	Description	Values per year
$M(0)$	Maternally-Derived-Immunity	82,010,000
$S(0)$	Susceptible	7,099,464,364
$I(0)$	Infected	254,918
$R(0)$	Recovered/Immune	118,270,718

	Total Population	7,300,000,000
[illegible]	Recruitment rate	139,000,000
[illegible]	Contact Rate	0.9
[illegible]	Death Rate due to Disease	0.53
[illegible]	Recovery Rate	0.47
	Natural Death Rate	0.008
μ	Vaccination Rate	0.85
θ	Immunity Rate	0.61

Source: [13]

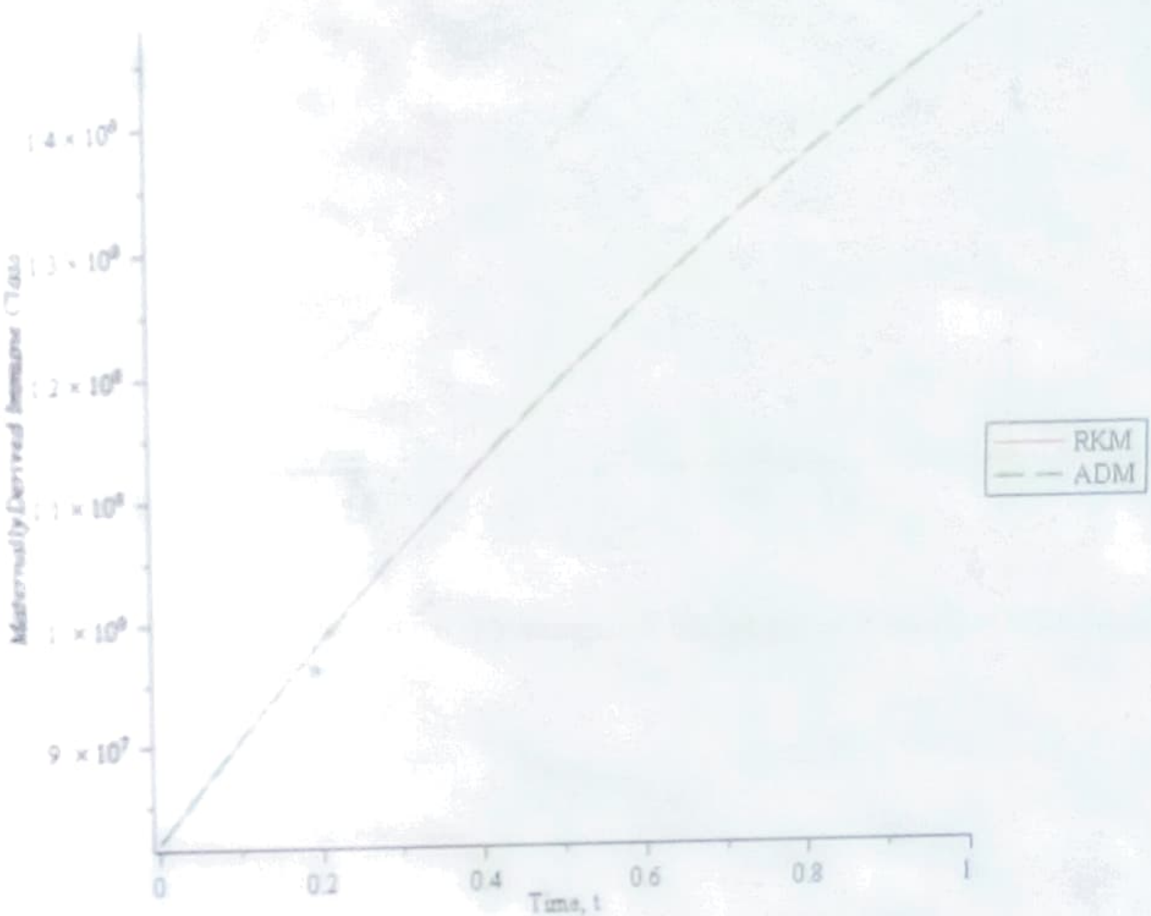

Figure 3.1: Graphical Solution for the Maternally Derived Immune Class

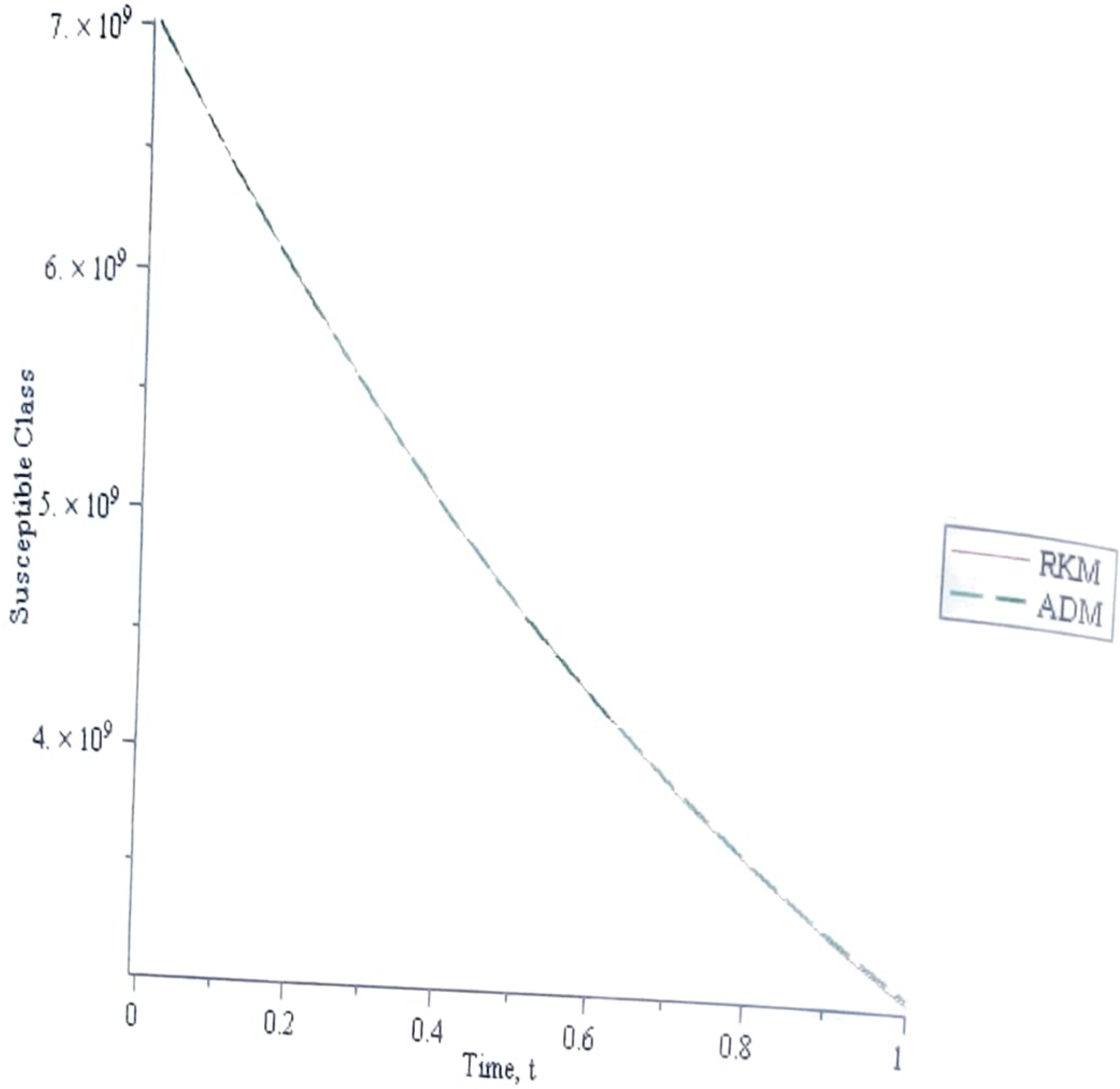

Figure 3.2: Graphical Solution for the Susceptible Class

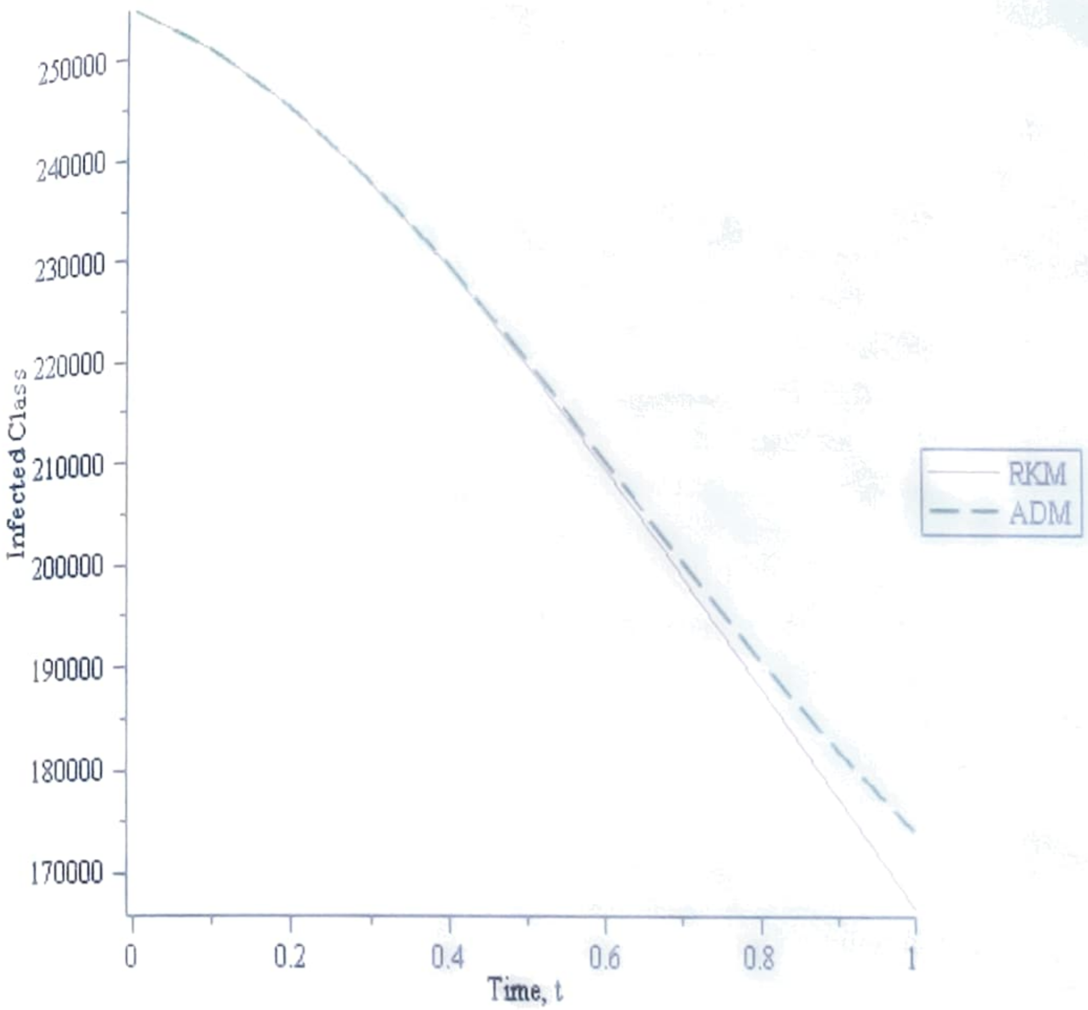

Figure 3.3: Graphical Solution for the Infected Class

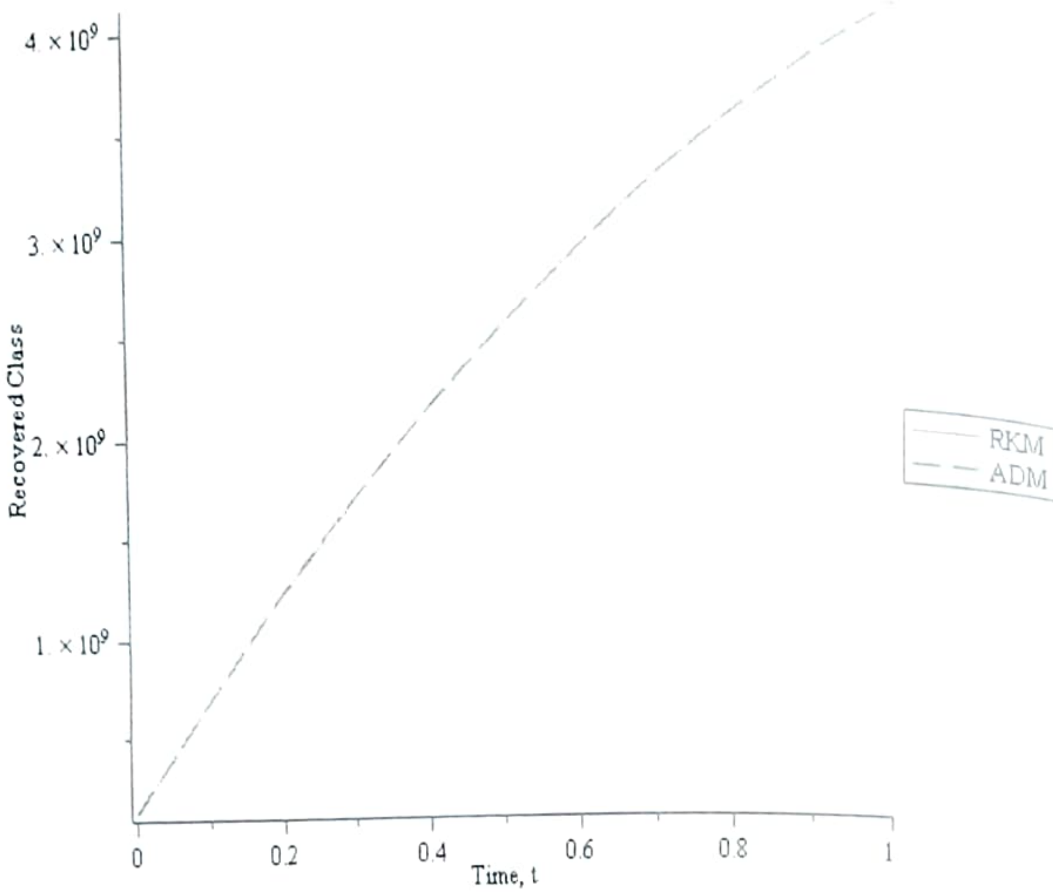

Figure 3.4: Graphical Solution for the Recovered Class

Figure 3.1 is the graphical solutions of Maternally-Derived-Immune class, it shows that the number of Maternally-Immune individual increase a little with time. This is because the new born are going into the class and they leave after nine month. It is observed from Figure 3.2 which is the graphical solution of Susceptible class that, the number decreases with time as a result of high vaccination rate. It is assumed that the susceptible individuals that are vaccinated move to Recovered class. In Figure 3.3, the number of infected decreases because as more susceptible are vaccinated the less the number of infected. Figure 3.4, the solution of Recovered class, it shows that the number increases with time. The increase is as a result of the vaccinated susceptible and treated infected individuals that move to the class.

4 Conclusion

The solution of ADM was validated with Runge-Kutta built-in in Maple software. It is observed from the result that they are in agreement. The increase and decrease in the classes are as result of high vaccination and recovery rate. Semi-analytical solutions give better understanding of the dynamics of infectious diseases.

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